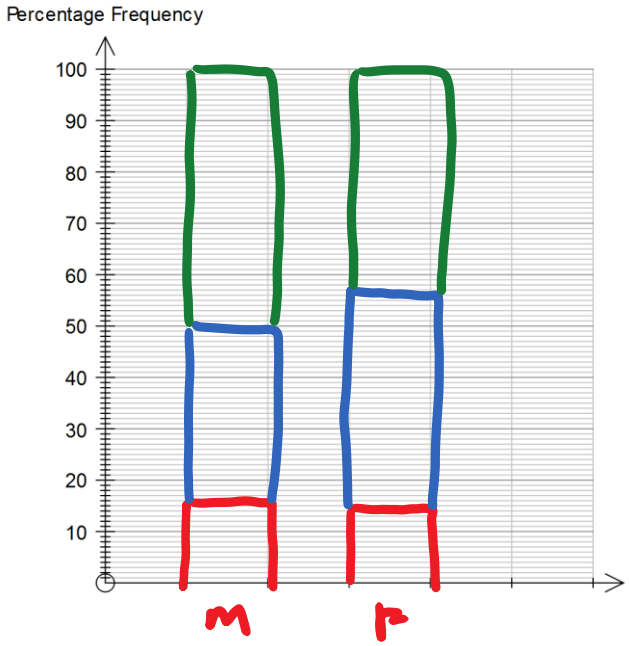
1. Data Analysis Answers

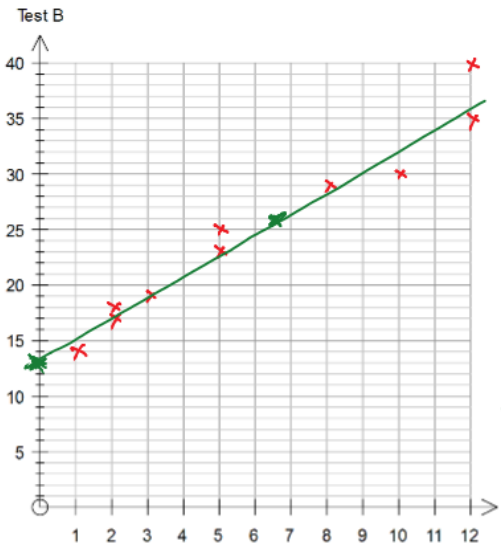
EXAMPLE 1

	> 90	80 to 90	< 90	
Male	12 – 17%	24 – 33%	36 – 50%	72
Female	16 – 15%	46 – 43%	46 – 43%	108
	28	70	82	180

Little or no association. Females slightly more likely to do better in mid-range scores.



EXAMPLE 2



$r \approx 0.978$

$[r^2 \approx 0.957]$

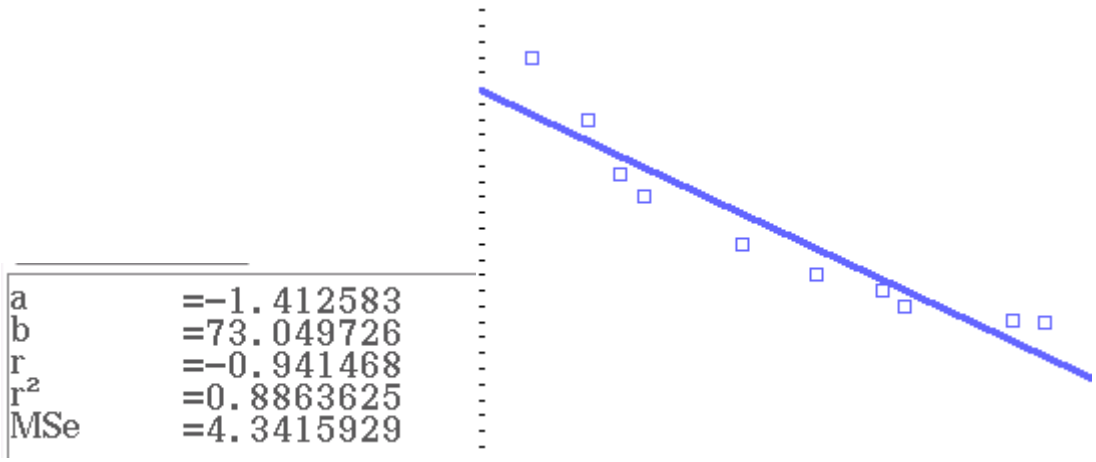
$\hat{y} = 1.96x + 13.26$

$(6.5, 25.98)$

$\hat{y}(7) = 1.96(7) + 13.26$

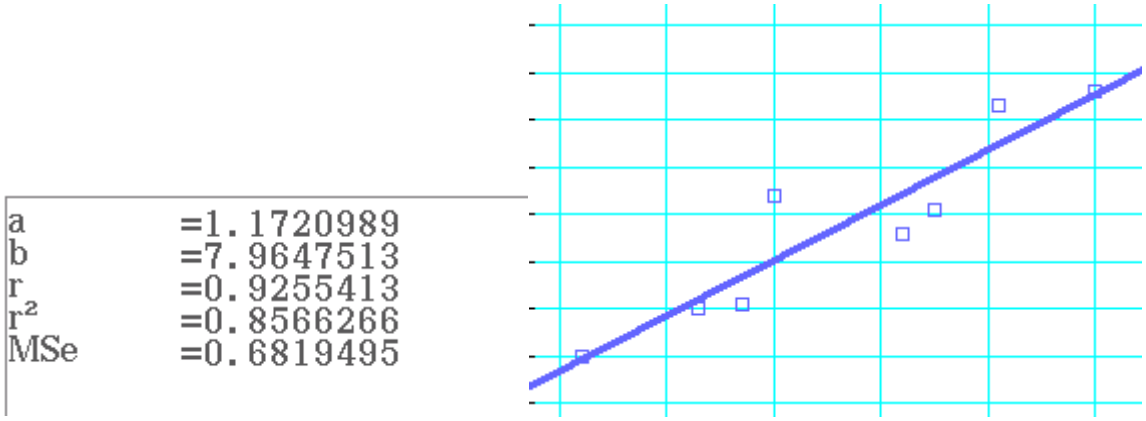
$= 27 \quad (26.98)$

EXAMPLE 3



Could be a non-linear relationship.
Residuals would be in a pattern.

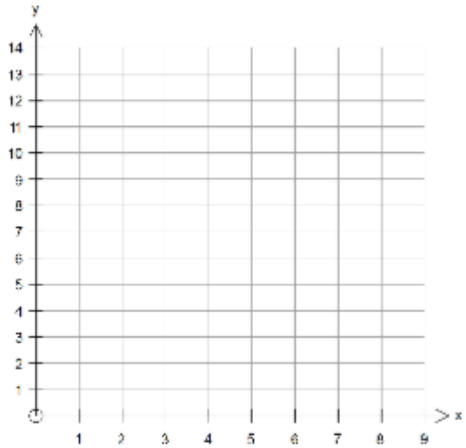
EXAMPLE 4



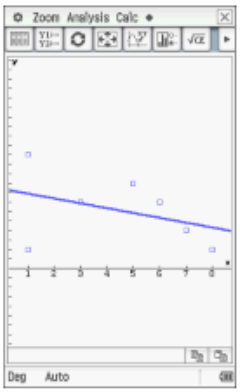
85.7% of the variation in maximum can be attributed to the minimum.

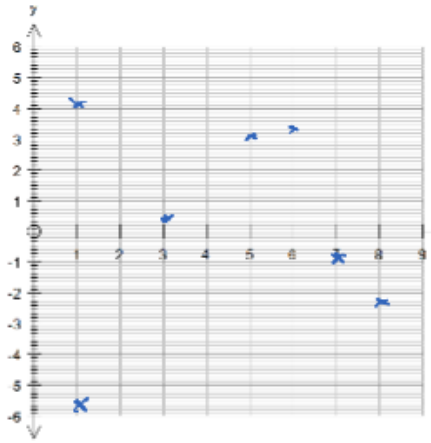
EXAMPLE 5

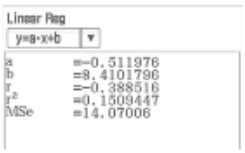
x	3	5	6	1	8	7	1
y	7	9	7	12	2	4	2
$\hat{y}$	6.88	5.86	5.35	7.9	4.33	4.84	7.9
Residual $(y-\hat{y})$	+0.12	+3.14	+1.65	+4.1	−2.33	−0.84	−5.9



$\hat{y} = -0.51x + 8.41$







The residual plot does not look entirely random (esp. if we ignore (1, +4.1)) It may be that a linear model is not appropriate.

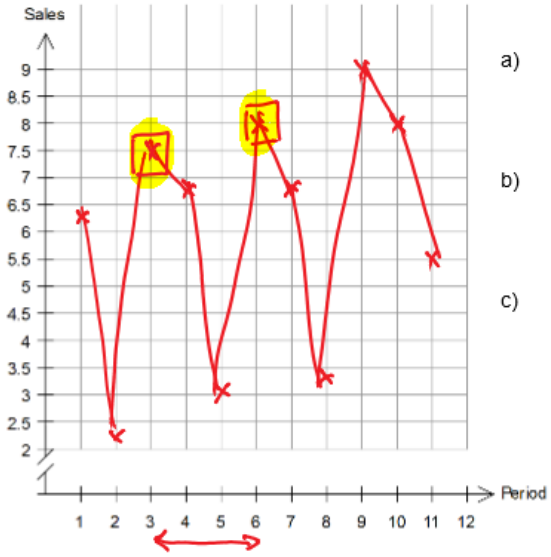
EXAMPLE 6

Slight upward trend. Clear seasons every 4 data points ie cycle of 4, period of one year. No irregular fluctuations.

EXAMPLE 7

Time Period (t)	Year	Sales (000's)	**A** pt moving Average
1	1992	6.3	
2	1993	2.1	**B** 5.3
3	1994	7.5	**C** 5.5
4	1995	6.8	**D** 5.8
5	1996	3	5.9
6	1997	8	6.0
7	1998	6.9	6.1
8	1999	3.4	6.4
9	2000	9	**E** 6.8
10	2001	8	7.5
11	2002	5.5	7.4
12	2003	**F**	8.7

A B C



a) Plot the known sales figures on the axes. [1]

b) Comment on the seasonality of the data. [2]

high every three years, dropping down slightly then more before increasing again

c) What is the period of the data? [1]

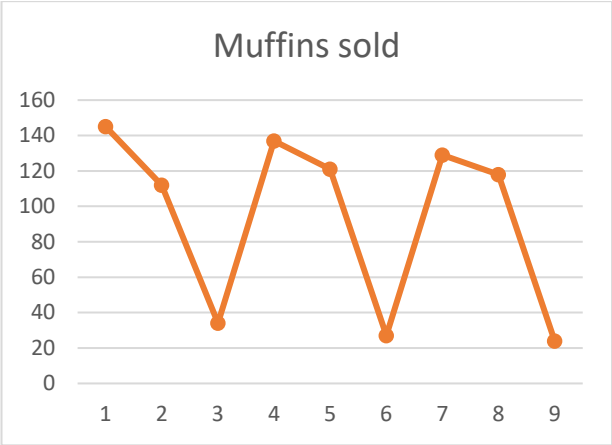
3 years

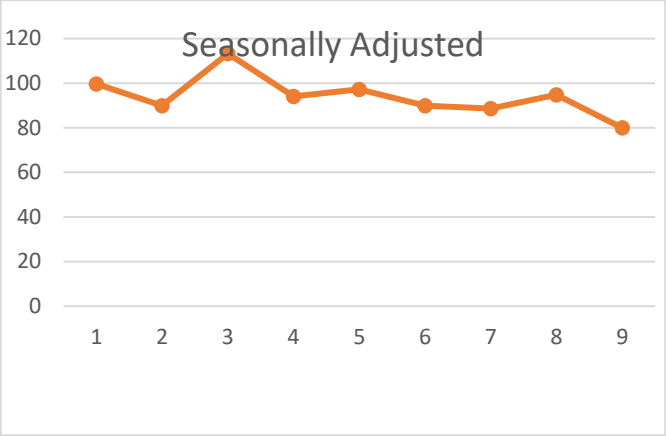
c) Determine the values of **A**, **B**, **C**, **D**, **E** and **F** in the table above.

3

Trend of the data is upwards. A positive, secular trend.

EXAMPLE 8





Date	Shift	Period	Muffins sold	3pt MA	Percentage of Daily Mean	Seasonally adjusted
2-Jul	Morning	1	145		149.5	99.7
	Afternoon	2	112	97.0	115.5	90.0
	Night	3	34	94.3	35.1	113.3
9-Jul	Morning	4	137	97.3	144.2	94.2
	Afternoon	5	121	95.0	127.4	97.2
	Night	6	27	92.3	28.4	90.0
16-Jul	Morning	7	129	91.3	142.9	88.7
	Afternoon	8	118	90.3	130.7	94.8
	Night	9	24		26.6	80.0

	2-Jul	9-Jul	16-Jul
Daily Mean	97	95	90.3

	Morning	Afternoon	Night
Seasonal Index	145.5	124.5	30

2. Sequences and Finance Answers

EXAMPLE 9

a) $t_{n+1} = -4 + t_n, \quad t_1 = 8$ [4]

n	1	2	3	4	5
t_n	8	4	0	−4	−8

∴ subtract 4 off previous term.

b) $b_n = 7 - b_{n-1}, \quad b_1 = 20$ [4]

n	1	2	3	4	5
b_n	20	−13	20	−13	20

c) $t_{n+3} = t_{n+1} - t_{n+2}, \quad t_1 = 5, t_2 = 1$ [4]

n	1	2	3	4	5
T_n	5	1	4	−3	7

d) $g_n = g_{n-1} - n, \quad g_1 = 4$ [4]

n	1	2	3	4	5
g_n	4	2	−1	−5	−10

$T_{n+1} = T_n + 4, T_1 = 5$ ✓✓ [6]

General Term / n^{th} term

$T_n = 5 + (n-1)4$

$= 5 + 4n - 4$

$= 4n + 1$ ✓✓

$T_{10} = 4(10) + 1$

$= 41$ ✓

$T_{100} = 4(100) + 1$ ✓

$= 401$

b) 50, 40, 30, 20, … [6]

$T_{n+1} = T_n - 10, T_1 = 50$ ✓✓

$T_n = 50 + (n-1)(-10)$ ✓✓

$= 50 - 10n + 10$

$T_n = -10n + 60$

$T_{10} = -10(10) + 60$

$= -40$ ✓

$T_{100} = -10(100) + 60$

$= -940$ ✓

3. Show clearly if 151 a term in the sequence -5, 1, 7, 13, …? If so, which term? [4]

$T_n = -5 + (n-1)6$

$= -5 + 6n - 6$

$= 6n - 11$

$151 = 6n - 11$

$162 = 6n$ $\therefore T_{27}$

$27 = n$ ✓ yes!

4. Which is the first term of the sequence 3, 11, 19, … that is larger than 500. [3]

$T_n = 3 + (n-1)8$

$T_n = 3 + 8n - 8$

$T_n = 8n - 5$

$8n - 5 = 500$

$\therefore 8n = 505$

$\therefore n = 63\frac{1}{8}$

$\therefore 64^{th}$ term.

EXAMPLE 10

[5]

i)

n	1	2	3	4	5
t_n	-2	2	6	10	14

(+4, +4, +4, +4)

ii) Arithmetic

iii) $T_n = -2 + (n-1)4$

$= -2 + 4n - 4$

$= 4n - 6$

iv) $T_{10} = 34$

v) $S_{10} = 160$

b) $b_n = 10b_{n-1}, \; b_1 = 4$ [5]

n	1	2	3	4	5
t_n	4	40	400	4000	40 000

Geometric $T_n = 4(10)^{n-1}$

$T_{10} = 4 \times 10^9$

$S_{10} = 4444444444$

c) $t_{n+3} = 2t_{n+1} - t_{n+2}, \; t_1 = 3, t_2 = 6$ [5]

Neither ✗

$T_{10} = 516$

$S_{10} = 381$

d) 50, 25, 12.5, ... , 6.25 , 3.125 [5]

Geometric $T_n = 50(0.5)^{n-1}$

$T_{10} = 0.09765625$

$S_{10} = 99.90234375$

e) 4, -6, 9, ... , 13.5 , [5]

Geometric $T_n = 4(-1.5)^{n-1}$

Ratio $= \frac{T_2}{T_1} = \frac{-6}{4}$ $T_{10} = -153.7734375$

$= -1.5$ $S_{10} = -90.6640625$

$\frac{T_3}{T_2} = -1.5$

2. A geometric sequence has a second term of 30 and a fifth term of 101.25. Determine a general term for the sequence. [4]

n	2		5
T_n	30		101.25

x x x

$T_2 = 30 = a \times r^1$

$T_5 = 101.25 = ar^4$

$\frac{101.25}{30} = \frac{ar^4}{ar^1}$

$3.375 = r^3$

$r = 1.5$

$T_n = 20(1.5)^{n-1}$

EXAMPLE 11

a) Determine the first 5 terms of each sequence. [3]

n	1	2	3	4	5	
A_n	2	9	16	23	30	✓
B_n	5	7	11	19	35	✓
C_n	20	28	33.6	37.52	40.26	✓

b) Only one of the recursion relations has a steady-state solution. Explain clearly which one it is and why the others do not have a steady-state solution. [2]

C_n ∵ r between −1 & 1.

both the others have r outside this

c) Determine the steady-state solution for the appropriate recursion relation above. [3]

$$X = 0.7X + 14$$

$$\therefore 0.3X = 14$$

$$\therefore X = \frac{14}{0.3}$$

$$\therefore X = 46\frac{2}{3}$$

A grain store has 4 tonnes of wheat and an extra 3 tonnes is added every month. If 25% of the grain is removed every month, determine the amount of grain that will eventually remain in the grain store.

$W_{n+1} = 0.75W_n + 3$ $T_1 = 4$

$$X = 0.75X + 3$$

$$0.25X = 3$$

$$X = 12 \text{ tonnes}$$

EXAMPLE 12

1. Write recurrence relations for each of the following situations:

a) $40,000 invested at interest rate of 3.5% p.a., compounded yearly. [2]

$A_{n+1} = A_n\left(1 + \frac{3.5}{100}\right) \quad A_0 = 40\,000$

b) $62,500 invested at interest rate of 4.25% p.a., compounded monthly. [2]

$A_{n+1} = A_n\left(1 + \frac{4.25}{(100\times12)}\right) \quad A_0 = 62\,500$

c) $850 invested at interest rate of 2.34% p.a., compounded 3 monthly. [2]

$A_{n+1} = A_n\left(1 + \frac{2.34}{(100\times4)}\right) \quad A_0 = 850$

2. Determine the amount of interest earned in each of the following scenarios by first writing a recurrence relation.

a) $80,000 invested for 10 years at an interest rate of 4.3% p.a., compounded yearly. [3]

$A_{n+1} = A_n\left(1 + \frac{4.3}{100(1)}\right) \quad A_0 = 80\,000$

$A_{10} = \$121\,880.18 - 80\,000$ for interest only.

Interest = $41 880.18

b) $350,000 invested for 40 months at an interest rate of 6.15% p.a., compounded monthly. [3]

$A_{n+1} = A_n\left(1 + \frac{6.15}{(100\times12)}\right) \quad A_0 = 350\,000$

$\therefore A_{40} = \$429\,408.91 - 350\,000$

$\therefore$ Interest = $79 408.91

3. Judy invests $120,000 at 5.1% p.a. for 16 years, compounded six monthly.
Show working to answer:

a) How much is the investment worth after 16 years? [3]

$A_{nm} = A_n\left(1 + \frac{5.1}{(100 \times 2)}\right)$ $A_0 = 120\,000$

$\therefore A_{32} = \$268\,610.21$

b) How much more does the investment earn in the last four years, compared to the first twelve years? [3]

$\therefore A_{24} = \$219602.46$

$\therefore \$49007.75$ Last 4 years

$\$99602.46$ First 12 years

$\therefore \$50\,594.71$ more in The first 12 years

	n	5	Effective	6.5	Effective	11	Effective
Annual	1	105	5	106.5	6.5	111	11
6 Monthly	2	105.0625	5.0625	106.605625	6.605625	111.3025	11.3025
Quarterly	4	105.0945	5.094534	106.660161	6.660161	111.462126	11.46213
Monthly	12	105.1162	5.11619	106.697185	6.697185	111.571884	11.57188
Weekly	52	105.1246	5.124584	106.711571	6.711571	111.614839	11.61484
Daily	365	105.1267	5.12675	106.715285	6.715285	111.625957	11.62596

EXAMPLE 13

a) Write a recurrence relation for this mortgage. [2]

$A_{n+1} = A_n\left(1 + \frac{7.2}{2 \times 100}\right) - 32\,000$

$A_0 = 430\,000$

b) When is the mortgage paid off? [1]

19 payment periods ∴ Jan-25

c) Show working to determine the final repayment. [2]

$\$21561.34 + 21561.34 \times \frac{7.2}{2 \times 100}$

$= 21561.34 + 776.21 =$

d) Determine the amount of interest paid on the loan. [2]

$18 \times 32000 + 22337.55$

$- 430\,000 = \$168\,337.55$

e) If all other parameters stay the same, how much should be repaid per time period so that the loan is paid off in exactly seven years? [1]

$\$39640.02$

a) monthly repayment if the loan is paid in 25 years, [1]

$4095.54

b) total amount repaid on the loan, [1]

$25 \times 12 \times \$4095.54 = \$1228662$

c) amount of interest paid on the loan. [1]

$-756\,000 = \$472\,662$

3. Guptill borrows $109,000 at 8.45% p.a. adjusted monthly. If I takes him 8 years to pay off the loan, how much will he have paid off his loan at the end of the third year? [3]

Payments = $1565.94

$A_x = \$76\,415.72$ ← Value after 3 years

$\$\,109\,000 - 76415.72 = \$32\,584.28$

4. Captain Starlight borrows $80,000 at 9.3% p.a. adjusted monthly to be repaid in exactly 10 years.

a) How much interest does he pay? [2]

$\$1026.44 \times 120$ ← Repayments in total

$-\,80\,000$ ← borrowed

$\$43172.80$

b) If he pays $65 a month extra, how much will he save over the lifetime of the loan? [4]

$\$1091.44 \quad \therefore N = 108.7 \rightarrow$ 108 full payments ↪ part payment

$\therefore \$798.78$ at the start of the 109^{th} time period

$\text{Last Payment} = 798.78 + 798.78 \times \frac{9.3}{12(100)}$

$= \$804.97$

$\therefore 108 \times 1091.44 + 804.97 - 80\,000$

$= \$38\,680.49$

$43172.80 - 38680.49$

$= \$4492.31$

EXAMPLE 14

3. Nic's everyday banking account is accidently credited with $825,000. He immediately removes the money to an off shore account and sets up an annuity. Answer the questions below for the given conditions:

a) If the annuity receives an annual interest rate of 4.65% p.a., how much can he withdraw per year if he wants it to last exactly 20 years? [1]

FINANCIAL $64 249.84 per year

b) In the situation in a), how much interest has he received after 10 years? [2]

Value after 10 years $504 662.08

$825 000 − 10x 64 249.84 = $182 501.60 ← difference is interest

∴ $322 160.48 interest

c) If her removes $6,000 per month with interest rate of 5.04% p.a. added monthly, how long will the annuity last and how much will his last withdrawal be? [3]

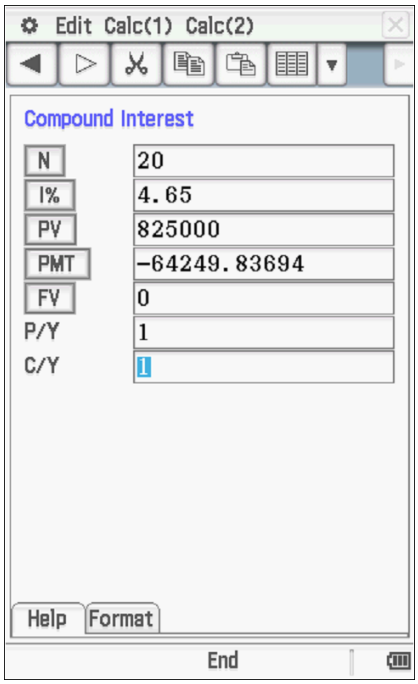
FINANCIAL N = 205.6

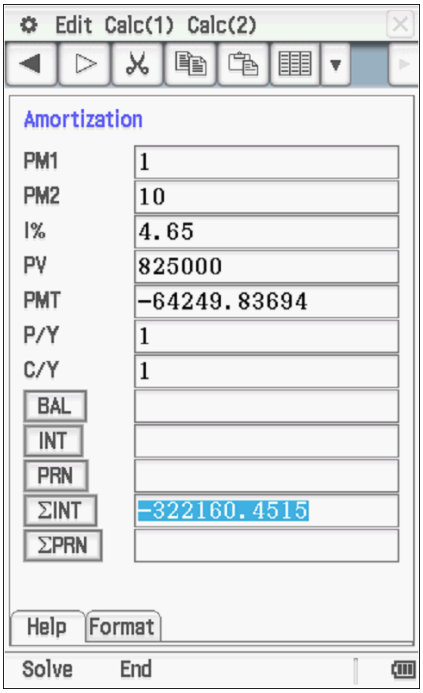
∴ 205 full + 1 smaller

After 205 ⇒ FV = $3380.12 + interest

∴ = $3394.32. last payment.

d) **$3465**

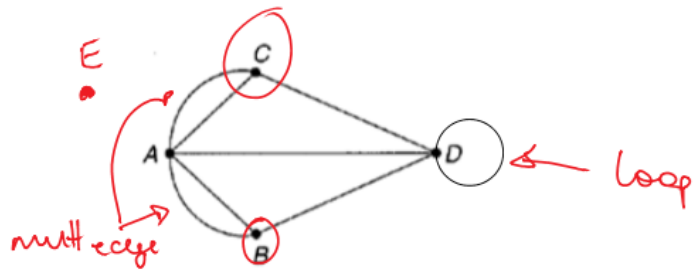




3. Graphs and Networks Answers

EXAMPLE 15

1.



Given the graph above, answer the questions below:

a) List all the vertices. A, B, C, D [1]

b) Add an isolated vertex E. [1]

c) How many arcs are there? [1]

8

d) Clearly label any loops and multiple edges on the graph. [2]

e) Give a definition of a graph and explain when it becomes a network. [2]

- Diagram with vertices connected by edges
- Practical application.

f) Give the degree of each vertex, A, B, C, D and E. [2]

5 3 3 5 0

g) Are all of the vertices adjacent to each other? Explain. [2]

No. C not connected directly to B.

[2]

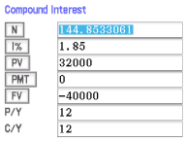
Compound Interest/Depreciation

1) Amos invests $32 000 in an account paying 1.85% interest compounded monthly

a) How much interest is earned in 5 years?

$3098.72

b) How long until the investment is worth $40 000?



2) After buying a car for $48 000, Amos finds that after three years it is only worth $27 150. Determine the average rate of depreciation as a percentage.

17.3% p.a.

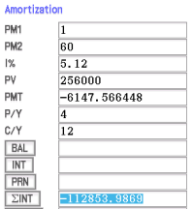
Reducing Balance Loans

Belinda borrows $256 000 over 15 years, with quarterly repayments and interest charged monthly at 5.12% p.a..

a) How much are the repayments?

6147.57

b) How much interest does she pay over the lifetime of the loan?



c) If she increases her repayments by $500 per quarter, how much will she save in time? How much is her final repayment?
54 payments, 6x3 = 18 months saved
$3297.87

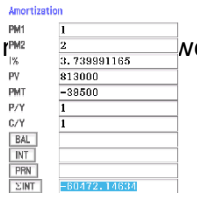
Annuities

Candice receives $813 000 in her grandfather's will. She plans to set up an annuity to receive an annual payment of $39 500 for the next 40 years from a financial account paying interest compounded yearly.

a) Determine the interest rate she requires.

3.74% p.a.

b) How much interest does she earn in two years?



$60472.15

c) Assuming the interest rate above, for how long will the annuity last of she withdraws $50 000 per year?

40

Perpetuities

Desmond plans to implement a scholarship in perpetuity at his former school paying one student $2 000 per month. How much will he need to invest at 2.85 % p.a. compounded monthly?

$842105.26

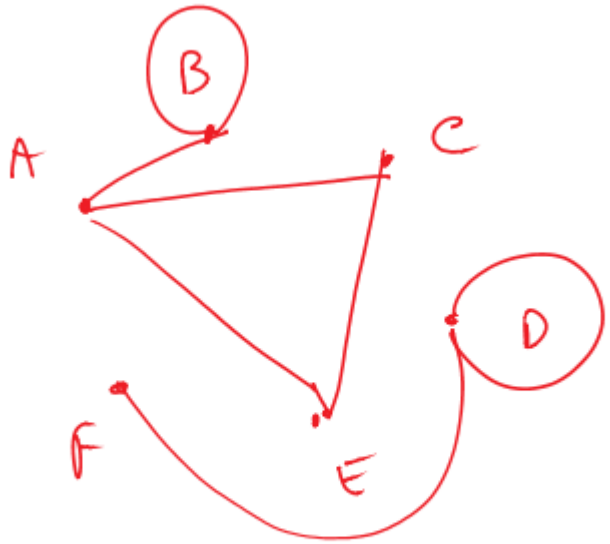
***Effective Rate of Interest**

What is the effective annual interest rate Desmond will receive?

2.8875% p.a.

2. Create a graph with V = {A, B, C, D, E, F} and E = {AB, BB, AC, AE, CE, DF, DD}

[4]



Resource Rich Section

3. Max has a windfall of $20000 which he invests at 8.4% p.a., compounded six monthly.
He also invests another $3000 each six months into the account.

a) Write a recursive relation to model this situation.

[3]

$T_{n+1} = T_n \times (1 + \frac{8.4}{2}) + 3000$, $T_0 = 20\,000$

(1.042)

b) How much is the account worth after the $3000 payment is made at the end of the third year?

[2]

$45599.01

c) How much interest has been earned in this time?

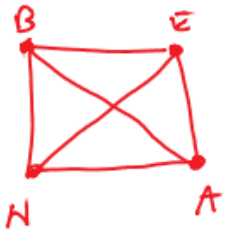
[2]

$20000 + 6 \times 3000$

$= 38000$

$7599.01

a) Draw a complete graph with 4 vertices, B, E, A and N. [2]



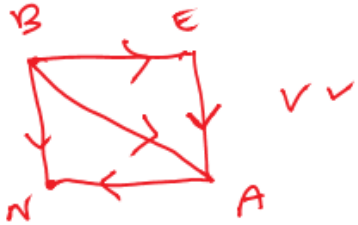
b) Use the complete graph rule to determine the number of edges in part a). [2]

Edges $= \frac{4(4-1)}{2}$

$= 6$

$\frac{n(n-1)}{2}$

a) Draw a directed graph that shows B influences E, A and N, E influences A, and A influences N [2]

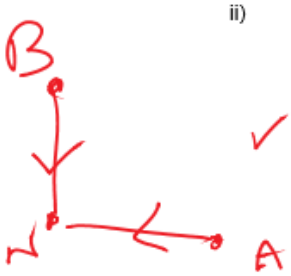


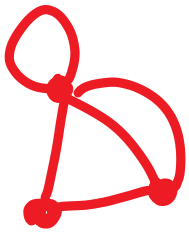
b) Are all of the vertices in a) adjacent to each other? Explain. [2]

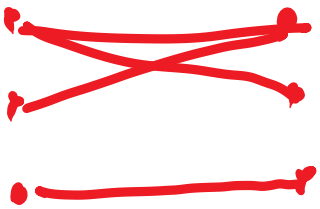
No. E is not adjacent to N.

c) Draw two subgraphs from the directed graph in part a), showing just the influences on: [2]

i) N ii) B







EXAMPLE 16

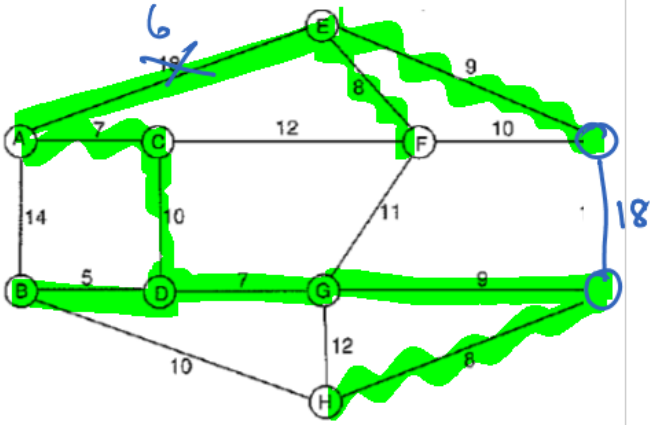
BACHDGFE (Also a cycle)
DHIJGFEABCD

EXAMPLE 17

Consider the network on the right where the numbers represent the distance, in kilometres, between adjacent vertices.

(a) Find the length of the minimum spanning tree of the given network, clearly indicating the tree on the diagram right. 74 km

(b) An error was made in measuring the distance between A and E. The correct distance is 6 km and not 18 km. How does this change length of the minimum spanning tree? Justify.



↓ by 5 no longer need GF
⇒ change to AE 6.

	1 Ex	3 K	5 MB	N	7 P	4 PH	2 O	6 W
Exmouth	-	315	600	605	400	510	110	440
Karratha	315	-	320	420	290	195	210	230
Marble Bar	600	320	-	250	315	153	490	190
Newman	605	420	250	-	220	365	520	195
Paraburdoo	400	290	315	220	-	345	320	130
Port Hedland	510	195	153	365	345	-	400	220
Onslow	110	210	490	520	320	400	-	350
Wittenoom	440	230	190	195	130	220	350	-

1. Ex - O 110
2. O - K 210
3. K - PH 195
4. PH - MB 153
5. MB - W 190
6. W - P 130
7. W - N 195

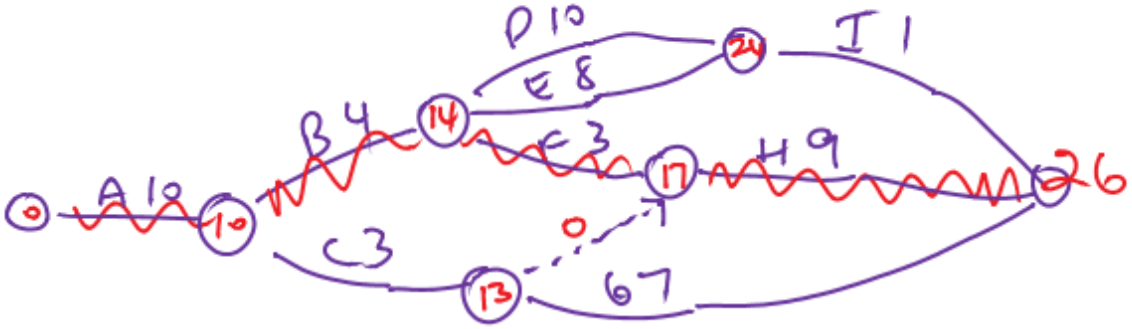
1183 km

EXAMPLE 18

1. The project below consists of activities A to I.

Task	Duration (days)	Predecessor(s)	Earliest Start Time	Latest Start Time	Float Time
A	10	-	0	0	0
B	4	A	10	10	0
C	3	A	10	14	4
D	10	B	14	15	1
E	8	B	14	17	3
F	3	B	14	14	0
G	7	C	13	19	6
H	9	C,F	17	17	0
I	1	D,E	24	25	1

(a) Draw a project network given the information above. [3]



(b) Determine the critical path and minimum completion time. [2]

A B F H 26 days.

(c) Complete the table. [4]

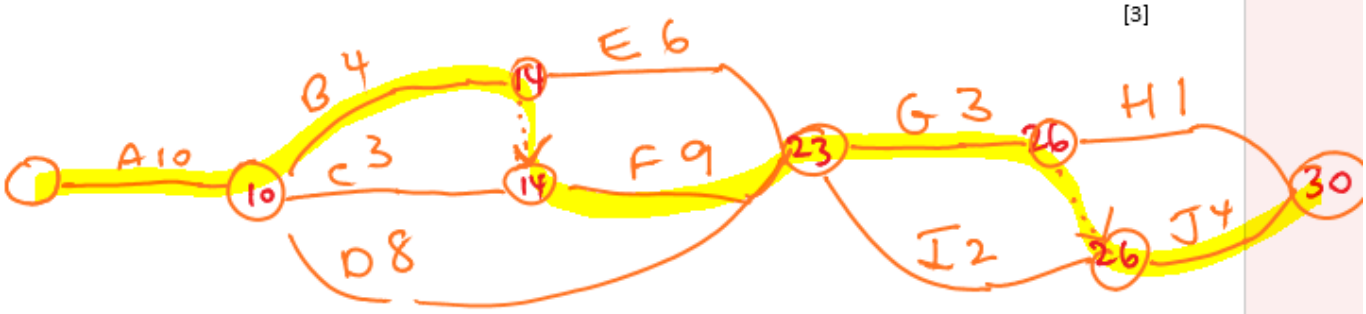
(d) How long can task C be delayed without effecting the minimum completion time? [1]

4 days

2. The project below consists of activities A to J.

Task	Duration (days)	Predecessor(s)	Earliest Start Time	Latest Start Time	Float Time
A	10	-	0	0	0
B	4	A	10	10	0
C	3	A	10	11	1
D	8	A	10	15	5
E	6	B	14	17	3
F	9	B,C	14	14	0
G	3	D,E,F	23	23	0
H	1	G	26	29	3
I	2	D,E,F	23	24	1
J	4	G,I	26	26	0

(a) Draw a project network given the information above. [3]



(b) Determine the critical path and minimum completion time. [2]

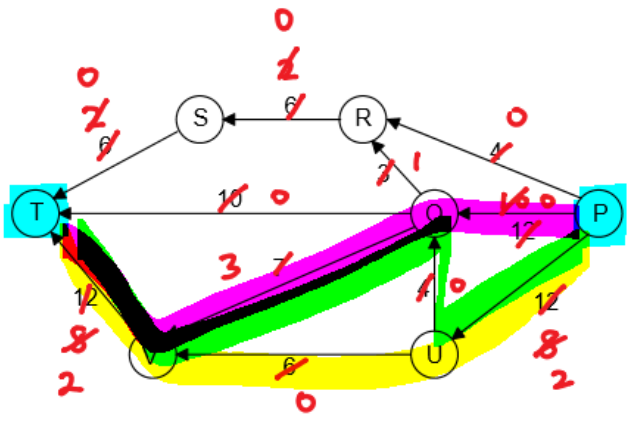
ABFGJ 30 days.

(c) Complete the table. [4]

(d) How long can task E be delayed without effecting the minimum completion time? [1]

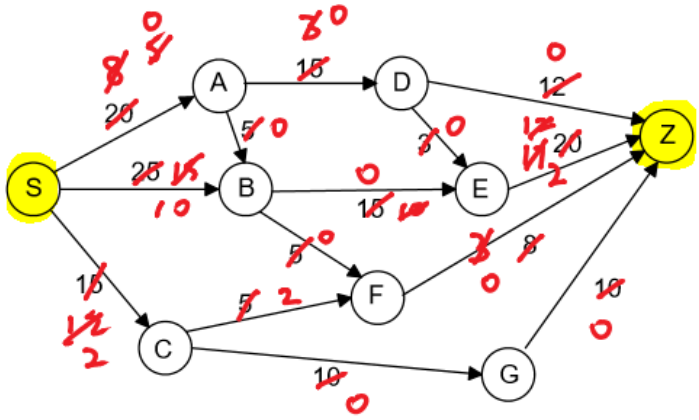
3 days.

Example 19

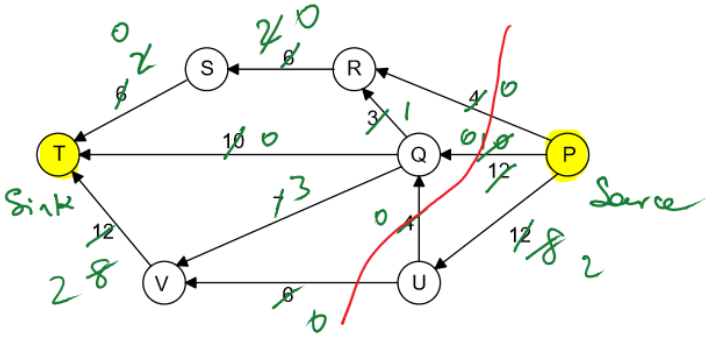


PRST 4
PQRST 2
PQT 10
PUQVT 4
PUVT 6
26 00t/day

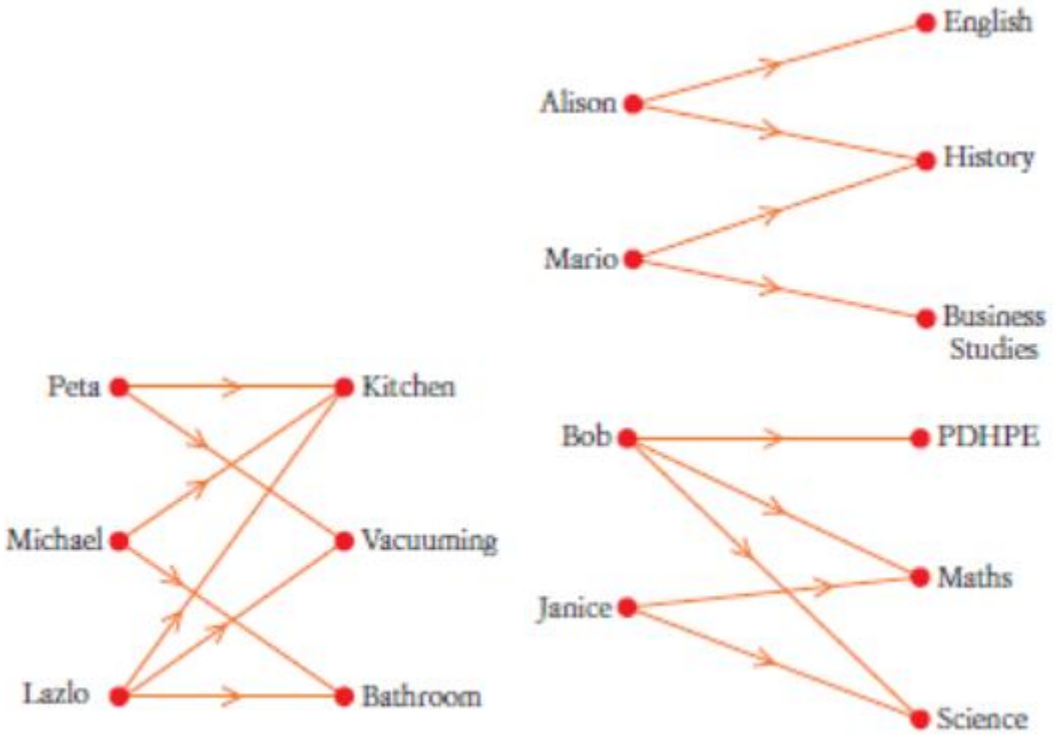
qua by 2
↑ uv by 2
↑ PQ by 2



SADZ 12
SADGZ 3
SABGZ 5
SBEZ 10
SBFZ 5
SCFZ 3
SCGZ 10
48



EXAMPLE 20



*E*1:Task 1; *E*2: Task 3; *E*3: Task 2; *E*4: Task 4.

a $\begin{bmatrix} 4 & 0 & 1 \\ 0 & 1 & 0 \\ 5 & 0 & 3 \end{bmatrix}$

b $\begin{bmatrix} 3 & 0 & 0 \\ 0 & 2 & 0 \\ 4 & 0 & 2 \end{bmatrix}$

c *A*3, *B*1, *C*2

d 18 minutes

a *A*2, *B*1, *C*4, *D*3

b 45 km