ATAR course examination, 2022
Question/Answer booklet

## MATHEMATICS APPLICATIONS

## Section Two: Calculator-assumed



In words

## Time allowed for this section

Reading time before commencing work: Working time:
ten minutes
one hundred minutes

Number of additional answer booklets used (if applicable):

## Materials required/recommended for this section

To be provided by the supervisor
This Question/Answer booklet
Formula sheet (retained from Section One)
To be provided by the candidate
Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: drawing instruments, templates, notes on two unfolded sheets of A4 paper, and up to three calculators, which can include scientific, graphic and Computer Algebra System (CAS) calculators, are permitted in this ATAR course examination

## Important note to candidates

No other items may be taken into the examination room. It is your responsibility to ensure that you do not have any unauthorised material. If you have any unauthorised material with you, hand it to the supervisor before reading any further.

## Structure of this paper

| Section | Number of <br> questions <br> available | Number of <br> questions to <br> be answered | Working <br> time <br> (minutes) | Marks <br> available | Percentage <br> of <br> examination |
| :--- | :---: | :---: | :---: | :---: | :---: |
| Section One: <br> Calculator-free | 7 | 7 | 50 | 54 | 35 |
| Section Two: <br> Calculator-assumed | 10 | 10 | 100 | 97 | 65 |
| Total |  |  |  |  | 100 |

## Instructions to candidates

1. The rules for the conduct of the Western Australian external examinations are detailed in the Year 12 Information Handbook 2022: Part II Examinations. Sitting this examination implies that you agree to abide by these rules.
2. Write your answers in this Question/Answer booklet preferably using a blue/black pen. Do not use erasable or gel pens.
3. You must be careful to confine your answers to the specific question asked and to follow any instructions that are specified to a particular question.
4. Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.
5. It is recommended that you do not use pencil, except in diagrams.
6. Supplementary pages for planning/continuing your answers to questions are provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.
7. The Formula sheet is not to be handed in with your Question/Answer booklet.

## Section Two：Calculator－assumed

This section has 10 questions．Answer all questions．Write your answers in the spaces provided．
Supplementary pages for planning／continuing your answers to questions are provided at the end of this Question／Answer booklet．If you use these pages to continue an answer，indicate at the original answer where the answer is continued，i．e．give the page number．

Working time： 100 minutes．

## Question 8

Yash is saving for a deposit on an apartment, which is about to begin construction.
(a) Two saving plans are found in advertisements in a newspaper.

Option 1: $3.05 \%$ per annum compounded at the end of each month. Option 2: $3.01 \%$ per annum compounded at the end of each day.

Which of these two options would give the better return? Justify your answer mathematically.

Yash locates a better investment option online. This offers 3.5\% per annum compounded at the end of each day. Yash will begin with an initial deposit of $\$ 12300$ and a weekly deposit of $\$ 300$. The deposit required to secure the apartment in 12 months is $\$ 30000$.
(b) (i) Show that this option will not be sufficient to reach the total needed to secure the apartment. State how far short of the required amount this option will be. (4 marks)
(ii) Determine what the weekly deposit needs to be for a total of $\$ 30000$ to be achieved by the end of 12 months.
(2 marks)

## Question 9

A study of a penguin colony on an island was conducted and it found the initial population size of 1200 was dropping by $14 \%$ each year due to the introduction of non-native predators.
(a) Explain why the population after $n$ weeks is $1200 \times(0.86)^{n}$ penguins.

After eight weeks, the Parks and Wildlife Service set traps to reduce the predator numbers. This saw the penguin population increase weekly by $6 \%$.
(b) State the recursive formula that models the new population growth.
(c) How many weeks will it take to get the population back up to the initial size?

Once the population returns to the initial size，it is further helped by the introduction of penguins from a breeding program at the zoo．

The new population growth model can be represented by
$P_{n+1}=-0.25 P_{n}+3000, P_{0}=1200$.
（d）Discuss the long－term behaviour of the penguin population，now that it is being supported by the breeding program．

The network shown below represents the cycle tracks connecting the various points of interest on an island. The numbers indicate the distance between points of interest to the nearest 100 metres.

(a) Each morning all tracks must have a safety check. Is it possible to check all individual tracks by an Eulerian trail? Justify your answer.
(b) Rubbish bins at each point of interest are emptied daily. The rubbish truck leaves from position C and must return to C after emptying all rubbish bins.
(i) What name is given to the type of path taken by the rubbish truck?
（ii）The rubbish truck wants to follow the path of least distance．State the path and its length．
（c）A tourist at $G$ wishes to travel to $C$ but path $A G$ is closed for maintenance．Determine the shortest path possible and its length．
（2 marks）

Nullah wanted to see if there was a relationship between outside temperature and the time taken to dry his laundry. The following data was collected over a 10 day period.


| Temperature ( ${ }^{\circ} \mathrm{C}$ ) | 28 | 35 | 42 | 31 | 24 | 21 | 21 | 35 | 32 | 28 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Time (minutes) | 29 | 20 | 15 | 27 | 34 | 38 | 35 | 18 |  |  |

(a) Complete the table by locating the data in the graph.
(b) Determine the equation of the least-squares line and state the correlation coefficient.
(c) Draw the least-squares line onto the graph above.
（d）Describe the association between the two variables in terms of direction and strength．
（e）What percentage of the variation in drying time can be explained by the variation in outside temperature？
（f）Identify at least one other factor that could explain the variation in drying time．（1 mark）
（g）The temperature on Day 11 is predicted to be $17^{\circ} \mathrm{C}$ ．
（i）Use the equation for the least－squares line from part（b）to predict the time Nullah should expect his laundry to dry on this day．
（1 mark）
（ii）Is this prediction reliable？Justify your answer．

## Question 12

The network below shows the different routes commuters can take, travelling from a bus stop to a designated train platform. The edges are a combination of stairs, escalators and covered walkways, with the numbers indicating the maximum number of people per minute that each route can manage.

(a) Which vertex represents
(i) the bus stop?
(ii) the train platform?
(b) Determine the maximum number of people per minute that can travel from the bus stop to the train platform, listing each path and the corresponding flow.
(3 marks)
(c) Draw the minimum cut that corresponds to the maximum flow on the network diagram.

(d) The Transport Authority has approved upgrades for one escalator connection to increase the capacity from 120 to 150 people per minute. Which connection should be upgraded? Justify your response.
(2 marks)

Data have been collected for nine suburbs within a city about the number of mobile phone towers and the number of births in the last 12 months for each suburb.

|  | Suburb |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| Number of mobile <br> phone towers $(\boldsymbol{n})$ | 4 | 6 | 7 | 8 | 6 | 10 | 5 | 8 | 7 |
| Number of births in <br> the last 12 months $(\boldsymbol{b})$ | 25 | 29 | 35 | 45 | 38 | 54 | 22 | 38 | 39 |

The data has a correlation coefficient of 0.92 , and the equation of the least-squares line is $b=5.13 n+1.31$.
(a) Interpret the gradient of the least-squares line in the context of the question. (2 marks)
(b) Explain the significance of the correlation coefficient in the context of the question.
(2 marks)
(c) (i) Predict the number of births for a suburb in this city that has nine mobile phone towers.
(ii) Comment on the validity of the prediction in part (c)(i). Justify your response.
(2 marks)
(d) Complete the residual plot below.

Residual


A spare grid is provided at the end of this Question/Answer booklet. If you need to use it, cross out this attempt and indicate that you have redrawn it on the spare grid.
(e) Based on the residual plot, comment on whether the least-squares line is a suitable model for these data.
(f) A 10th suburb has a data point $(5,12)$ which has been verified as correct. State a practical explanation of how this could be a correct data point.
(g) A journalist has followed the mathematics involved in working with bivariate data and is writing a report for a newspaper. What is a valid statement that could be made about the observed association between the number of mobile phone towers and the number of births in the last 12 months for suburbs within the city?

Electric wiring is being installed to six powered sites (A-F) at a new caravan park from the main power source. The distance, in metres, between each site is shown below.

(a) Complete the minimum spanning tree below.

(b) The cost of installing the electric wiring is $\$ 14.20$ per metre. If the wiring must include the connection between D and the main power source
(i) calculate the minimum length of wiring required.
(ii) calculate the total cost of installing the wiring.

It is discovered that between Sites B and F there is a large section of rock．The owner can still install the wiring directly between these two sites but the cost for that section will double in price．
（c）What effect，if any，will this have on the minimum spanning tree and total cost of the installation？A copy of the original graph has been provided below．


Paris, a retired hairdresser, has a superannuation balance of \$775 320.
(a) She intends to set up an annual scholarship of $\$ 5000$ to be awarded to a worthy apprentice. Paris is able to negotiate a fixed annual interest rate of $7.2 \%$ through a share trading company. Show that $\$ 69444.44$ (to the nearest cent) of Paris's superannuation is needed to be invested in this company to meet the requirements of the annual scholarship.

After the scholarship is in place, Paris considers her future investment plans with the remaining balance. Assume a financial advisor has indicated an interest rate of $6.1 \%$ per annum, added monthly, can be locked in.
(b) Paris decides on receiving a monthly annuity of $\$ 7000$, with the interest added before the annuity is paid, at the end of each month.
(i) The annuity can be modelled by the recurrence relation $T_{n+1}=(1+0.00508) T_{n}-7000, T_{0}=a$.
Using the information given in the question, show how to obtain the value 0.00508 and state the value of $a$.
(ii) For how many months will Paris be able to receive the annuity of $\$ 7000$ ? (1 mark)
（iii）Determine the amount of the final payment．
（iv）Determine the total interest received on the annuity．
（c）Paris also considers receiving a perpetuity after the scholarship is in place．Determine the monthly amount she would receive．
（3 marks）

After paying a deposit for his new apartment, Declan obtains a bank loan for the remaining amount of $\$ 112000$ at $3.26 \%$ per annum compounded monthly. He can currently afford to repay $\$ 970$ per month at the end of every month.
(a) Calculate how much he would owe after the 40th repayment.
(b) Declan decided to deposit a one-off extra amount of $\$ 1600$, after the 16th repayment. Calculate the new amount he would owe after the 40th repayment.

## Question 17

Indie was in a line with 24 other people for a slide at a water park. She noticed that the approximate number of people $(P)$ in the line for the slide increased by $1.5 \%$ every minute ( $m$ ).
(a) Write an exponential equation in the form $P=a r^{m}$ to represent this situation. (2 marks)
(b) Determine the approximate number of people in the line after 2 hours.

After 3 hours, the line started to decrease by 1\% per minute.
(c) Using this new information, calculate the approximate number of people in line, 5 hours after Indie initially lined up.

Supplementary page
Question number:

Supplementary page
Question number：

Supplementary page
Question number:

Supplementary page
Question number：

Supplementary page
Question number:

Spare grid for Question 13(d)


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