Asset Pricing, the Fama-French Factor Model and the Implications of Quantile Regression Analysis

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ABSTRACT

In traditional tests of asset pricing theory Ordinary Least Squares (OLS) regression methods are used in empirical tests of factor models, which implies a focus on the means of the distributions of covariates. The work of Koenker and Basset (1982) and Koenker (2005) provides an alternative via Quantile regression featuring inference about conditional quantile functions. This study empirically examines the behaviour of the three risk factors from Fama-French Three Factor model of stock returns, beyond the mean of the distribution, by using quantile regressions and a US data set. The study not only shows that the factor models does not necessarily follow a linear relationship but also shows that the traditional method of OLS becomes less effective when it comes to analysing the extremes within a distribution, which is often of key interest to investors and risk managers.

Keywords: Factor models; OLS; Quantile regression

1. INTRODUCTION

Traditionally regression based factor analysis is extensively used in quantitative finance to analyse the performance of the factors in different factor models. These factor models assume that, the expected return is linearly dependent on the risk factors, and hence Ordinary Least Square (OLS), is widely used to model the risk distribution for these models. When it comes to risk assessment, the parts of the return distributions in which the investor and risk managers are often interested, such as extreme outcomes in the tails, which go beyond the mean values, are not well analysed by means of OLS.

Quantile regression promises to be a more effective tool than OLS, when it comes to analysing the extremes of a distribution. The behaviour of the tails of a distribution is more efficiently described by quantile regression. In this paper, we analyse the expected return distribution of 30 stocks of Dow Jones Industrial average, obtained from the Fama-French three factor model, using Quantile Regression techniques.

The paper is divided into six sections, following this introductory section we briefly review the Fama-French three factor model, quantile regression is introduced in sections three, the data and research method follows in section four, the results and presented in section five, and a brief conclusion is provided in section six.

2. THE FAMA-FRENCH THREE FACTOR MODEL

Volatility is widely accepted measure of risk, which is the amount an asset's return varies through successive time periods. Volatility is most commonly quoted in terms of standard deviation of returns. There is a greater risk involved for asset whose return fluctuates more dramatically than the other. The familiar beta from the CAPM equation is a widely accepted measure of systematic risk. whilst unsystematic risk is captured by the error term of the OLS application of CAPM. Beta is a measure of the risk contribution of an individual security to a well diversified portfolio as measured below;

$$\beta_A = \frac{cov(r_A, r_M)}{{\sigma_M}^2} \tag{1}$$

where

r_A is the return of the asset

r_M is the return of the market

${\sigma_M}^2$ is the variance of the return of the market, and

cov(r_A, r_M) is covariance between the return of the market and the return of the asset.

Jack Treynor (1961, 1962), William Sharpe (1964), John Lintner (1965) and Jan Mossin (1966) independently, proposed Capital Asset Pricing Theory, (CAPM), to quantify the relationship between beta of an asset and its corresponding return. CAPM stands on a broad assumption that, that only one risk factor is common to a broad-based market portfolio, which is beta. Modelling of CAPM using OLS assumes that the relationship between return and beta is linear, as given in equation (2).

$$r_A = r_f + \beta_A(r_M - r_F) + \alpha + e \qquad (2)$$

where

r_A is the return of the asset

r_M is the return of the market

r_f is the risk free rate of return

α is the intercept of regression

e is the standard error of regression

Fama and French (1992,1993) extended the basic CAPM to include size and book-to-market as explanatory factors in explaining the cross-section of stock returns. SMB, which stands for Small Minus Big, is designed to measure the additional return investors have historically received from investing in stocks of companies with relatively small market capitalization. This additional return is often referred to as the "size premium." HML, which is short for High Minus Low, has been constructed to measure the "value premium" provided to investors for investing in companies with high book-to-market values (essentially, the value placed on the company by accountants as a ratio relative to the value the public markets placed on the company, commonly expressed as B/M).

SMB is a measure of "size risk", and reflects the view that, small companies logically, should be expected to be more sensitive to many risk factors as a result of their relatively undiversified nature and their reduced ability to absorb negative financial events. On the other hand, the HML factor suggests higher risk exposure for typical "value" stocks (high B/M) versus "growth" stocks (low B/M). This makes sense intuitively because companies need to reach a minimum size in order to execute an Initial Public Offering; and if we later observe them in the bucket of high B/M, this is usually an indication that their public market value has plummeted because of hard times or doubt regarding future earnings.On the other hand, the HML factor suggests higher risk exposure for typical "value" stocks (high B/M) versus "growth" stocks (low B/M). This makes sense intuitively because companies need to reach a minimum size in order to execute an Initial Public Offering; and if we later observe them in the bucket of high B/M, this is usually an indication that their public market value has plummeted because of hard times or doubt regarding future earnings.

The three factor Fama-French model is written as;

$$r_A = r_f + \beta_A(r_M - r_F) + s_A SMB + h_A HML + \alpha + e \qquad (3)$$

where $S_A$and H_A capture the security's sensitivity to these two additional factors.

2.1 DOUBTS ABOUT THE MODEL

Black (1993) suggested that the Fama- French results might be the effect of data mining. Kothari, Shanken and Sloan (1995) suggest that the use of annual returns provides stronger evidence in favour of the influence of beta. Levhari and Levy (1977) show that beta coefficients estimated with monthly returns are not the same as betas estimated with annual returns. There is an abundance of evidence that stock returns distributions have fat tails. Knez and Ready (1997) undertook tests of the model after removing extreme observations from their data sets using a least trimmed squares technique (LTS). They trimmed one percent of the extreme returns in their monthly data and found that this greatly reduced the size effect. Horowitz et al (2000) suggest that the size effect is not robust across different sample periods and argue that it may have disappeared since 1982. In this paper we follow a lead first suggested by Chan and Lakonishok (1992) and apply robust methods to explore the efficacy of the three factor model using quantile regressions.

3. QUANTILE REGRESSION

Linear regression represents the dependent variable, as a linear function of one or more independent variable, subject to a random 'disturbance' or 'error'. It estimates the mean value of the dependent variable for given levels of the independent variables. For this type of regression, where we want to understand the central tendency in a dataset, OLS is an effective method. OLS loses its effectiveness when we try to go beyond the median value or towards the extremes of a data set.

Quantile regression as introduced in Koenker and Bassett (1978) is an extension of classical least squares estimation of conditional mean models to the estimation of an ensemble of models for conditional quantile functions. The central special case is the median regression estimator that minimizes a sum of absolute errors. The remaining conditional quantile functions are estimated by minimizing an asymmetrically weighted sum of absolute errors. Taken together the ensemble of estimated conditional quantile functions offers a much more complete view of the effect of covariates on the location, scale and shape of the distribution of the response variable.

In linear regression, the regression coefficient represents the change in the response variable produced by a one unit change in the predictor variable associated with that coefficient. The quantile regression parameter estimates the change in a specified quantile of the response variable produced by a one unit change in the predictor variable.

The quantiles, or percentiles, or occasionally fractiles, refer to the general case of dividing a dataset into parts. Quantile regression seeks to extend these ideas to the estimation of conditional quantile functions - models in which quantiles of the conditional distribution of the response variable are expressed as functions of observed covariates.

In quantile regression, the median estimator minimizes the symmetrically weighted sum of absolute errors (where the weight is equal to 0.5) to estimate the conditional median function, other conditional quantile functions are estimated by minimizing an asymmetrically weighted sum of absolute errors, where the weights are functions of the quantile of interest. This makes quantile regression robust to the presence of outliers.

We can define the quantiles through a simple alternative expedient as an optimization problem. Just as we can define the sample mean as the solution to the problem of minimizing a sum of squared residuals, we can define the median as the solution to the problem of minimizing a sum of absolute residuals. The symmetry of the piecewise linear absolute value function implies that the minimization of the sum of absolute residuals must equate the number of positive and negative residuals, thus assuring that there are the same number of observations above and below the median.

The other quantile values can be obtained by minimizing a sum of asymmetrically weighted absolute residuals, (giving different weights to positive and negative residuals). Solving

$$\min_{\xi \epsilon \mathcal{R}} \sum \rho_\tau (y_i - \xi) \qquad (4)$$

where $\rho_\tau(\cdot)$ is the tilted absolute value function as shown in Figure 1, this gives the τth sample quantile with its solution. To see that this problem yields the sample quantiles as its solutions, it is only necessary to compute the directional derivative of the objective function with respect to ξ, taken from the left and from the right.

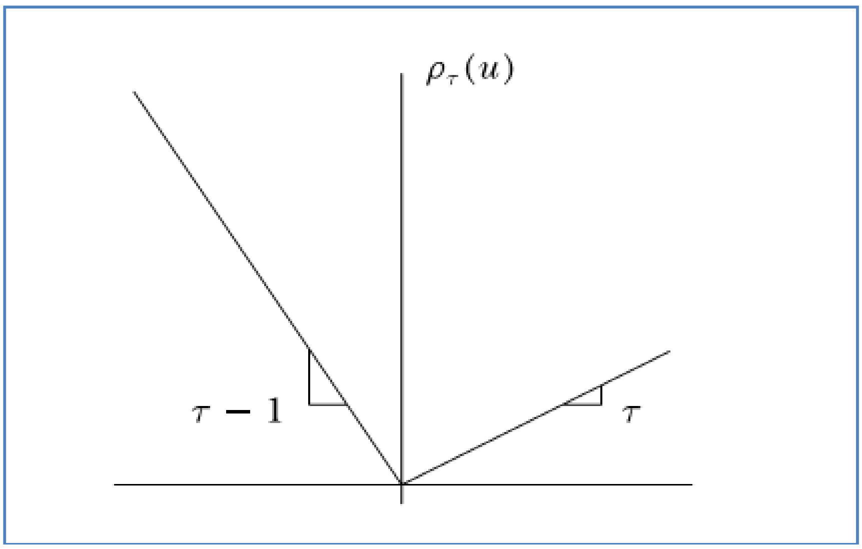

Figure 1: Quantile Regression ρ Function

After defining the unconditional quantiles as an optimization problem, it is easy to define conditional quantiles in an analogous fashion. Least squares regression offers a model for how to proceed. If, we have a random sample,$\{y_1, y_2, \dots, y_n\}$, we solve

$$\min_{\mu \epsilon \mathcal{R}} \sum_{i=1}^{n} (y_i - \mu)^2 \qquad (5)$$

we obtain the sample mean, an estimate of the unconditional population mean, EY. If we now replace the scalar μ by a parametric function $\mu(x, \beta)$ and solve

$$\min_{\mu \epsilon \mathcal{R}^p} \sum_{i=1}^{n} (y_i - \mu(x_i, \beta))^2 \qquad (6)$$

we obtain an estimate of the conditional expectation function $E(Y|x)$.

We proceed exactly the same way in quantile regression. To obtain an estimate of the conditional median function, we simply replace the scalar ξ in the first equation by the parametric function $\xi(x_t, \beta)$ and set τ to $\frac{1}{2}$. Variants of this idea were proposed in the mid-eighteenth century by Boscovich and subsequently investigated by Laplace and Edgeworth, among others. To obtain estimates of the other conditional quantile functions, we replace absolute values by $\rho_\tau(\cdot)$ and solve

$$\min_{\xi \epsilon \mathcal{R}^p} \sum \rho_\tau (y_i - \xi(x_i, \beta)) \qquad (7)$$

The resulting minimization problem, when $\xi(x, \beta)$ is formulated as a linear function of parameters, can be solved very efficiently by linear programming methods.

This technique has been used widely in the past decade in many areas of applied econometrics; applications include investigations of wage structure (Buchinsky and Leslie 1997), earnings mobility (Eide and Showalter 1999; Buchinsky and Hahn 1998), and educational attainment (Eide and Showalter 1998). Financial applications include Engle and Manganelli (1999) and Morillo (2000) to the problems of Value at Risk and option pricing respectively. Barnes, Hughes (2002), applied quantile regression to study CAPM, in their work on cross section of stock market returns.

4. DATA & METHODOLOGY

The study uses daily prices of the 30 Dow Jones Industrial Average Stocks, for a period from January 2002-May 2009, along with the Fama-French factors for the same period, obtained from French's website to calculate the Fama-French coefficients. (http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html#International) Table 1, gives the 30 stocks traded at Dow Jones Industrial Average and used in this study.

Table 1 : Dow Jones Industrial 30 Stocks used in the study.

3M	EI DU PONT DE NEMOURS	KRAFT FOODS
ALCOA	EXXON MOBILE	MCDONALDS
AMERICAN EXPRESS	GENERAL ELECTRIC	MERCK & CO.
AT&T	GENERAL MOTORS	MICROSOFT
BANK OF AMERICA	HEWLETT-PACKARD	PFIZER
BOEING	HOME DEPOT	PROCTER&GAMBLE
CATERPILLAR	INTEL	UNITED TECHNOLOGIES
CHEVRON	INTERNATIONAL BUS.MCHS.	VERIZON COMMUNICATIONS
CITIGROUP	JOHNSON & JOHNSON	WAL MART STORES
COCA COLA	JP MORGAN CHASE & CO.	WALT DISNEY

The approach here is to study the behaviour of the return distribution along the quantiles, by using quantile regression. The coefficients for all the three factors of the model are calculated both by means of OLS and quantile regressions. While OLS calculates the coefficients along the median (0.50), quantile regression calculates the values for .05, .25, .50, .75 and .95 quantiles, at 95 percentile confidence levels. We also plot the fitted values using OLS and the two extreme quantile values to examine the behaviour of fitted and actual values. The sequence of coefficients along the quantiles is also plotted to show the non-linear relationship between the factors and the return. (Both open source statistical software, GRETL, and STAT is used to calculate the Fama-French Coefficients for the OLS and quantile regression analyses.)

5. RESULTS: QUANTILE ANALYSIS OF FAMA-FRENCH FACTORS

We have been emphasizing of the fact that, when it comes to risk assessment, the tail distributions become more interesting for an investor or risk manager. Here we present the results of an analysis of the three factors, based both on their coefficients as obtained from OLS and quantile regressions to examine whether OLS is able to capture the extreme tail distributions and to explore whether the two techniques provided different insights.
Table 2, Table 3 and Table 4, provide the Fama-French risk coefficients obtained for the stocks for the year 2006, using both OLS and quantile regressions at the 0.05 and 95 quantiles respectively.

Figure 2, Figure 3, Figure 4, respectively provide an example of the values of the individual coefficients beta , s , h, across different quantiles plotted against the values obtained from OLS. The plots clearly show that, when it comes to boundary values in a distribution the OLS method becomes inefficient. The plots show that the return of a security is not linearly dependent on these factors around the whole distribution. The coefficients are calculated within a 95% confidence band, and within this confidence level, OLS is unable to capture the distribution of historical returns for tail distributions.

In Figure 2 which depicts the relationship between the market factor beta and Alcoa, the slope of the relationship changes across the quantiles moving from positive to negative; the expected positive relationship between beta and return only holds across the first few quantiles for Alcoa in this sample. Similarly, the coefficient on the size factor *s* is insignificant and constant in the lower quantiles, but then becomes significant and positive in the higher quantiles. Finally, the coefficient on book to market, whilst initially having a negative slope, moves to a neutral constant relationship and then becomes negative again as we move up the quantiles.

Table 2 : Fama-French Risk Coefficients from OLS

Stocks	$\boldsymbol{\beta}$	**t-ratio**	***s***	**t-ratio**	***h***	**t-ratio**
3M	0.861433	6.784	-0.3064	-1.662	-0.43424	-1.556
ALCOA	1.29545	7.604	0.535942	2.167	1.4032	3.749
American EX	1.05774	11.5	-0.39726	-2.977	-0.56408	-2.792
AT&T	0.907788	7.682	-0.36426	-2.124	0.203955	0.7856
BOA	0.980515	12.89	-0.5335	-4.83	0.171075	1.023
Boeing	1.12187	7.422	-0.17483	-0.7969	-0.41978	-1.264
Caterpillar	1.27797	6.844	0.476337	1.757	0.178799	0.4358
Chevron	1.31899	10.72	-0.30614	-1.715	2.31344	8.561
Citi Grp	1.07535	12.49	-0.62878	-5.031	-0.20935	-1.107
Coca Cola	0.727359	10.7	-0.4558	-4.621	-0.2842	-1.903
Ei Du PONT	0.954334	9.135	-0.17851	-1.177	-0.15368	-0.6695
Exxon	1.35761	12.08	-0.40948	-2.51	1.60937	6.518
General Electric	0.828127	10.78	-0.56913	-5.102	-0.78315	-4.638
GM	0.943521	3.034	0.23487	0.5203	0.613318	0.8976
HP	1.16367	6.681	-0.37682	-1.491	-1.22986	-3.214
Home Depot	0.99401	7.532	-0.29788	-1.555	-1.0519	-3.628
Intel	1.36821	8.115	-0.49473	-2.022	-1.92767	-5.204
IBM	0.834319	9.799	-0.47419	-3.837	-1.04999	-5.613
J&J	0.626409	7.95	-0.81103	-7.091	-0.7378	-4.262
JP MORGAN	1.35867	15.08	-0.45604	-3.487	0.082624	0.4173
Kraft Food	0.452821	3.575	-0.27933	-1.519	-0.11306	-0.4062
Mac Donalds	0.762379	6.202	-0.07155	-0.401	-0.25684	-0.9511
Merk & Co	0.988362	7.433	-0.62663	-3.247	-0.40021	-1.37
Microsoft	0.949205	6.555	-0.883	-4.201	-1.39182	-4.375
Pfizer	1.02723	6.895	-0.76543	-3.539	-0.42427	-1.296
P&G	0.612559	6.61	-0.37983	-2.824	-0.56196	-2.76
United Tech	0.863363	7.604	0.264874	1.607	0.037011	0.1484
Verizon	0.988217	9.306	-0.42441	-2.754	-0.37263	-1.597
Wal Mart	0.770144	6.793	-0.26923	-1.636	-0.91555	-3.675
Walt Disney	0.856893	6.422	-0.25322	-1.307	-0.39954	-1.363

Table 3: Fama-French Risk Coefficients from Quantile Regression (0.05)

Stocks	β	t-ratio	s	t-ratio	h	t-ratio
3M	0.87204	2.26051	-0.5269	-0.94096	-0.57551	-0.67901
ALCOA	1.18791	2.85562	0.588687	0.97493	2.36129	2.58358
American EX	0.864705	5.96864	-0.24107	-1.14639	-0.46551	-1.4625
AT&T	1.17791	3.7648	-0.50548	-1.11304	1.26947	1.84676
BOA	0.716948	6.0502	-0.79574	-4.62624	0.191382	0.735091
Boeing	0.853578	5.59013	0.018835	0.08498	-0.13962	-0.41619
Caterpillar	1.22212	4.24452	0.689138	1.6489	1.33572	2.11149
Chevron	0.973393	5.55154	-0.34748	-1.36528	2.33514	6.06172
Citi Grp	1.1633	7.71822	-0.88868	-4.06203	0.126894	0.383198
Coca Cola	0.631032	8.05259	-0.44245	-3.88974	-0.47857	-2.77962
Ei Du PONT	0.875708	4.61191	-0.12179	-0.44187	-0.01468	-0.03518
Exxon	1.04406	2.94557	-0.29185	-0.56726	1.47583	1.89512
General Electric	1.04546	8.93606	-0.8351	-4.91756	-1.17848	-4.58477
GM	0.567122	0.534743	1.26481	0.821611	0.751434	0.32249
HP	1.49214	4.32834	-0.43895	-0.87721	-1.11902	-1.47743
Home Depot	0.863297	3.98355	-0.67734	-2.15324	-0.64507	-1.35479
Intel	1.49894	7.67671	-0.26167	-0.92323	-1.8261	-4.2567
IBM	0.647236	5.21346	-0.60064	-3.33311	-1.74028	-6.38026
J&J	0.690067	5.41373	-1.15228	-6.22784	-1.48702	-5.30982
JP MORGAN	1.16703	8.69032	-0.65883	-3.37985	0.403075	1.36614
Kraft Food	0.559433	4.16925	-0.59142	-3.03651	0.343988	1.16684
Mac Donalds	0.930027	3.34017	0.272431	0.674065	0.411963	0.673423
Merk & Co	1.30209	9.57915	-0.79226	-4.01536	-0.71424	-2.39157
Microsoft	0.661151	3.0969	-0.6191	-1.99785	-2.33957	-4.98793
Pfizer	0.655839	3.98412	-0.6634	-2.77639	-1.11997	-3.09669
P&G	0.584348	5.50808	-0.67821	-4.40416	-0.65575	-2.81334
United Tech	0.930752	6.30333	0.318567	1.48631	-0.63194	-1.9479
Verizon	0.934861	2.10958	-0.35657	-0.55434	-0.13032	-0.13385
WalMart	0.652872	4.60605	0.040349	0.196111	-0.11176	-0.35888
Walt Disney	1.56589	9.90239	-0.99119	-4.31824	0.129304	0.372175

Table 4: Fama-French Risk Coefficients from Quantile Regression (0.95)

Stocks	β	t-ratio	s	t-ratio	h	t-raio
3M	0.801955	7.71975	-0.36829	-2.44242	-0.07688	-0.33685
ALCOA	0.650295	4.44094	0.906195	4.26343	0.70458	2.19004
American EX	1.17947	5.205	-0.16356	-0.49725	0.156435	0.314213
AT&T	0.947283	2.99554	-0.42657	-0.92931	-1.30077	-1.8722
BOA	1.0973	3.41942	-0.8827	-1.89502	0.030389	0.043102
Boeing	1.39033	4.8707	-0.48082	-1.16047	-0.47805	-0.76225
Caterpillar	1.31213	5.58564	0.596229	1.74856	-0.10548	-0.20437
Chevron	1.29154	5.0939	-0.58191	-1.58114	1.49052	2.67571
Citi Grp	1.04364	3.701	-0.44377	-1.08418	-0.17367	-0.28031
Coca Cola	0.738879	4.13137	-0.249	-0.95915	0.079608	0.202597
Ei Du PONT	0.964043	3.7992	-0.57177	-1.55236	-0.4336	-0.77776
Exxon	1.33279	4.73872	-0.77013	-1.8864	1.29133	2.08974
General Electric	0.743803	2.87296	-0.62394	-1.6603	-0.71401	-1.25526
GM	0.162238	0.530055	1.17895	2.65361	1.51127	2.24733
HP	1.46965	4.32544	-0.88291	-1.79023	-1.91572	-2.5663
Home Depot	0.737246	2.56655	0.175047	0.419822	-1.96623	-3.1155
Intel	1.38265	7.3273	-0.5464	-1.99485	-2.64584	-6.38191
IBM	0.824713	3.9382	-0.06601	-0.21717	-1.20615	-2.62153
J&J	0.667347	2.97667	-0.92862	-2.85357	-0.79836	-1.62082
JP MORGAN	1.64231	4.57376	-0.58757	-1.12733	0.659379	0.835814
Kraft Food	0.710951	2.70263	-0.4679	-1.22539	0.435444	0.753418
MacD	0.882037	4.54509	0.131172	0.465662	-0.49492	-1.16076
Merk & Co	0.80509	7.10293	-0.51005	-3.10012	-0.40883	-1.64168
Microsoft	1.36497	5.91946	-1.53312	-4.58043	-0.89978	-1.77604
Pfizer	1.58166	4.42855	-0.94971	-1.83194	0.397758	0.506903
P&G	0.709654	3.20818	-0.28361	-0.8833	-0.70983	-1.46058
United Tech	1.10386	3.96697	-0.04976	-0.1232	0.358124	0.585783
Verizon	1.11802	27.314	-0.57456	-9.67029	-1.12131	-12.4686
WalMart	0.962667	6.87505	-0.38106	-1.87487	-1.60325	-5.21145
Walt Disney	0.969899	3.01824	-0.29676	-0.63623	0.11207	0.158735

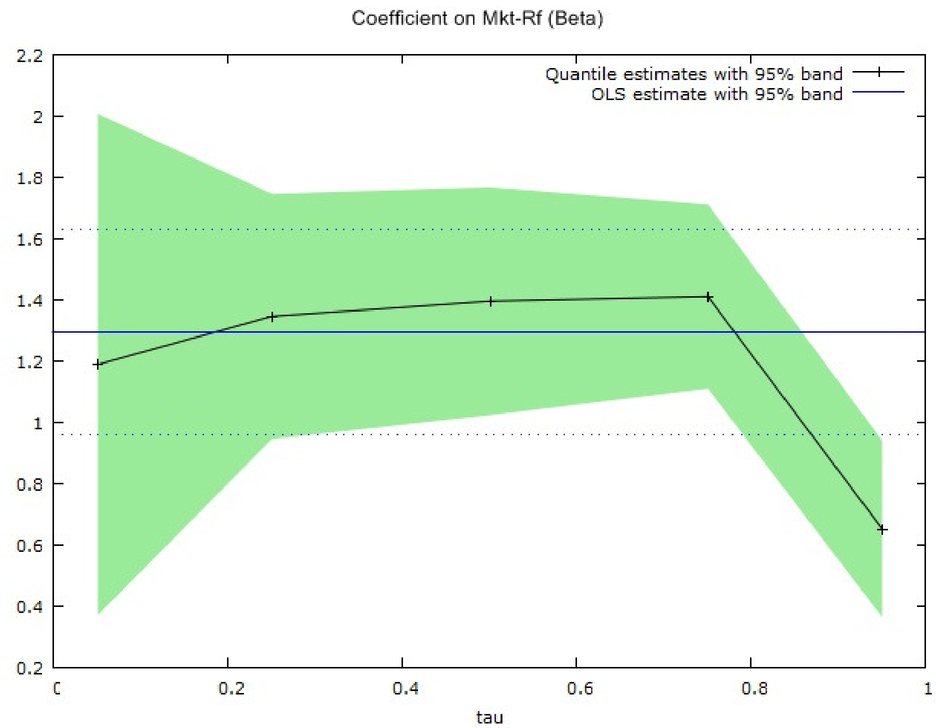

Figure 2: Values of Beta across quantiles (Alcoa).

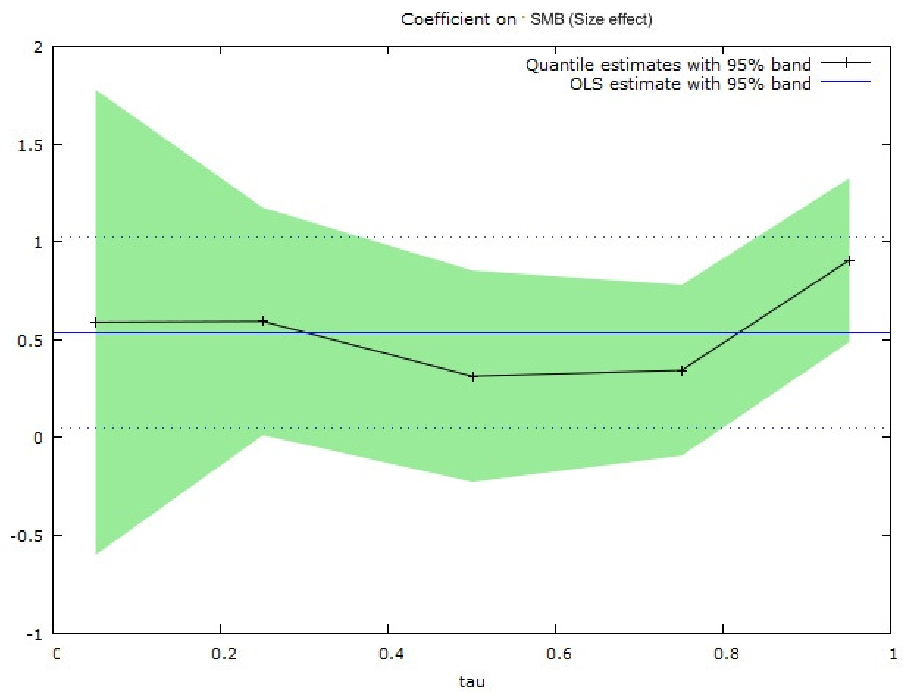

Figure 3: Values of s across quantiles (Alcoa)

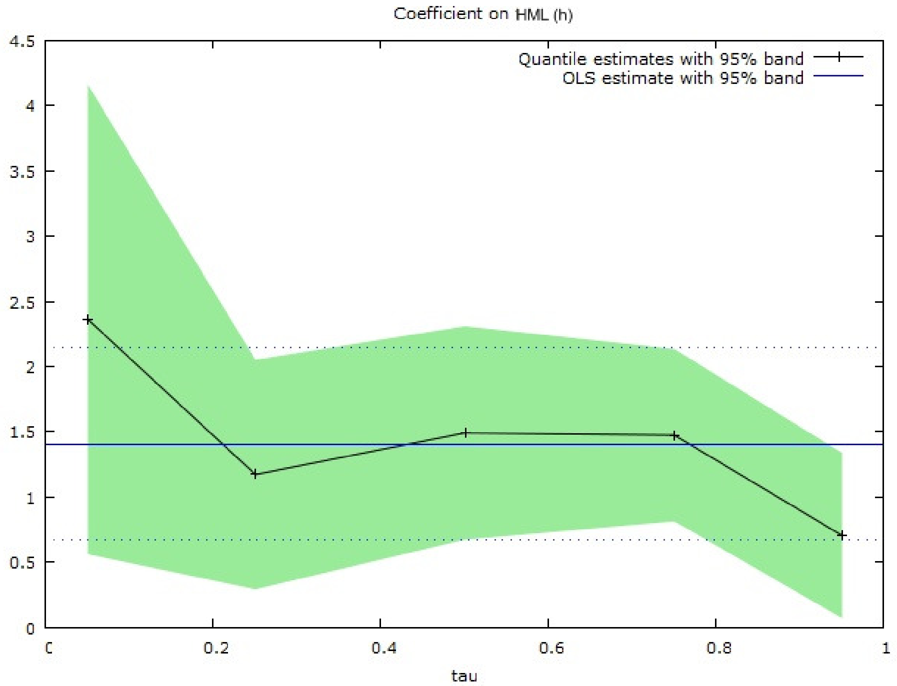

Figure 4: Values of h across quantiles (Alcoa)

Another approach to understand this is to plot the actual and fitted values obtained from both of the regression methods for an actual stock. Figure 5, Figure 6 and Figure 7, plot the fitted and actual values obtained from historical daily returns for a year using OLS, and the .05 and .95 quantiles respectively. The plotted values clearly show that, OLS is unable to cover the extreme values in the distribution. When it comes to efficient risk assessment, it becomes important for an investor or risk manager to account for the extreme tails of a distribution, which is captured by the quantile regression fitted values.

Through this analysis we are able to show that, quantile regression gives more efficient results when it comes to the boundary values of a distribution.

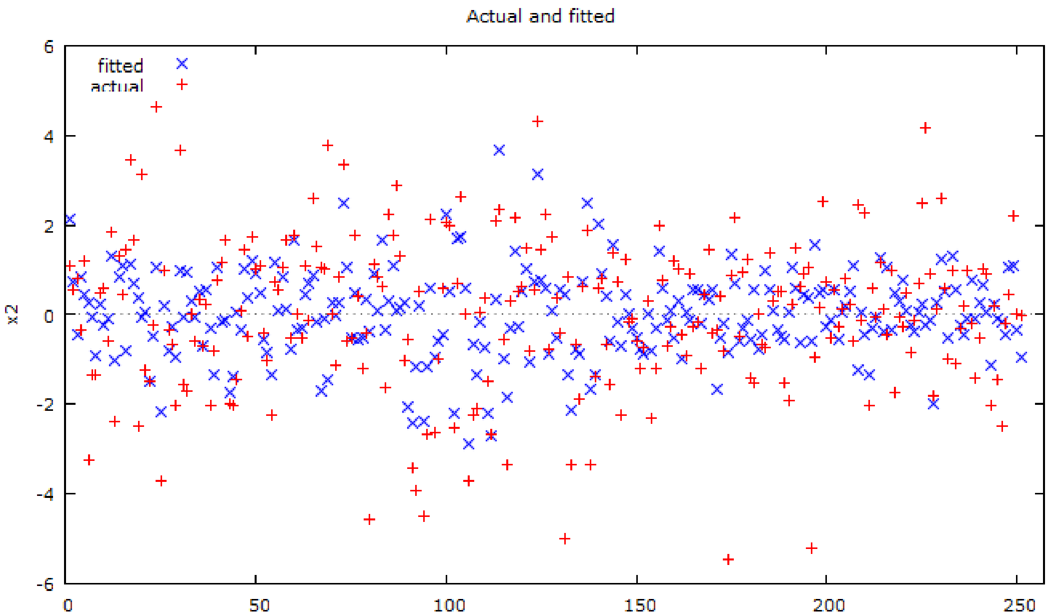

Figure 5: Actual Versus Fitted Values of Expected Return for OLS

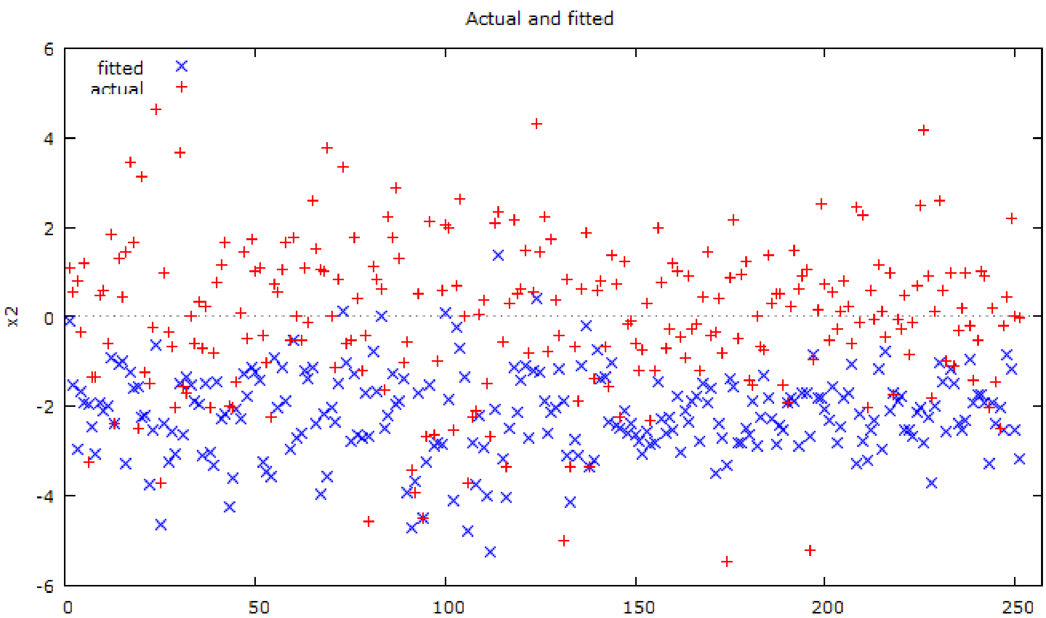

Figure 6: Actual Versus Fitted Values of Expected Return for Quantile Regression (0.05)

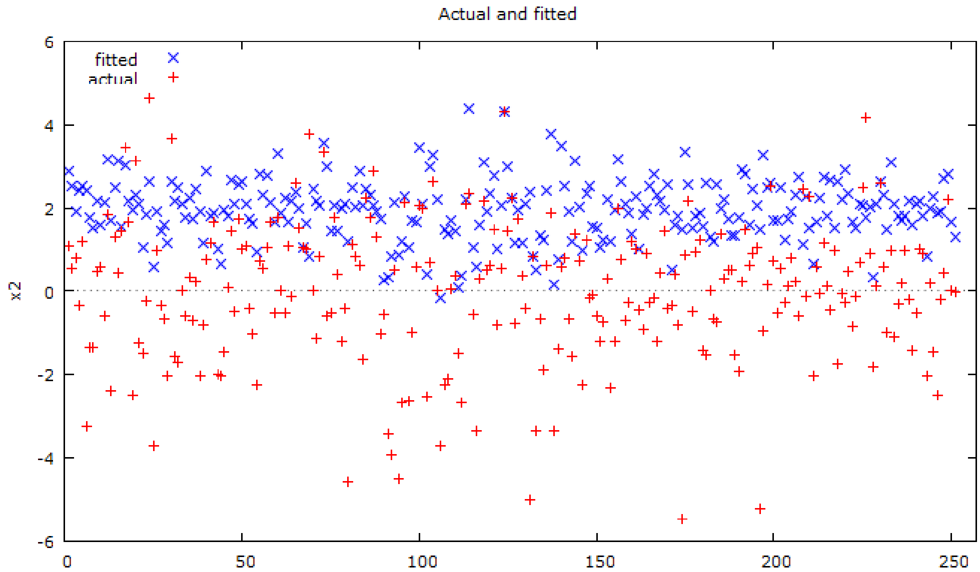

Figure 7: Actual Versus Fitted Values of Expected Return for Quantile Regression (0.95)

To further illustrate the additional information garnered by the technique we present some three-dimensional graphs of how the loadings on the three factors vary from year to year across the quantiles. Standard OLS and asset pricing techniques simply capture the average; clearly the behaviour is much more complex than perusal of the averages would suggest. The results also suggest why simple reliance on standard OLS based asset pricing models as benchmarks to capture abnormal returns may produce highly inconsistent results.

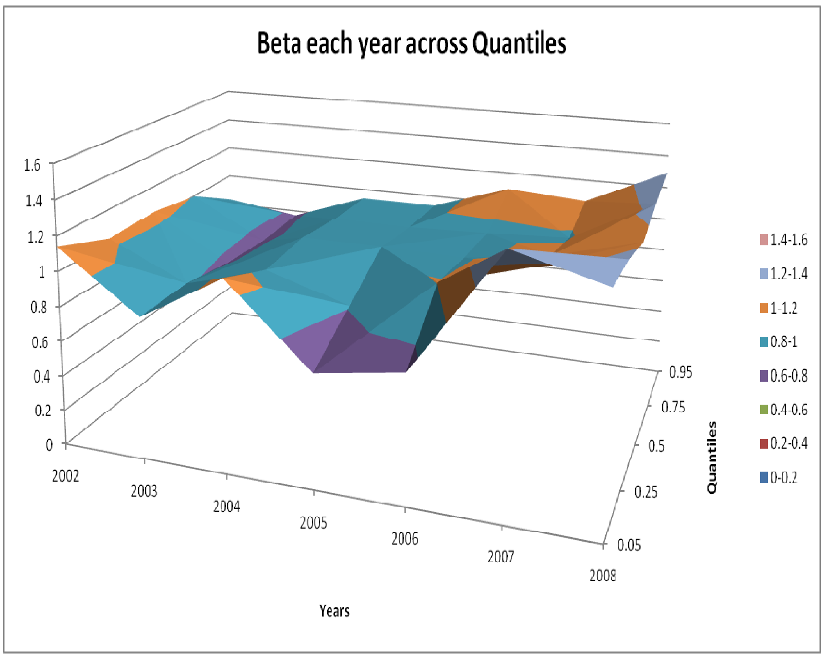

Figure 8. Annual Beta Estimates across the Quantiles for Bank of America.

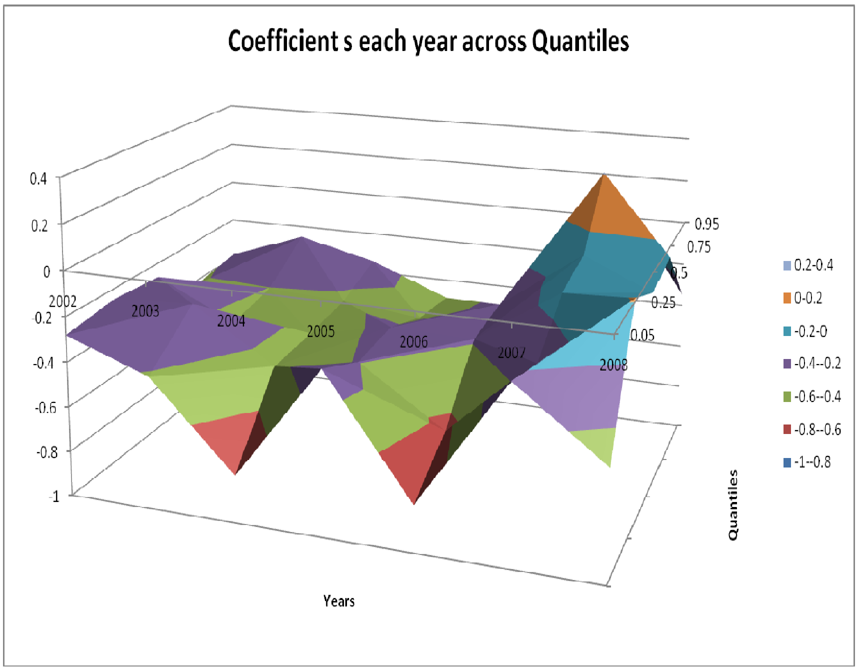

Figure 9. Annual estimates of coefficients *s* (SMB) for Bank of America

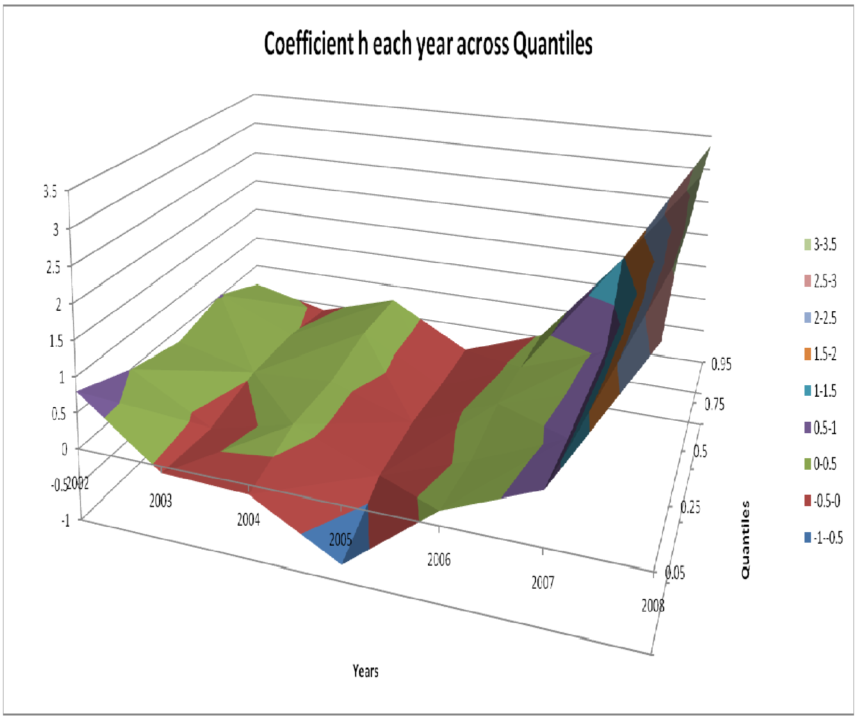

Figure 10. Annual estimates of coefficients *h* (HML) for Bank of America

The interesting feature of all three figures is that the direction or slope of the coefficients across the quantiles is not consistent and frequently changes from positive to negative as we move from the lowest to the highest quantiles. To be consistent with asset pricing theory and the implications of OLS analysis these estimates should be a constant. This is clearly not the case.

As a further test we ran some tests of the equivalence of slopes estimated across different quantiles. We used a bootstrapping technique to test the significances of the differences across the slopes. The results are reported in the Appendix. These just provide the results across the whole sample period, for reasons of economy. Around nine percent of the slope estimates across quantiles were significantly different at a ten percent level or better across the whole sample period.

6. Conclusion

We have explored the relationship between a set of returns of the 30 Dow Jones Index stocks and the three factor model using factors obtained from Professor Ken French's website for the period 2002 to 2009 and quantile regression analysis. We report the assumptions of OLS, as criticised originally by Frances Galton's famous exclamation against his statistical colleagues who: "limited their inquiries to averages and do not seem to revel in more comprehensive views", appear also to apply in finance in the large literature on testing asset pricing models. Our results reveal large and sometimes significant differences between returns and these three factors both across quantiles and through time. The picture that results from quantile regression analysis is far more complex than the assumptions inherent in OLS would lead us to believe.

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Appendix 1
Bootstrapped tests of the equivalence of slopes for 0.05 and 0.095 quantiles
For sample period 2004 -2008

	Rm-Rf	**SMB**	**HML**
3M	-0.01753	-0.04076	-0.02799
P>t	0.793	0.779	0.843
ALCOA	0.39625	-0.25665	-1.13105
P>t	0.001	0.345	0.006
American EX	0.057244	0.220976	0.162383
P>t	0.586	0.21	0.47
AT&T	0.03082	-0.2486	-0.09982
P>t	0.692	0.112	0.544
BOA	0.044157	0.100558	1.704162
P>t	0.685	0.609	0
Boeing	-0.07032	0.069527	0.197475
P>t	0.385	0.67	0.307
Caterpillar	0.092061	0.228815	-0.18698
P>t	0.4	0.207	0.264
Chevron	0.037604	-0.28524	-0.25619
P>t	0.606	0.125	0.17
Citi Grp	0.196229	0.100393	1.028629
P>t	0.231	0.828	0.006
Coca Cola	-0.04611	0.142811	-0.00975
P>t	0.596	0.466	0.971
Ei Du PONT	0.013873	0.151119	-0.0373
P>t	0.854	0.18	0.812
Exxon	-0.00631	-0.14074	-0.1723
P>t	0.912	0.492	0.212
General Electric	-0.02262	-0.01135	0.279718
P>t	0.765	0.958	0.077
GM	-0.11413	0.519874	0.371921
P>t	0.67	0.218	0.428
HP	-0.05644	-0.21838	-0.02956
P>t	0.516	0.295	0.87
Home Depot	-0.00143	0.123126	0.301347
P>t	0.982	0.552	0.058
Intel	-0.16474	0.154044	0.187521
P>t	0.122	0.637	0.442
IBM	-0.06498	0.010448	0.003313
P>t	0.255	0.925	0.983
J&J	0.070358	0.016706	-0.01663
P>t	0.521	0.879	0.928
JP MORGAN	-0.2141	0.351248	1.461143
P>t	0.088	0.13	0

Kraft Food	-0.0851	-0.04495	0.012665
P>t	0.383	0.778	0.941
MacD	-0.09526	-0.1119	0.154789
P>t	0.187	0.523	0.368
Merk & Co	0.121172	0.047442	-0.11001
P>t	0.185	0.691	0.563
Microsoft	-0.00668	-0.11403	-0.13767
P>t	0.945	0.478	0.42
Pfizer	-0.09415	-0.1507	0.289669
P>t	0.182	0.263	0.025
P&G	-0.00025	-0.05096	0.023961
P>t	0.996	0.694	0.857
United Tech	0.087084	0.073079	0.05069
P>t	0.178	0.685	0.753
Verizon	-0.07961	0.103571	0.059251
P>t	0.453	0.56	0.778
WalMart	-0.03076	0.16796	0.143322
P>t	0.748	0.338	0.442
Walt Disney	0.041348	0.19643	-0.17033
P>t	0.626	0.19	0.297

NB: Differences significant at .10 or better marked in yellow.