Edith Cowan University

2022 ATAR Revision Seminar

ATAR Mathematics Applications

Curriculum Dot points

Examination and study tips

Revision notes Examination questions

Examination marker comments

Prepared and presented by

Sean Ebert

Statistics 1

- The association and relationship between two variables
- Explanatory and response variable

Sequences 1

- Recursive and general terms
- Arithmetic, geometric and first order recurrence relations

Networks 1

- Terminology and definitions

Statistics 2

- Time series
- Seasonal adjustment and prediction

Sequences 2

- Finance: compound interest, reducible interest, annuities
- eActivity, Sequence and Financial

Networks 2

- Minimum spanning trees - Prim's algorithm, gossip
- Project network - slow cooker
- Maximum flow - freeway
- Hungarian algorithm - The Bachelor

Mr Ebert

Six Big Ideas

eActivity – Communication Setup – Open – 3pt Cable

4 Hours – 5 minute breaks at 9:30am, 10:30am and 11:30am

https://charliewatson.com/casio/cpatar.php

Yr12 Apps/General U3&4

Versions for new CPII

- AppsU34-(eActivity)

Applications Unit 3 Content

An understanding of the Year 11 content is assumed knowledge for students in Year 12. It is recommended that students studying Unit 3 and Unit 4 have completed Unit 1 and Unit 2.

This unit includes the knowledge, understandings and skills described below. This is the examinable content.

Topic 3.1: Bivariate data analysis (20 hours)

The statistical investigation process

3.1.1 review the statistical investigation process: identify a problem; pose a statistical question; collect or obtain data; analyse data; interpret and communicate results

Identifying and describing associations between two categorical variables

3.1.2 construct two-way frequency tables and determine the associated row and column sums and percentages

3.1.3 use an appropriately percentaged two-way frequency table to identify patterns that suggest the presence of an association

3.1.4 describe an association in terms of differences observed in percentages across categories in a systematic and concise manner, and interpret this in the context of the data

Identifying and describing associations between two numerical variables

3.1.5 construct a scatterplot to identify patterns in the data suggesting the presence of an association

3.1.6 describe an association between two numerical variables in terms of direction (positive/negative), form (linear/non-linear) and strength (strong/moderate/weak)

3.1.7 calculate, using technology, and interpret the correlation coefficient (r) to quantify the strength of a linear association

Fitting a linear model to numerical data

3.1.8 identify the response variable and the explanatory variable for primary and secondary data

3.1.9 use a scatterplot to identify the nature of the relationship between variables

3.1.10 model a linear relationship by fitting a least-squares line to the data

3.1.11 use a residual plot to assess the appropriateness of fitting a linear model to the data

3.1.12 interpret the intercept and slope of the fitted line

3.1.13 use the coefficient of determination to assess the strength of a linear association in terms of the explained variation

3.1.14 use the equation of a fitted line to make predictions

3.1.15 distinguish between interpolation and extrapolation when using the fitted line to make predictions, recognising the potential dangers of extrapolation

3.1.16 write up the results of the above analysis in a systematic and concise manner

Association and causation

3.1.17 recognise that an observed association between two variables does not necessarily mean that there is a causal relationship between them

3.1.18 identify possible non-causal explanations for an association, including coincidence and confounding due to a common response to another variable, and communicate these explanations in a systematic and concise manner

The data investigation process

3.1.19 implement the statistical investigation process to answer questions that involve identifying, analysing and describing associations between two categorical variables or between two numerical variables

Topic 3.2: Growth and decay in sequences (15 hours)

The arithmetic sequence

3.2.1 use recursion to generate an arithmetic sequence

3.2.2 display the terms of an arithmetic sequence in both tabular and graphical form and demonstrate that arithmetic sequences can be used to model linear growth and decay in discrete situations

3.2.3 deduce a rule for the n^{th} term of a particular arithmetic sequence from the pattern of the terms in an arithmetic sequence, and use this rule to make predictions

3.2.4 use arithmetic sequences to model and analyse practical situations involving linear growth or decay

The geometric sequence

3.2.5 use recursion to generate a geometric sequence

3.2.6 display the terms of a geometric sequence in both tabular and graphical form and demonstrate that geometric sequences can be used to model exponential growth and decay in discrete situations

3.2.7 deduce a rule for the n^{th} term of a particular geometric sequence from the pattern of the terms in the sequence, and use this rule to make predictions

3.2.8 use geometric sequences to model and analyse (numerically, or graphically only) practical problems involving geometric growth and decay

Sequences generated by first-order linear recurrence relations

3.2.9 use a general first-order linear recurrence relation to generate the terms of a sequence and to display it in both tabular and graphical form

3.2.10 generate a sequence defined by a first-order linear recurrence relation that gives long term increasing, decreasing or steady-state solutions

3.2.11 use first-order linear recurrence relations to model and analyse (numerically or graphically only) practical problems

Topic 3.3: Graphs and networks (20 hours)

The definition of a graph and associated terminology

3.3.1 demonstrate the meanings of, and use, the terms: graph, edge, vertex, loop, degree of a vertex, subgraph, simple graph, complete graph, bipartite graph, directed graph (digraph), arc, weighted graph, and network

3.3.2 identify practical situations that can be represented by a network, and construct such networks

3.3.3 construct an adjacency matrix from a given graph or digraph and use the matrix to solve associated problems

Planar graphs

3.3.4 demonstrate the meanings of, and use, the terms: planar graph and face

3.3.5 apply Euler's formula, $v + f - e = 2$ to solve problems relating to planar graphs.

Paths and cycles

3.3.6 demonstrate the meanings of, and use, the terms: walk, trail, path, closed walk, closed trail, cycle, connected graph, and bridge

3.3.7 investigate and solve practical problems to determine the shortest path between two vertices in a weighted graph (by trial-and-error methods only)

3.3.8 demonstrate the meanings of, and use, the terms: Eulerian graph, Eulerian trail, semi-Eulerian graph, semi-Eulerian trail and the conditions for their existence, and use these concepts to investigate and solve practical problems

3.3.9 demonstrate the meanings of, and use, the terms: Hamiltonian graph and semi-Hamiltonian graph, and use these concepts to investigate and solve practical problems

Applications Unit 4 Content

This unit builds on the content covered in Unit 3.

This unit includes the knowledge, understandings and skills described below. This is the examinable content.

Topic 4.1: Time series analysis (15 hours)

Describing and interpreting patterns in time series data

4.1.1 construct time series plots

4.1.2 describe time series plots by identifying features such as trend (long term direction), seasonality (systematic, calendar-related movements), and irregular fluctuations (unsystematic, short term fluctuations), and recognise when there are outliers

Analysing time series data

4.1.3 smooth time series data by using a simple moving average, including the use of spreadsheets to implement this process

4.1.4 calculate seasonal indices by using the average percentage method

4.1.5 deseasonalise a time series by using a seasonal index, including the use of spreadsheets to implement this process

4.1.6 fit a least-squares line to model long-term trends in time series data

4.1.7 predict from regression lines, making seasonal adjustments for periodic data

The data investigation process

4.1.8 implement the statistical investigation process to answer questions that involve the analysis of time series data

Topic 4.2: Loans, investments and annuities (20 hours)

Compound interest loans and investments

4.2.1 use a recurrence relation to model a compound interest loan or investment and investigate (numerically or graphically) the effect of the interest rate and the number of compounding periods on the future value of the loan or investment

4.2.2 calculate the effective annual rate of interest and use the results to compare investment returns and cost of loans when interest is paid or charged daily, monthly, quarterly or six-monthly

4.2.3 with the aid of a calculator or computer-based financial software, solve problems involving compound interest loans, investments and depreciating assets

Reducing balance loans (compound interest loans with periodic repayments)

4.2.4 use a recurrence relation to model a reducing balance loan and investigate (numerically or graphically) the effect of the interest rate and repayment amount on the time taken to repay the loan

4.2.5 with the aid of a financial calculator or computer-based financial software, solve problems involving reducing balance loans

Annuities and perpetuities (compound interest investments with periodic payments made from the investment)

4.2.6 use a recurrence relation to model an annuity, and investigate (numerically or graphically) the effect of the amount invested, the interest rate, and the payment amount on the duration of the annuity

4.2.7 with the aid of a financial calculator or computer-based financial software, solve problems involving annuities (including perpetuities as a special case)

Topic 4.3: Networks and decision mathematics (20 hours)

Trees and minimum connector problems

4.3.1 identify practical examples that can be represented by trees and spanning trees

4.3.2 identify a minimum spanning tree in a weighted connected graph, either by inspection or by using Prim's algorithm

4.3.3 use minimal spanning trees to solve minimal connector problems

Project planning and scheduling using critical path analysis (CPA)

4.3.4 construct a network to represent the durations and interdependencies of activities that must be completed during the project

4.3.5 use forward and backward scanning to determine the earliest starting time (EST) and latest starting times (LST) for each activity in the project

4.3.6 use ESTs and LSTs to locate the critical path(s) for the project

4.3.7 use the critical path to determine the minimum time for a project to be completed

4.3.8 calculate float times for non-critical activities

Flow networks

4.3.9 solve small-scale network flow problems, including the use of the 'maximum flow-minimum cut' theorem

Assignment problems

4.3.10 use a bipartite graph and/or its tabular or matrix form to represent an assignment/ allocation problem

4.3.11 determine the optimum assignment(s), by inspection for small-scale problems, or by use of the Hungarian algorithm for larger problems

The SIX big ideas in the Year 12 Applications course are:

-
-
-

1. Data Analysis

The statistical investigation process

3.1.1 review the statistical investigation process: identify a problem; pose a statistical question; collect or obtain data; analyse data; interpret and communicate results

Identifying and describing associations between two categorical variables

3.1.2 construct two-way frequency tables and determine the associated row and column sums and percentages

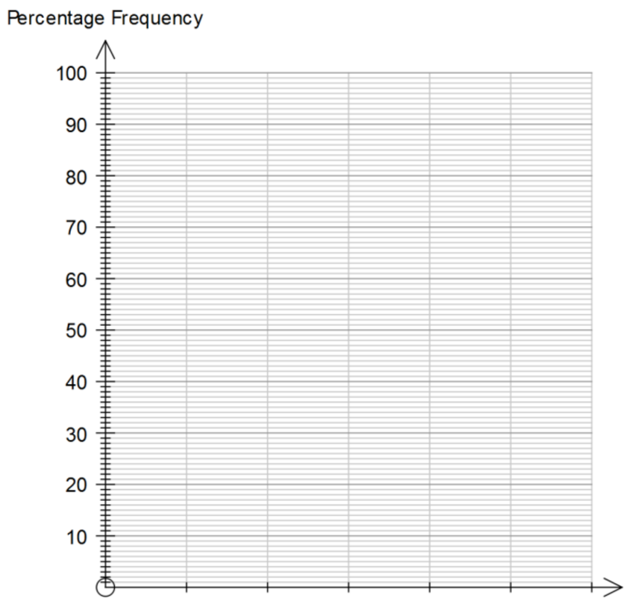
3.1.3 use an appropriately percentaged two-way frequency table to identify patterns that suggest the presence of an association

3.1.4 describe an association in terms of differences observed in percentages across categories in a systematic and concise manner, and interpret this in the context of the data

EXAMPLE 1

In 2015 at a coeducational school in Perth, 180 students received an ATAR. Sixty percent of the ATAR students were female and of these 28 received a score above 90, 70 between 80 and 90 and the rest less than 80. Of the males, 12 scored above 90 and half scored less than 80.

Use the syllabus reference points above to determine whether an association exists.



Identifying and describing associations between two numerical variables

3.1.5 construct a scatterplot to identify patterns in the data suggesting the presence of an association

3.1.6 describe an association between two numerical variables in terms of direction (positive/negative), form (linear/non-linear) and strength (strong/moderate/weak)

3.1.7 calculate, using technology, and interpret the correlation coefficient (*r*) to quantify the strength of a linear association

EXAMPLE 2

Ten people were given two tests, A and B and the marks obtained were as follows:

Test A	1	2	2	3	5	5	8	10	12	12
Test B	14	17	18	19	23	25	29	30	35	40

(a) Plot the data as a scattergraph with the test A results on the horizontal axis and the test B results on the vertical axis.

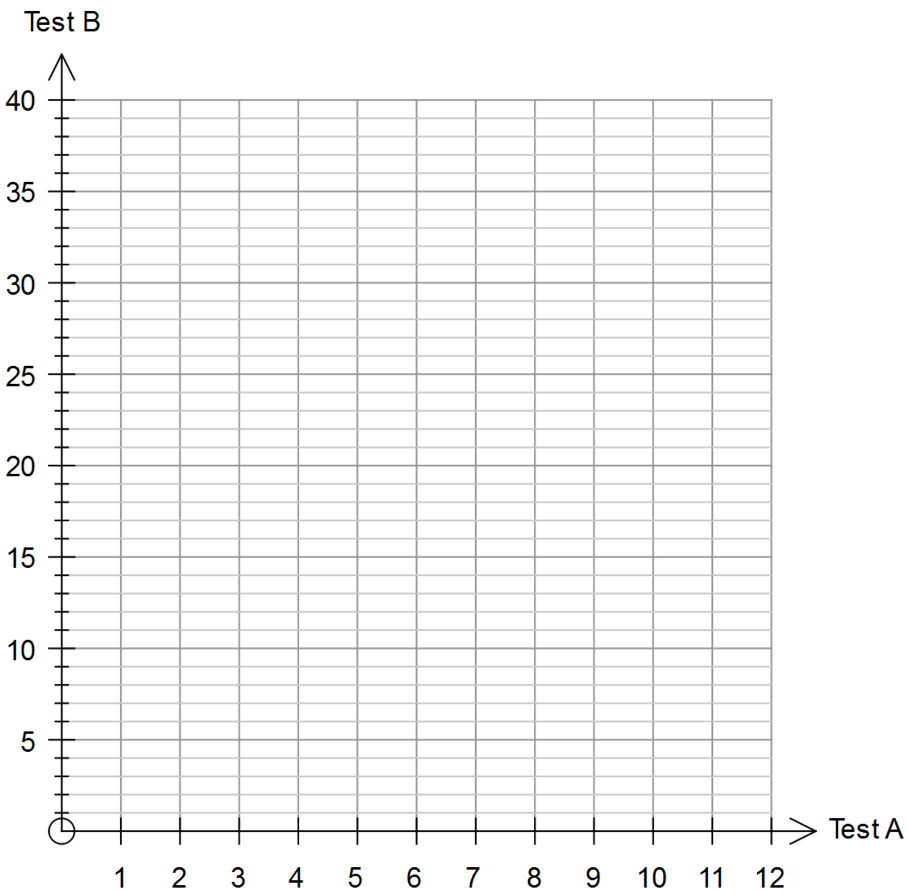
(b) Suggest whether or not the data seems suited to the use of a linear model.

(c) Determine the correlation coefficient for the 10 pairs of results.

(d) Determine the equation of the line of best fit in the form

$$\text{Test B score} = a \times \text{Test A score} + b$$

(e) Predict the Test B score for someone with Test A score of 7 and comment on the validity of your prediction.



Fitting a linear model to numerical data

3.1.13 identify the response variable and the explanatory variable for primary and secondary data

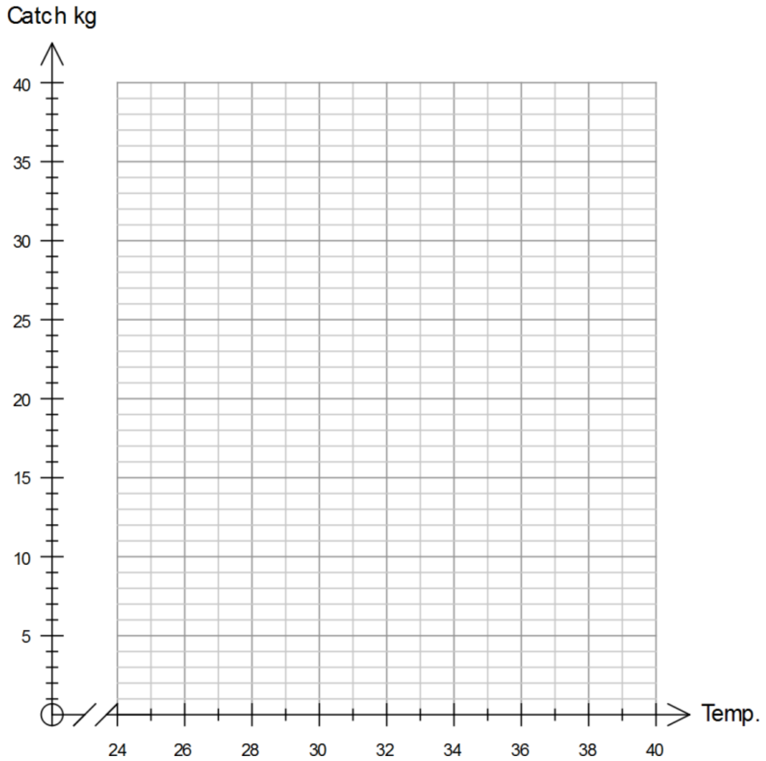
3.1.14 use a scatterplot to identify the nature of the relationship between variables

3.1.15 model a linear relationship by fitting a least-squares line to the data

EXAMPLE 3

The weight of abalone in kilograms caught off Trigg Beach and the daily temperature (°C) is shown below.

Daily Temp. (°C)	26	27.2	27.9	28.4	30.5	32.1	34	36.3	37	33.5
Catch (kg)	39.9	36	32.5	31	28	26	24	23.1	23	25



Comment on the relationship between the two variables using the graph.

Comment on the relationship between the two variables using the correlation coefficient, r.

Determine and draw a line of best fit and hence predict the catch for a 40°C day. Explain how confident you are with your prediction.

What would a residual plot tell you about the relationship between the two variables?

3.1.11 use a residual plot to assess the appropriateness of fitting a linear model to the data

3.1.12 interpret the intercept and slope of the fitted line

3.1.13 use the coefficient of determination to assess the strength of a linear association in terms of the explained variation

3.1.14 use the equation of a fitted line to make predictions

3.1.15 distinguish between interpolation and extrapolation when using the fitted line to make predictions, recognising the potential dangers of extrapolation

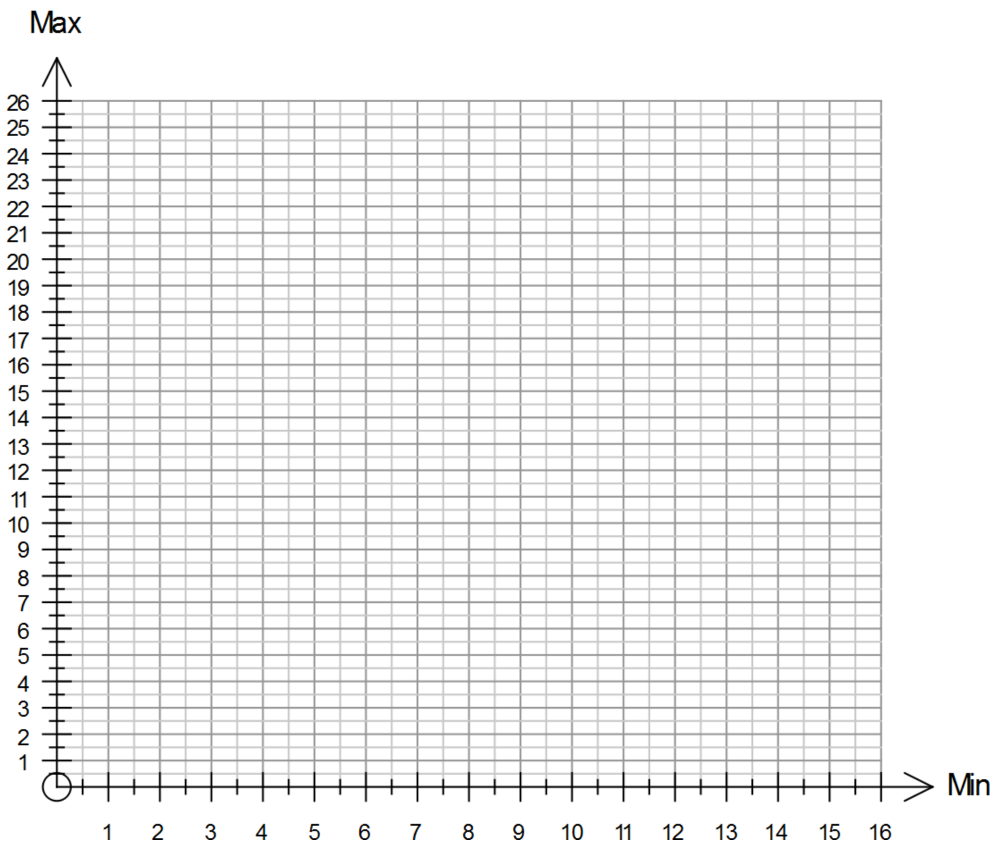
3.1.16 write up the results of the above analysis in a systematic and concise manner

EXAMPLE 4

Data was collected to investigate the relationship between the minimum daily temperature and the maximum daily temperature and is displayed in the table below.

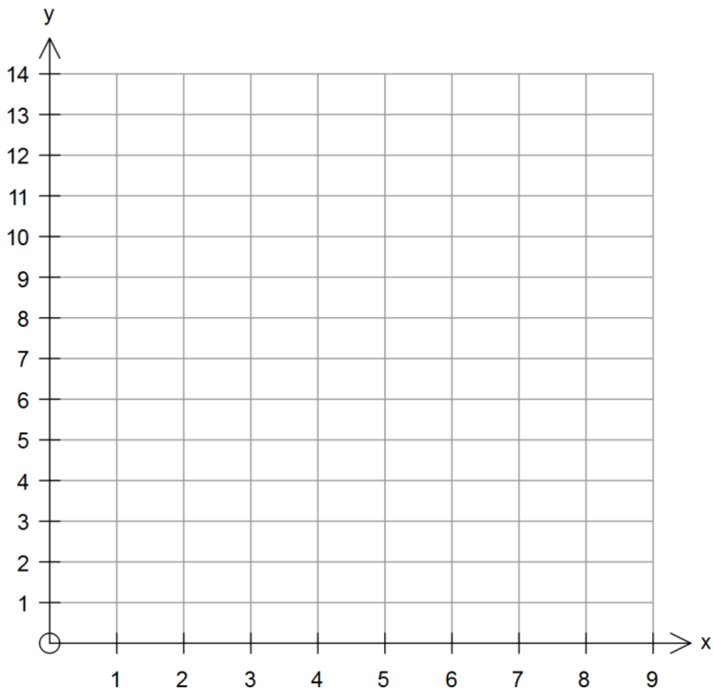
Minimum temperature (°C)	12	15	13.5	14.1	10.2	11.7	13.2	11.3
Maximum temperature (°C)	23.4	25.6	23.1	25.3	20	21.1	22.6	21

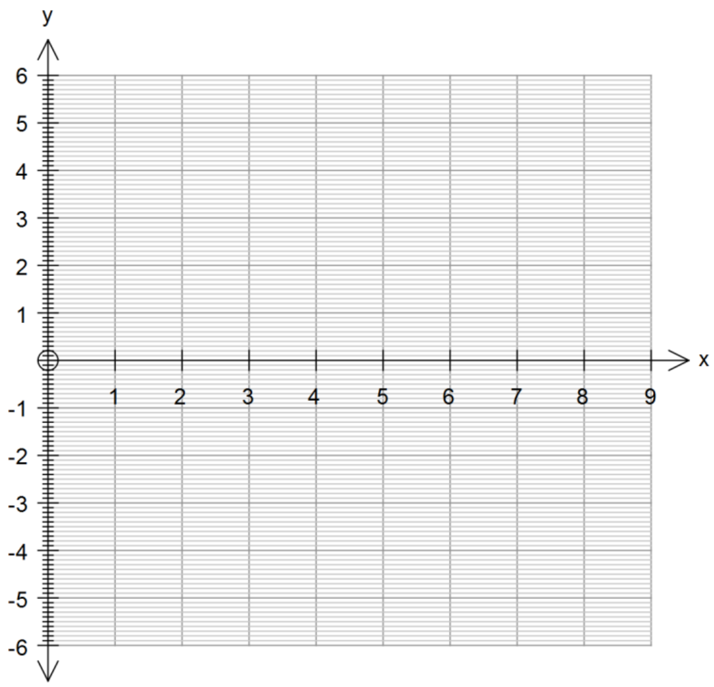
a Assuming that Minimum temperature is the explanatory variable, calculate the coefficient of determination, correct to 3 decimal places.
b Interpret the coefficient of determination.
c State the regression line that models this data.
d Do you think that the model is appropriate? Justify your answer.



EXAMPLE 5

x	3	5	6	1	8	7	1
y	7	9	7	12	2	4	2





Association and causation

3.1.17 recognise that an observed association between two variables does not necessarily mean that there is a causal relationship between them

3.1.18 identify possible non-causal explanations for an association, including coincidence and confounding due to a common response to another variable, and communicate these explanations in a systematic and concise manner

The data investigation process

3.1.19 implement the statistical investigation process to answer questions that involve identifying, analysing and describing associations between two categorical variables or between two numerical variables

Value of r	What it means
1	Perfect positive relationship
$0.75 \leq r < 1$	Strong positive relationship
$0.5 \leq r < 0.75$	Moderate positive relationship
$0.25 \leq r < 0.5$	Weak positive relationship
$-0.25 < r < 0.25$	No relationship
$-0.5 < r \leq -0.25$	Weak negative relationship
$-0.75 < r \leq -0.5$	Moderate negative relationship
$-1 < r \leq 0.75$	Strong negative relationship
-1	Perfect negative relationship

Describing and interpreting patterns in time series data

4.1.2 construct time series plots

4.1.2 describe time series plots by identifying features such as trend (long term direction), seasonality (systematic, calendar-related movements), and irregular fluctuations (unsystematic, short term fluctuations), and recognise when there are outliers

EXAMPLE 6

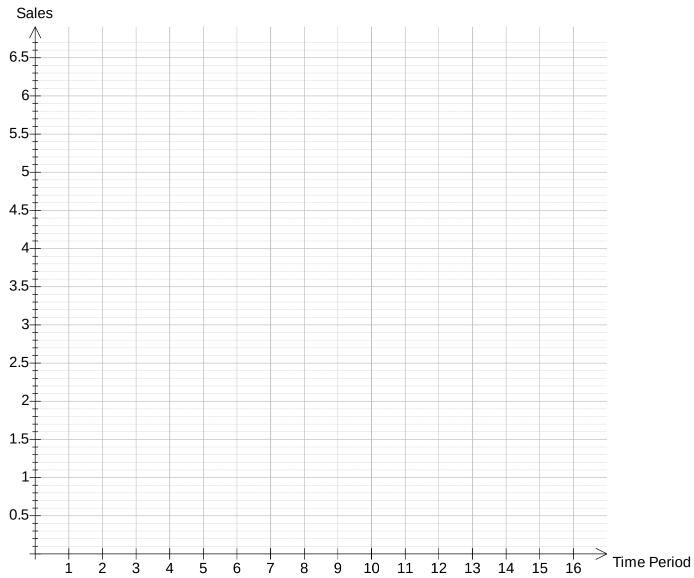
Time Period	Quarter	Ice Cream Sales (000s kg)	
1	Mar-13	2.1	
2	Jun-13	1.6	
3	Sep-13	3.0	
4	Dec-13	3.1	
5	Mar-14	3.2	
6	Jun-14	2.2	
7	Sep-14	4.0	
8	Dec-14	4.1	
9	Mar-15	4.1	
10	Jun-15	3.0	
11	Sep-15	5.5	
12	Dec-15	5.7	

Construct a time series plot on the axes below.

Trend

Seasonality

Irregular fluctuations



Analysing time series data

4.1.3 smooth time series data by using a simple moving average, including the use of spreadsheets to implement this process

4.1.5 calculate seasonal indices by using the average percentage method

4.1.5 deseasonalise a time series by using a seasonal index, including the use of spreadsheets to implement this process

4.1.7 fit a least-squares line to model long-term trends in time series data

4.1.7 predict from regression lines, making seasonal adjustments for periodic data

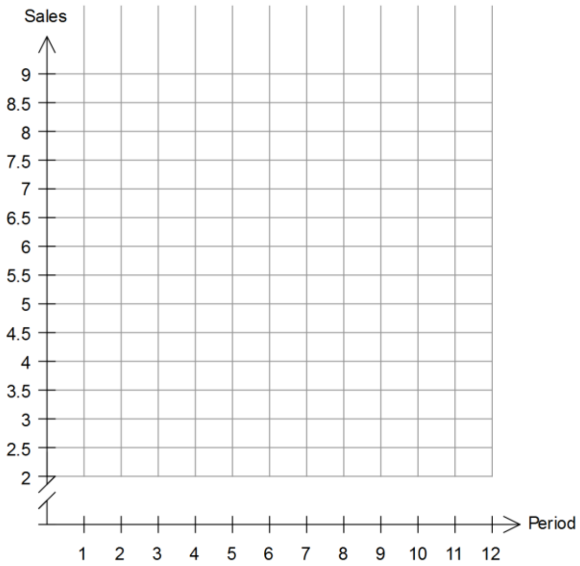
The data investigation process

4.1.8 implement the statistical investigation process to answer questions that involve the analysis of time series data

EXAMPLE 7

1. Sales figures of the 'HOMER' automobile are shown in the table and graph below.

Time Period (t)	Year	Sales (000's)	**A** pt moving Average
1	1992	6.3	
2	1993	2.1	**B**
3	1994	7.5	**C**
4	1995	6.8	**D**
5	1996	3	5.9
6	1997	8	6.0
7	1998	6.9	6.1
8	1999	3.4	6.4
9	2000	9	**E**
10	2001	8	7.5
11	2002	5.5	7.4
12	2003	**F**	



a) Plot the known sales figures on the axes. [1]

b) Comment on the seasonality of the data. [2]

c) What is the period of the data? [1]

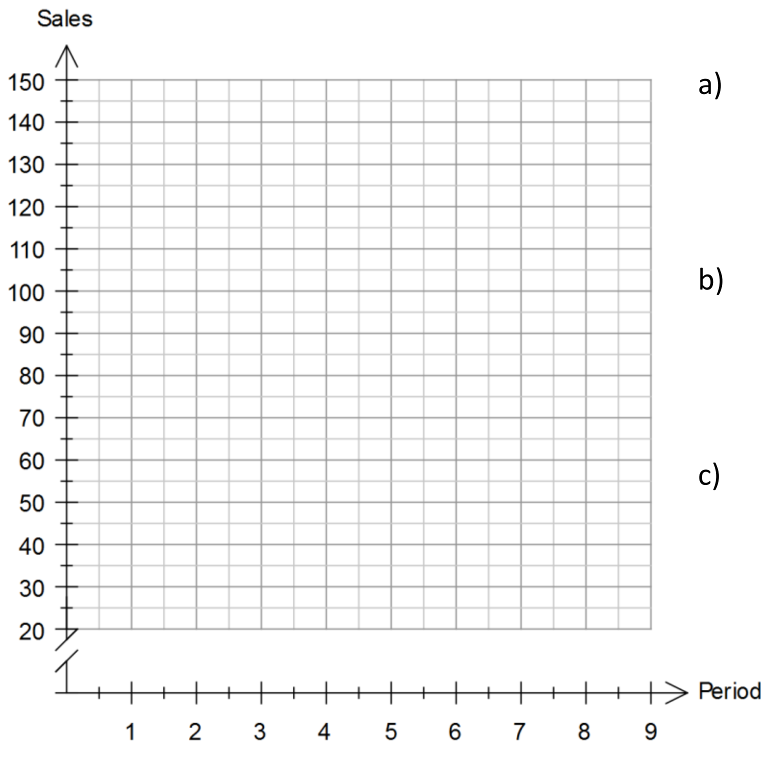
d) Determine the values of **A**, **B**, **C**, **D**, **E** and **F** in the table above.

e) Plot the moving averages on the axes above. Describe the trend of the data.

EXAMPLE 8

Saturday Muffin sales at Café de Pascal are shown below:

Date	Shift	Period	Muffins sold	3pt MA	Percentage of Daily Mean	Seasonally adjusted
2 July	Morning	1	145			
	Afternoon	2	112			
	Night	3	34			
9 July	Morning	4	137			
	Afternoon	5	121			
	Night	6	27			
16 July	Morning	7	129			
	Afternoon	8	118			
	Night	9	24			



a) Plot the muffin sales figures on the axes. [1]

b) Comment on the seasonality of the data. [2]

c) What is the period of the data? [1]

d) Complete all values in the table above.

Average for each cycle

Cycle			
Average			

Percentage of daily mean

Seasonal index

Morning	
Afternoon	
Night	

Seasonally adjusted figures

2. Sequences and Finance

The arithmetic sequence

3.2.2 use recursion to generate an arithmetic sequence

3.2.2 display the terms of an arithmetic sequence in both tabular and graphical form and demonstrate that arithmetic sequences can be used to model linear growth and decay in discrete situations

3.2.3 deduce a rule for the n^{th} term of a particular arithmetic sequence from the pattern of the terms in an arithmetic sequence, and use this rule to make predictions

3.2.4 use arithmetic sequences to model and analyse practical situations involving linear growth or decay

EXAMPLE 9

Resource Free Section

1. For each of the following sequences, find the first five terms (DRAW A TABLE):

a) $t_{n+1} = t_n - 4, \quad t_1 = 8$

[4]

n	
t_n	

b) $b_n = b_{n-1} + 7, \quad b_1 = 20$

[4]

c) $t_{n+3} = 2 + t_{n+2}, \quad t_1 = 5$

[4]

d) $g_n - g_{n-1} = -7, \quad g_1 = 4$

[4]

2. For each of the following, determine a recursive equation, a general equation for the n^{th} term and hence determine T_{10} and T_{100}.

a) 5, 9, 13, 17, …

[6]

b) 50, 40, 30, 20, …

[6]

c) 4, -2, -8, -14, …

[6]

3. Show clearly if 151 a term in the sequence -5, 1, 7, 13, …? If so, which term?

[4]

4. Which is the first term of the sequence 3, 11, 19, … that is larger than 500? What is the term?

[4]

The geometric sequence

3.2.6 use recursion to generate a geometric sequence

3.2.6 display the terms of a geometric sequence in both tabular and graphical form and demonstrate that geometric sequences can be used to model exponential growth and decay in discrete situations

3.2.7 deduce a rule for the n^{th} term of a particular geometric sequence from the pattern of the terms in the sequence, and use this rule to make predictions

3.2.8 use geometric sequences to model and analyse (numerically, or graphically only) practical problems involving geometric growth and decay

EXAMPLE 10

1. For each of the following sequences, determine

   i) the first five terms,

   ii) whether the sequence is arithmetic, geometric or neither,

   iii) a general term (if the sequence is arithmetic or geometric)

   iv) T_{10} using the general term

   v) The sum of the first 10 terms, S_{10}.

   a) $t_{n+1} = t_n + 4, \quad t_1 = -2$

[5]

n	
t_n	

   b) $b_n = 10b_{n-1}, \quad b_1 = 4$

[5]

   c) $t_{n+3} = 2t_{n+1} - t_{n+2}, \quad t_1 = 3, t_2 = 6$

[5]

d) 50, 25, 12.5, …

[5]

e) 4, -6, 9, …

[5]

2. A geometric sequence has a second term of 30 and a fifth term of 101.25.
Determine a general term for the sequence.

[4]

Sequences generated by first-order linear recurrence relations

3.2.9 use a general first-order linear recurrence relation to generate the terms of a sequence and to display it in both tabular and graphical form

3.2.10 generate a sequence defined by a first-order linear recurrence relation that gives long term increasing, decreasing or steady-state solutions

3.2.11 use first-order linear recurrence relations to model and analyse (numerically or graphically only) practical problems

EXAMPLE 11

Three recursion relations are given below.

$$A_{n+1} = A_n + 7, A_1 = 2$$
$$B_n = 2B_{n-1} - 3, B_1 = 5$$
$$C_{n+1} = 0.7C_n + 14, C_1 = 20$$

a) Determine the first 5 terms of each sequence.

[3]

b) Only one of the recursion relations has a steady-state solution. Explain clearly which one it is and why the others do not have a steady-state solution.

[2]

c) Determine the steady-state solution for the appropriate recursion relation above.

[3]

2. (5 marks)

A grain store has 4 tonnes of wheat and an extra 3 tonnes is added every month. If 25% of the grain is removed every month, determine the amount of grain that will eventually remain in the grain store.

Compound interest loans and investments

4.2.1 use a recurrence relation to model a compound interest loan or investment and investigate (numerically or graphically) the effect of the interest rate and the number of compounding periods on the future value of the loan or investment

4.2.2 calculate the effective annual rate of interest and use the results to compare investment returns and cost of loans when interest is paid or charged daily, monthly, quarterly or six-monthly

4.2.3 with the aid of a calculator or computer-based financial software, solve problems involving compound interest loans, investments and depreciating assets

EXAMPLE 12

1. Write recurrence relations for each of the following situations:

   a) $40,000 invested at interest rate of 3.5% p.a., compounded yearly. [2]

   b) $62,500 invested at interest rate of 4.25% p.a., compounded monthly. [2]

   c) $850 invested at interest rate of 2.34% p.a., compounded 3 monthly. [2]

2. Determine the amount of interest earned in each of the following scenarios by first writing a recurrence relation.

   a) $80,000 invested for 10 years at an interest rate of 4.3% p.a., compounded yearly. [3]

   b) $350,000 invested for 40 months at an interest rate of 6.15% p.a., compounded monthly. [3]

3. Judy invests $120,000 at 5.1% p.a. for 16 years, compounded six monthly.
Show working to answer:

a) How much is the investment worth after 16 years?

[3]

b) What is the difference between what the investment earns in the last four years, compared to the first twelve years?

[3]

The effective annual rate of interest $i_{effective}$ is used to compare the interest paid on loans (or investments) with the same nominal annual interest rate i but with different compounding periods (daily, monthly, quarterly, annually, other).

What does this mean?

Complete the following table, using the eActivity, when $100 is invested for one year for different compounding periods.

	n	5% p.a.	Effective	6.5% p.a.	Effective	11% p.a.	Effective
Annual							
6 Monthly							
Quarterly							
Monthly							
Weekly							
Daily							

Reducing balance loans (compound interest loans with periodic repayments)

4.2.4 use a recurrence relation to model a reducing balance loan and investigate (numerically or graphically) the effect of the interest rate and repayment amount on the time taken to repay the loan

4.2.5 with the aid of a financial calculator or computer-based financial software, solve problems involving reducing balance loans

EXAMPLE 13

1. A loan is repaid according to the schedule below:

Period	Date	Owing	Interest	Repayment	Owing
1	Jan-16	$430,000.00	$ 15,480.00	$ 32,000.00	$ 413,480.00
2	Jul-16	$413,480.00	$ 14,885.28	$ 32,000.00	$ 396,365.28
3	Jan-17	$396,365.28	$ 14,269.15	$ 32,000.00	$ 378,634.43
4	Jul-17	$378,634.43	$ 13,630.84	$ 32,000.00	$ 360,265.27
5	Jan-18	$360,265.27	$ 12,969.55	$ 32,000.00	$ 341,234.82

a) Write a recurrence relation for this mortgage.

[2]

b) When is the mortgage paid off?

[1]

c) Show working to determine the final repayment.

[2]

d) Determine the amount of interest paid on the loan.

[2]

e) If all other parameters stay the same, how much should be repaid per time period so that the loan is paid off in exactly seven years?

[1]

2. Nora borrows $756,000 from a bank at 4.25% p.a. compound interest, adjusted monthly.
Find the:

a) monthly repayment if the loan is paid in 25 years,

[1]

b) total amount repaid on the loan,

[1]

c) amount of interest paid on the loan.

[1]

3. Guptill borrows $109,000 at 8.45% p.a. adjusted monthly. If I takes him 8 years to pay off the loan, how much will he have paid off his loan at the end of the third year?

[3]

4. Captain Starlight borrows $80,000 at 9.3% p.a. adjusted monthly to be repaid in exactly 10 years.

a) How much interest does he pay?

[2]

b) If he pays $65 a month extra, how much will he save over the lifetime of the loan?

[4]

Annuities and perpetuities (compound interest investments with periodic payments made from the investment)

4.2.6 use a recurrence relation to model an annuity, and investigate (numerically or graphically) the effect of the amount invested, the interest rate, and the payment amount on the duration of the annuity

4.2.7 with the aid of a financial calculator or computer-based financial software, solve problems involving annuities (including perpetuities as a special case)

EXAMPLE 14

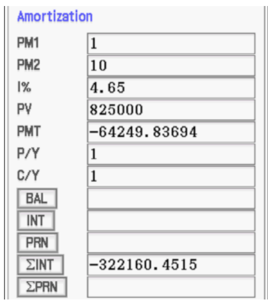
Nic's everyday banking account is accidently credited with $825,000. He immediately removes the money to an off shore account and sets up an annuity. Answer the questions below for the given conditions:

a) If the annuity receives an annual interest rate of 4.65% p.a., how much can he withdraw per year if he wants it to last exactly 20 years?

[1]

b) In the situation in a), how much interest has he received after 10 years?

[2]

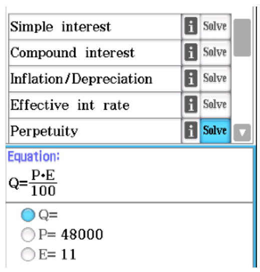


c) If he removes $6,000 per month with interest rate of 5.04% p.a. added monthly, how long will the annuity last and how much will his last withdrawal be?

[3]

d) How much can he remove per year if he wants the perpetuity to last forever?

[2]



Compound Interest/Depreciation

1) Amos invests $32 000 in an account paying 1.85% interest compounded monthly

a) How much interest is earned in 5 years?

b) How long until the investment is worth $40 000?

2) After buying a car for $48 000, Amos finds that after three years it is only worth $27 150. Determine the average rate of depreciation as a percentage.

Reducing Balance Loans

Belinda borrows $256 000 over 15 years, with quarterly repayments and interest charged monthly at 5.12% p.a..

a) How much are the repayments?

b) How much interest does she pay over the lifetime of the loan?

c) If she increases her repayments by $500 per quarter, how much will she save in time? How much is her final repayment?

Annuities

Candice receives $813 000 in her grandfather's will. She plans to set up an annuity to receive an annual payment of $39 500 for the next 40 years from a financial account paying interest compounded yearly.

a) Determine the interest rate she requires.

b) How much interest does she earn in the first two years?

c) Assuming the interest rate above, for how long will the annuity last of she withdraws $50 000 per year?

Perpetuities

Desmond plans to implement a scholarship in perpetuity at his former school paying one student $2 000 per month. How much will he need to invest at 2.85 % p.a. compounded monthly?

***Effective Rate of Interest**

What is the effective annual interest rate Desmond will receive?

Compound Interest/Depreciation

Compound Interest

N	
I%	
PV	
PMT	
FV	
P/Y	
C/Y	

Reducing Balance Loans

Compound Interest

N	
I%	
PV	
PMT	
FV	
P/Y	
C/Y	

$$A_{n+1} = A_n\left(1 + \frac{\text{rate}}{(100 \times \text{n})}\right) - \text{repayment}, \quad A_0 = \text{starting value}$$

Annuities

Compound Interest

N	
I%	
PV	
PMT	
FV	
P/Y	
C/Y	

Perpetuities

Compound Interest

N	
I%	
PV	
PMT	
FV	
P/Y	
C/Y	

3. Graphs and Networks

The definition of a graph and associated terminology

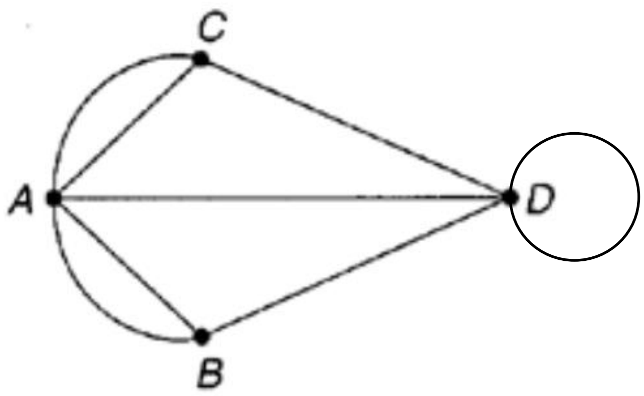
3.3.1 demonstrate the meanings of, and use, the terms: graph, edge, vertex, loop, degree of a vertex, subgraph, simple graph, complete graph, bipartite graph, directed graph (digraph), arc, weighted graph, and network

3.3.2 identify practical situations that can be represented by a network, and construct such networks

3.3.3 construct an adjacency matrix from a given graph or digraph and use the matrix to solve associated problems

EXAMPLE 15

1.



Given the graph above, answer the questions below:

a) List all the vertices.

[1]

b) Add an isolated vertex E.

[1]

c) How many arcs are there?

[1]

d) Clearly label any loops and multiple edges on the graph.

[2]

e) Give a definition of a graph and explain when it becomes a network.

[2]

f) Give the degree of each vertex, A, B, C, D and E. [2]

g) Are all of the vertices adjacent to each other? Explain. [2]

2. Create a graph with V = {A, B, C, D, E, F} and E = {AB, BB, AC, AE, CE, DF, DD} [4]

3. a) Draw a complete graph with 4 vertices, B, E, A and N. [2]

b) Use the complete graph rule to determine the number of edges in part a). [2]

4. a) Draw a directed graph that shows B influences E, A and N, E influences A, and A influences N

[2]

b) Are all of the vertices in a) adjacent to each other? Explain.

[2]

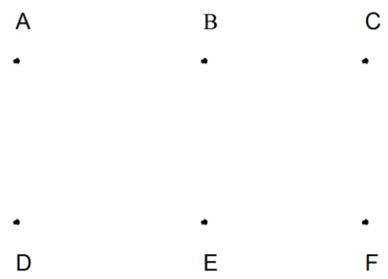
c) Draw two subgraphs from the directed graph in part a), showing just the influences on:

[2]

i) N   ii) B

5. a) Create a graph with V = {A, B, C, D, E, F} and E = {AD, BE, BC, AE, CF, CE, BF}

[2]



b) Is this graph bipartite? If not, which edge should you remove to make it bipartite? Explain.

[3]

6. Show an example of a graph that is:

a) connected but not simple.

[2]

b) bipartite, but not connected

[2]

Planar graphs

3.3.4 demonstrate the meanings of, and use, the terms: planar graph and face

3.3.5 apply Euler's formula, $v + f - e = 2$ to solve problems relating to planar graphs.

Paths and cycles

3.3.6 demonstrate the meanings of, and use, the terms: walk, trail, path, closed walk, closed trail, cycle, connected graph, and bridge

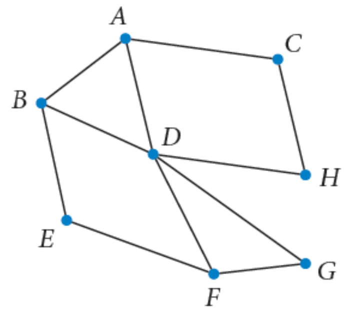
3.3.7 investigate and solve practical problems to determine the shortest path between two vertices in a weighted graph (by trial-and-error methods only)

3.3.8 demonstrate the meanings of, and use, the terms: Eulerian graph, Eulerian trail, semi-Eulerian graph, semi-Eulerian trail and the conditions for their existence, and use these concepts to investigate and solve practical problems

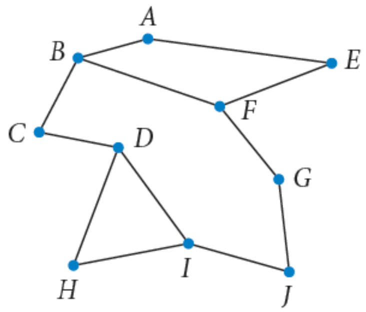
3.3.9 demonstrate the meanings of, and use, the terms: Hamiltonian graph and semi-Hamiltonian graph, and use these concepts to investigate and solve practical problems

EXAMPLE 16

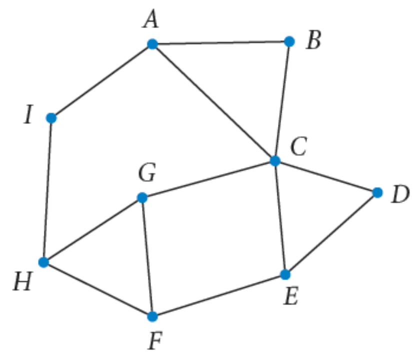
List a Hamiltonian path for the graph below.



List a Hamiltonian cycle for the graph below.



A salesman is travelling to seven country towns to sell flour to the local bakeries. A map of the towns is shown below, with the vertices representing the towns and the edges representing the roads connecting the towns.

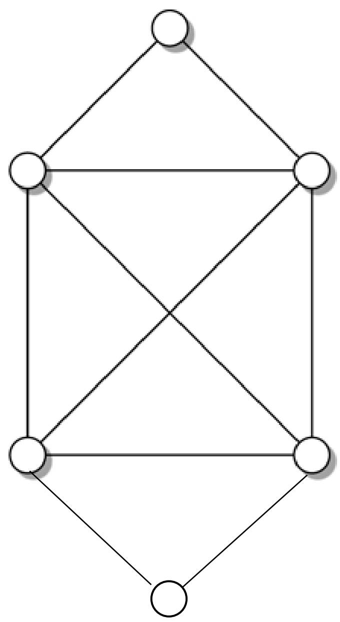


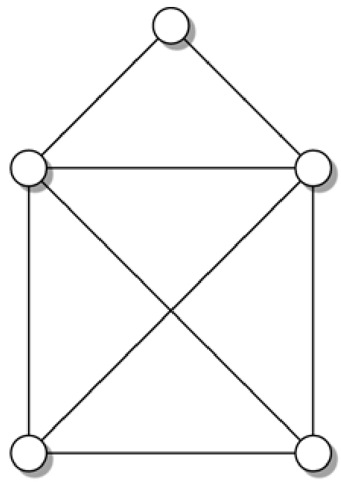
The salesman wants to travel from town A to visit every town once on his journey and then return back to town A.

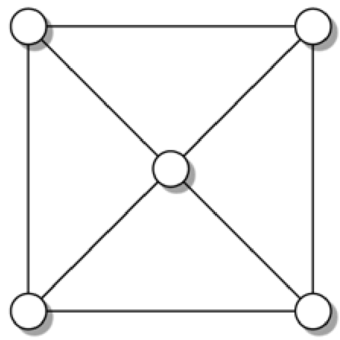
a What is this walk an example of?

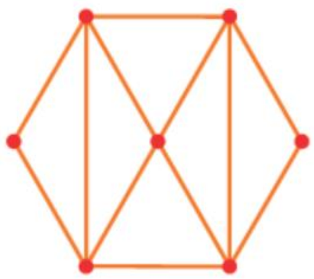
b Describe a route the salesman could take.

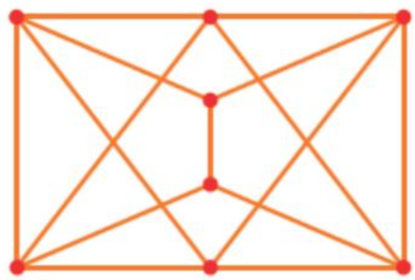
c List the edges that the salesman does not travel on.











Term	Diagram	Definition
GRAPH		
NETWORK		
EDGE/ARC		
ISOLATED VERTEX		
MULTIPLE EDGES		
DEGREE OF A VERTEX		
VERTEX		

Term	Diagram	Definition

Trees and minimum connector problems

Trees and minimum connector problems

4.3.1 identify practical examples that can be represented by trees and spanning trees

4.3.2 identify a minimum spanning tree in a weighted connected graph, either by inspection or by using Prim's algorithm

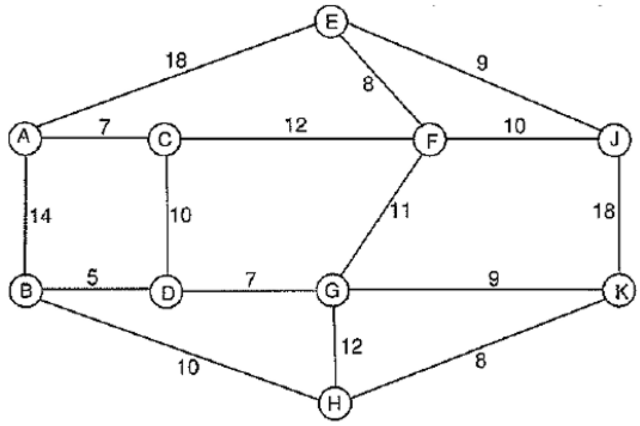
4.3.3 use minimal spanning trees to solve minimal connector problems

EXAMPLE 17

Consider the network on the right where the numbers represent the distance, in kilometres, between adjacent vertices.

(a) Find the length of the minimum spanning tree of the given network, clearly indicating the tree on the diagram right.

(b) An error was made in measuring the distance between A and E. The correct distance is 6 km and not 18 km. How does this change length of the minimum spanning tree? Justify.



The table below gives the flying distances in kilometres between certain towns.

(a) Find the combination of air routes which connects all these towns using the shortest length.

(b) Draw the minimum spanning tree indicating the arc lengths.

	Ex	K	MB	N	P	PH	O	W
Exmouth	-	315	600	605	400	510	110	440
Karratha	315	-	320	420	290	195	210	230
Marble Bar	600	320	-	250	315	153	490	190
Newman	605	420	250	-	220	365	520	195
Paraburdoo	400	290	315	220	-	345	320	130
Port Hedland	510	195	153	365	345	-	400	220
Onslow	110	210	490	520	320	400	-	350
Wittenoom	440	230	190	195	130	220	350	-

Project planning and scheduling using critical path analysis (CPA)

4.3.4 construct a network to represent the durations and interdependencies of activities that must be completed during the project

4.3.5 use forward and backward scanning to determine the earliest starting time (EST) and latest starting times (LST) for each activity in the project

4.3.8 use ESTs and LSTs to locate the critical path(s) for the project

4.3.9 use the critical path to determine the minimum time for a project to be completed

4.3.8 calculate float times for non-critical activities

EXAMPLE 18

1. The project below consists of activities A to I.

Task	Duration (days)	Predecessor(s)	Earliest Start Time	Latest Start Time	Float Time
A	10	-			
B	4	A			
C	3	A			
D	10	B			
E	8	B			
F	3	B			
G	7	C			
H	9	C,F			
I	1	D,E			

(a) Draw a project network given the information above. [3]

(b) Determine the critical path and minimum completion time. [2]

(c) Complete the table. [4]

(d) How long can task C be delayed without effecting the minimum completion time? [1]

2. The project below consists of activities A to J.

Task	Duration (days)	Predecessor(s)	Earliest Start Time	Latest Start Time	Float Time
A	10	-			
B	4	A			
C	3	A			
D	8	A			
E	6	B			
F	9	B,C			
G	3	D,E,F			
H	1	G			
I	2	D,E,F			
J	4	G,I			

(a) Draw a project network given the information above.

[3]

(b) Determine the critical path and minimum completion time.

[2]

(c) Complete the table.

[4]

(d) How long can task E be delayed without effecting the minimum completion time?

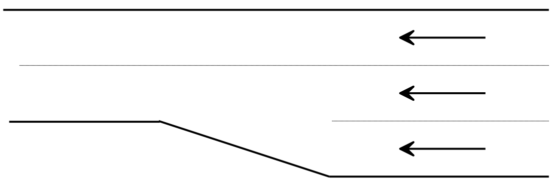
[1]

Flow networks

4.3.9 solve small-scale network flow problems, including the use of the 'maximum flow-minimum cut' theorem

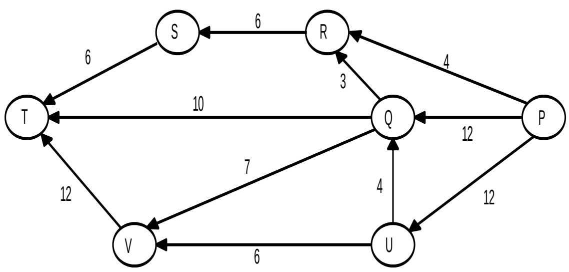
Drainage systems, electricity grids, pipes carrying water and oil and transportation networks (roads, railways, shipping) are concerned with carrying capacity or flow.

The diagram shows a three lane highway in which the two left lanes must merge to cross a bridge.



Example 19

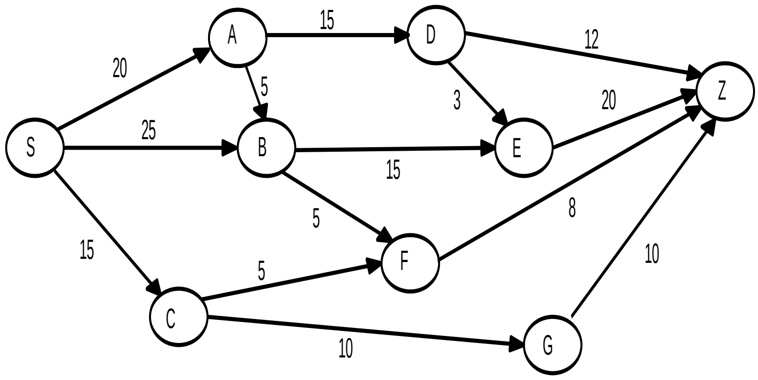
The network shows a rail system which connects the towns P and T. The numbers give the maximum volume of freight (in hundreds of tonnes per day) that can be carried on each section of the system.



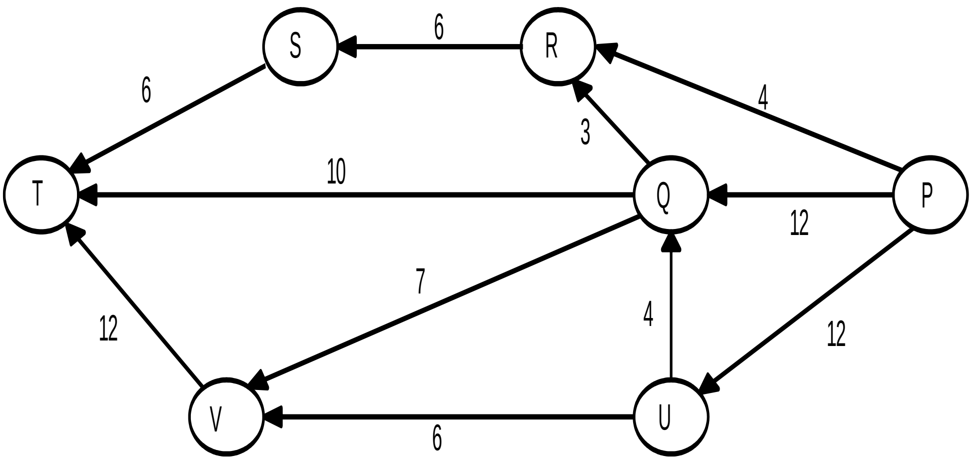
(a) What is the maximum volume of freight that can be carried from P to T per day?

(b) The system needs to be upgraded to increase the capacity by 200 tonnes per day. For practical reasons only one section of the track can be upgraded. Which section should it be?

Find the maximum flow from S to Z through the following network. Which one section could be closed without affecting this flow?



The example of the network showing a rail system which connected the towns P and T had maximum volume of freight that was 2600 tonnes/day. Use cuts to check this result.



Assignment problems

4.3.12 use a bipartite graph and/or its tabular or matrix form to represent an assignment/ allocation problem

4.3.13 determine the optimum assignment(s), by inspection for small-scale problems, or by use of the Hungarian algorithm for larger problems

EXAMPLE 20

Draw a bipartite graph for each of the following scenarios.

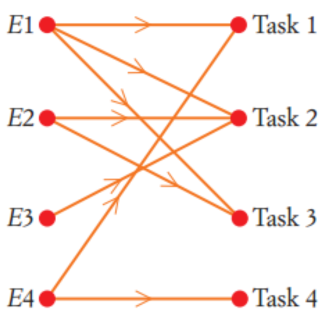
a Three housemates are sharing cleaning tasks as follows.

- Peta will clean the kitchen or do the vacuuming.
- Michael will clean the kitchen or the bathroom.
- Lazlo will do any of the three tasks.

b Four teachers teach different subjects as follows.

- Alison teaches English and History.
- Mario teaches History and Business Studies.
- Bob teaches PDHPE, Maths and Science.
- Janice teaches Maths and Science.

Determine allocations to the four employees:



The times taken for three nurses (A, B, C) to carry out three different procedures (1, 2, 3) are recorded in the following table.

	1	2	3
A	7	3	8
B	5	6	9
C	10	5	12

a Find the table that results from a row and column reduction.

b Use the Hungarian algorithm to produce a table from which an optimum allocation can be made.

c State the optimum allocation.

d Calculate the total working time for the three procedures.

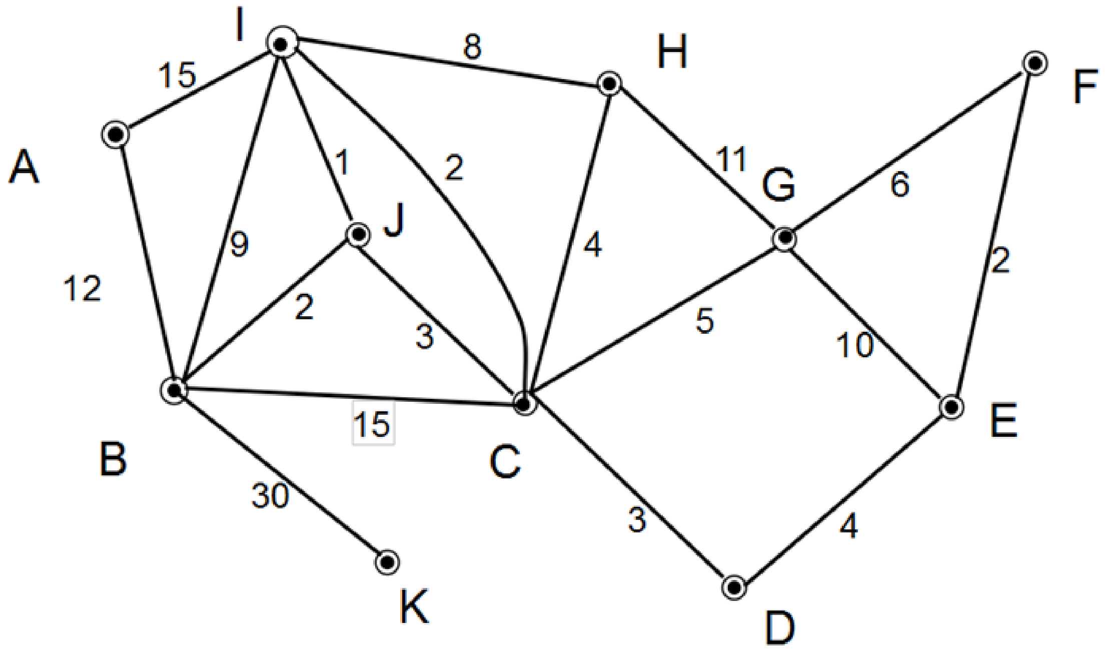
A taxi company has four taxis available, A, B, C and D, and there are four customers requiring a taxi each. The distance (km) that each taxi must travel to reach the customers is shown in the following table.

	1	2	3	4
A	12	13	6	22
B	11	15	13	23
C	23	22	12	15
D	19	17	6	21

a Use the Hungarian algorithm to find the optimum allocation of taxis to customers that minimises the distance travelled.

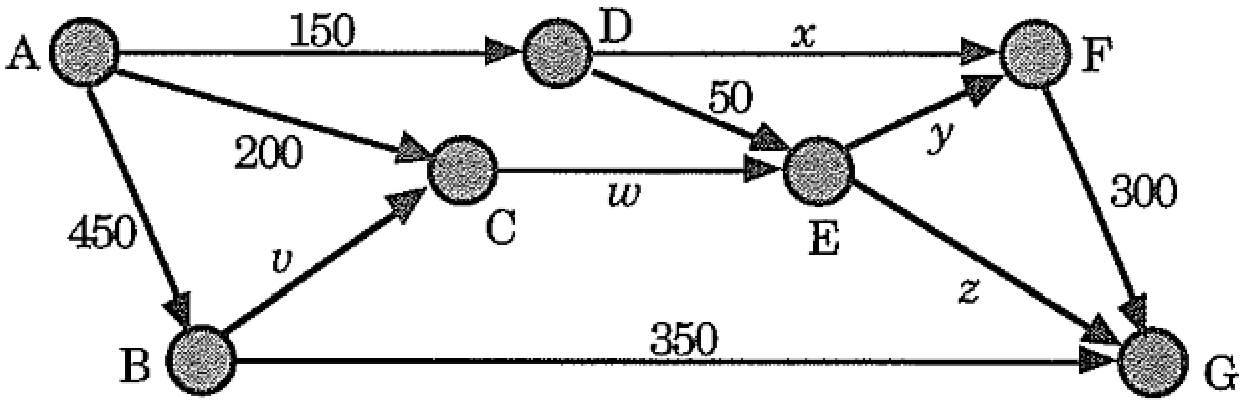
b If this allocation is implemented, find the total distance travelled.

The following graph shows the number of minutes required for the members of a friendship group to pass on a secret they weren't supposed to divulge.



What is the shortest time for everyone to find out the secret?

Go to shops and buy ingredients (30 minutes)	Cook meat in slow cooker (240 minutes)
Eat meal (15 minutes)	Cook beans (5 minutes)
Cook carrots, potatoes and pumpkin in slow cooker (120 minutes)	Serve food (10 minutes)
Boil water for beans and gravy (4 minutes)	Brown meat in frying pan (6 minutes)
Carve meat (8 minutes)	Slice and butter bread (5 minutes)
Prepare carrots, potatoes and pumpkin (12 minutes)	Preheat slow cooker (20 minutes)
Set table (4 minutes)	Make gravy (3 minutes)



In The Bachelor house there are four tasks that need to be done every day.

- **R**eading out the bachelor dates for the day
- **C**ooking
- **W**axing
- **M**aking nasty comments on the video diary

Contestant	Task they are happy to do
Keira 	• **W**axing • **M**aking nasty comments on the video diary
Nikki 	• **R**eading out the bachelor dates for the day • **C**ooking • **W**axing
Alex 	• **R**eading out the bachelor dates for the day • **M**aking nasty comments on the video diary
Faith 	• **W**axing

The BIG Ideas Year 12 Mathematics Applications

Statistics 1

Sequences 1

Networks 1

Statistics 2

Sequences 2

Networks 2

Congratulations! You have now completed your revision booklet!

Edith Cowan University would like to wish all students the best of luck with their future exams!