EDITH COWAN UNIVERSITY

Edith Cowan University

2025 ATAR Revision Seminars

ATAR Mathematics Methods

Curriculum Dot points

Examination and study tips

Revision notes Examination questions

Examination marker comments

Prepared and presented by

Question Number	Topic	Marks	Resource Free/Assumed
1	General Calculus	10	Free
2	General Calculus	6	Free
3	General Calculus	5	Free
4	General Calculus	6	Free
5	1st and 2nd derivatives	8	Free
6	1st and 2nd derivatives	5	Free
7	Tangents	5	Free
8	General Calculus	5	Free
9	Exponential growth	8	Assumed
10	Rectilinear motion	6	Assumed
11	Small change	7	Assumed
12	Area under a curve	5	Free
13	Total change from rate of change	5	Assumed
14	Rectilinear Motion	9	Assumed
15	Area between curves	5	Assumed
16	Binomial distribution	9	Assumed
17	Discrete random variables	9	Assumed
18	Discrete random variables	9	Assumed
19	Logarithmic Equations	11	Free
20	Rectilinear Motion	7	Free
21	Area under a curve	7	Free
22	Logarithmic functions	7	Assumed
23	Area between curves	7	Assumed
24	Uniform CRV	6	Free
25	Continuous random variables	6	Free
26	Sampling	12	Assumed
27	Normal distribution	8	Assumed
28	Continuous random variables	11	Assumed
29	Estimating areas	9	Assumed
30	Optimisation	9	Free
31	Optimisation	7	Assumed
32	Sampling	9	Free
TOTAL		238	

Question 1 (10 marks)
Determine:

(i) $\int 2x(3 - x^2)^6 dx$ [2]

(ii) $\frac{d}{dx}\left[\int_3^x 8 - 7t \, dt\right]$ [1]

(iii) $\int_{-1}^{2} 4e^{-t} \, dt$ [3]

(iv) the possible values of k, such that $\int_{\frac{\pi}{4}}^{k} \sin 2x \; dx = -0.5$ and $-\pi \leq k \leq 2\pi$. [4]

Question 2 (6 marks)
Determine:

(i) $\frac{d}{dx}(\frac{2\ln x}{3x})$ [2]

(ii) $\int \frac{0.5-x}{x^2-x} dx$ [2]

(iii) $\frac{d}{dx}(\int_0^{\ln x} 2t\ dt)$ [2]

Question 3 (5 marks)

(a) Show that $\frac{d}{dx}(xe^{3x}) = e^{3x} + 3xe^{3x}$ [1]

(b) Using your answer from part (a) determine $\int xe^{3x}\, dx$. [4]

Question 4 (6 marks)

Given that $\int_{-2}^{6} f(x)dx = 5$, $\int_{-2}^{9} f(x)dx = -20$ and $\int_{-2}^{10} f(x)dx = -7$, determine:

(i) $\int_{9}^{10} f(x)dx$ [2]

(ii) $\int_{6}^{10} \frac{f(x)+x}{2} dx$. [4]

Question 5 (8 marks)

(a) Determine the gradient of the curve $y = \frac{e^x}{x^3+1}$ at the point where $x = 2$. [4]

(b) Determine the x coordinates of the stationary points on the function $y = x(2x-1)^3$. [4]

Question 6 (5 marks)

Consider the function $g(x) = x^3 + x - 3$.

(a) Using calculus, show that the function has zero stationary points. [2]

(b) Determine the coordinates of the point of inflection on $g(x)$, using calculus to prove that it is a non-stationary point of inflection. Clearly state your reasoning. [3]

Question 7 (5 marks)
Determine the equation of the tangent to the curve $f(x) = e^x \sin(5x)$ at the point where $x = \pi$.

Question 8 (5 marks)

The function $y = ax^2 + bx^4$ has a gradient of -17 at the point $(1, \frac{-11}{2})$. Determine the values of a and b.

Question 9 (8 marks) CA

A foreign substance is introduced into a pond. It affects the population of the fish in the pond so that the population, P varies with t according to the rule

$$\frac{dP}{dt} = -0.15P$$

where t is the number of hours since the substance was introduced.

(a) If the initial population of fish was 300 determine:

(i) an expression for P in terms of t. [1]

(ii) the time taken for the population to halve. [2]

(iii) the rate of change of the population 1 day after the substance was introduced. [2]

(b) Determine the initial population if, after 50 hours, the population was decreasing by 0.025 fish per hour. [3]

Question 10 (6 marks) CA

A particle is moving in a straight line such that its displacement, x cm, at time t seconds is given by

$$x = \cos t + 0.1t^3 \quad \text{for } t \geq 0.$$

(a) Determine the displacement of the particle when $t = 3$. [1]

(b) Determine the velocity of the particle when it has an acceleration of $50cms^{-2}$ for the first time. [3]

(c) At what time(s) is the speed of the particle $0.5cms^{-1}$? [2]

Question 11 (7 marks) CA

(a) Given that $y = \sqrt{x} + x$, use calculus to approximate the small change in y when x changes from 5 to 5.01. [2]

(b) The incremental formula, $\frac{\delta y}{\delta x} \approx \frac{dy}{dx}$, is used to approximate the small change in the function $y = e^{ax} + bx$. It approximates that the small change in y when x changes from 2 to 2.05 is 1.5 and the small change in y when x changes from 10 to 10.05 is 3.5. Determine the values of a and b, correct to 3 significant figures. [5]

Question 12 (5 marks)

Determine the area between the curve $y = x^2 + 3x - 4$ and the x axis between $x = 0$ and $x = 2$.

Question 13 (5 marks) CA

The volume of a balloon, $V\ cm^3$, is varying such that after t minutes, $\frac{dV}{dt} = \frac{3}{\sqrt{t}}$.

(a) Determine, correct to 5 significant figures, the change in volume from $t = 10$ to $t = 16$. [2]

(b) Determine the time time taken, to the nearest second, from when $t = 50$ for the volume to increase by $40\ cm^3$. [3]

Question 14 (9 marks) CA

A particle, with an initial displacement of -0.25 meters, is moving in a straight line such that at time t seconds, its velocity is given by $v(t) = 4\sin(\frac{t}{6})$ meters per second, where $t \geq 0$.

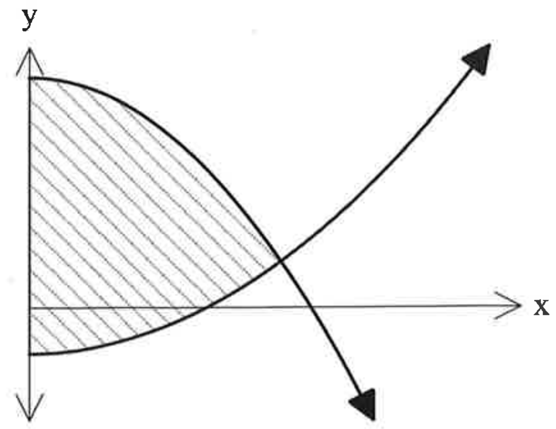
(a) Determine the average speed of the body in the first minute. [2]

(b) Determine the smallest value of q, such that the average velocity from $t = 1$ to $t = q$ is $1.75\ m/s$. [3]

(c) Determine the acceleration of the particle when it is at the origin for the first time. [4]

Question 15 (5 marks) CA

The graph below shows the functions $f(x) = 3x^2 - 1$ and $g(x) = ax^2 + b$, intersecting at $x = \frac{\sqrt{6}}{3}$. Given that the shaded area is equal to $\frac{4\sqrt{6}}{3}$ units squared, determine the values of a and b.



Question 16 (9 marks) CA

When throwing a dart at a dart board, Taylor has a 0.18 chance of hitting the bulls eye.

(a) If Taylor has 30 throws at the dart board determine the probability that he hits:

(i) exactly 2 bulls eyes. [2]

(ii) more than 3 bulls eyes. [1]

(iii) no more than 6 bulls eyes, given he hits more than 3 bulls eyes. [2]

(b) Taylor plays 5 games of darts, each time throwing 30 darts. Determine the probability he hits exactly 2 bulls eyes in at least 3 of the 5 games. [2]

(c) How many darts would Taylor need to throw so that he has at least a 75% chance of hitting at least one bulls eye? [2]

Question 17 (9 marks) CA

Consider the probability function shown below.

x	1	3	m	8
$P(X = x)$	0.15	n	0.3	0.35

(a) Determine n. [1]

(b) If the expected value of X is 5.65, determine:

(i) m. [2]

(ii) $Var(X)$. [2]

(iii) $E(3 - 1.5X)$. [1]

(iv) a and b such that $E(a + bX) = 12$ and $Var(a + bx) = 26$ and $b > 0$.

[3]

Question 18 (9 marks) CA

At the Royal Show, a game stall operator offers prizes of \$1, \$3, and \$10 with probabilities of 0.45, 0.2 and 0.05 respectively. The operator charges \$2 per game.

(a) Show that the probability of not winning a prize is 0.3. [1]

(b) Determine the probability of a player winning \$3, given that that win a prize. [2]

(c) After 200 plays of the game, how much should the game stall operator expect to profit? [2]

(d) If Sydney plays the game 35 times, what is the probability she gets a prize of exactly \$3 more than 5 times? [2]

(e) How much should the game stall operator charge if she wishes to make \$150 profit in 200 plays of the game? [2]

Question 19 (11 marks)

(a) Determine the exact value of x in each of the following equations.

(i) $3^x = 2^{5-x}$ [3]

(ii) $\ln(\tan x) + 0.5\ln 3 = 0$ for $0 < x < \frac{\pi}{2}$. [2]

(b) If $\log_2 9 = a$ and $\log_2 6 = b$ express the following in terms of a and b.

(i) $-\log_2 54$. [2]

(ii) $\log_2 36$. [1]

(iii) $\log_2 0.75$ [3]

Question 20 (7 marks)

A particle moves in a straight line such that its displacement, x metres, at time t seconds is given by

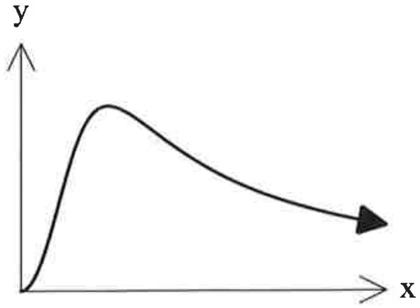
$$x(t) = -1 + \ln(t^2 - 2t + 2) \text{ for } t \geq 0.$$

(a) Determine when the particle is stationary and its distance from the origin at this time. [4]

(b) Determine the acceleration of the particle when $t = 3$. [3]

Question 21 (7 marks)

Consider $f(x) = \frac{3x^2}{x^3+1}$ graphed below.



(a) Determine the area bound by $f(x)$ and the x-axis from $x = 1$ to $x = 3$, expressing your answer as a single logarithm. [3]

(b) The area bound by $f(x)$ and the x-axis from $x = 1$ to $x = k$ is $\ln 63$ units squared. Determine the value of k. [4]

Question 22 (7 marks) CA

The loudness of a sound, L decibels, is related to the intensity of the sound, I by the equation

$$L = 10\log\frac{I}{I_0}$$

where I_0 is a constant.

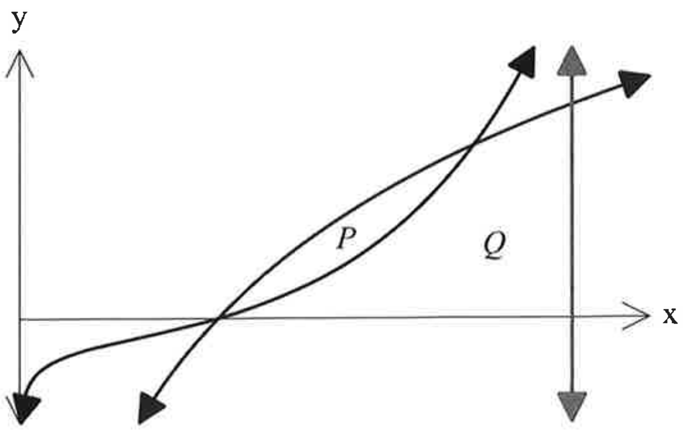
(a) Determine the loudness of a sound when its intensity is $150I_0$. [2]

(b) How many times more intense is a sound with a loudness of 75 decibels to that of a sound with a loudness of 48 decibels, correct to 3 significant figures. [2]

(c) Use calculus to approximate the small change in L when I increases to 1.1×10^{-4} from 10^{-4}. [3]

Question 23 (7 marks) CA

The functions $f(x) = 5\ln x$ and $g(x) = \frac{e^x \ln x}{2}$ and the line $x = b$ are graphed on the axes below. P is a region bounded by $f(x)$ and $g(x)$ and Q is the region bounded by the two curves, the line $x = b$ and the x-axis.



(a) Determine an integral that will give the exact area of region P, and state this area correct to 3 significant figures. [4]

(b) Region Q is 5 units squared. Determine the value of b correct to 3 significant figures. [3]

Question 24 (6 marks)

The probability density function of a continuous random variable X is given by

$$f(x) = \begin{cases} 0.2 & for\ \ 0 \leq x \leq 5 \\ 0 & elsewhere. \end{cases}$$

(a) Graph this distribution on the axes below, showing clearly the scale on each axis. [1]

(b) What type of distribution is this. [1]

(c) Determine:

(i) $P(X \leq 4)$. [1]

(ii) $P(X \leq 4 | X \geq 3)$. [1]

(iii) t such that $P(t < X < 2t) = \frac{1}{3}$. [2]

Question 25 (6 marks)

The probability density function for the continuous random variable X is given by

$$f(x) = \begin{cases} mx + c, & 0 \leq x \leq 3 \\ 0 \; elsewhere \end{cases}$$

Given than $E(X) = 1$, determine the values of m and c.

Question 26 (12 marks) CA

A random sample of 400 fish from a large pond is caught. 15 of these fish have a defect with their tail.

(a) Determine a point estimate for the proportion of fish in the entire pond that have a defect with their tail. [1]

(b) Determine a 95% confidence interval for the proportion of fish in the pond that have a defect with their tail. [3]

(c) From the sample taken, Andrea is $x\%$ confident that the proportion of fish in the pond with a tail defect lies between 0.0175 and 0.0575. Determine the value of x correct to one decimal place. [5]

(d) Seventy samples are taken and the 97% confidence intervals for the proportion of fish in the pond with a tail defect, p, are taken. If $X =$ the number of samples who confidence interval contains the true value of p, state the distribution of X, stating its parameters and mean and standard deviation. [3]

Question 27 (8 marks) CA

A tackle company sells sinkers. The weight of the sinkers is normally distributed with a mean of $80g$ and a standard deviation of $2g$.

A sinker is selected at random.

(a) Determine the probability that the sinker weighs:

(i) more than $78g$, [1]

(ii) less than $81g$, given that it weighs more than $78g$. [2]

(b) If 70 sinkers were selected at random, determine the probability that exactly 50 of them weigh more than $78g$. [2]

The machine that produces the sinkers is set to $80g$. What ever amount the machine is set to will be the mean of the normal distribution, with the standard deviation always remaining at $2g$.

(c) What should the machine be set to, if the company wants 98.5% of the sinkers produced to be more than $80g$? [3]

Question 28 (11 marks) CA

The continuous random variable, X, has probability density function

$$f(x) = \begin{cases} \dfrac{\sqrt{3x}}{6}, & for\ \ 0 \le x \le 3 \\ 0\ for\ all\ other\ values\ of\ x. \end{cases}$$

(a) Determine $P(1 < X < 2)$. [1]

(b) Determine q such that $P(0 < X < q) = 0.5$ [2]

(c) Determine:

(i) $E(X)$ [2]

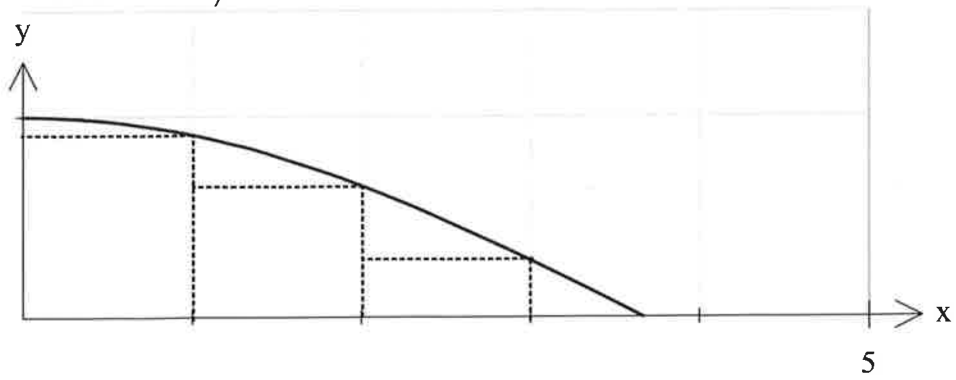
(ii) $Var(X)$ [2]

(iii) $E(5X - 3)$ [1]

(d) Determine the cumulative distribution function for X. [3]

Question 29 (9 marks) CA

The function $f(x) = \cos\frac{3x}{7}$ is graphed below.



(a) Complete the table below. [1]

x	0	1	2	3
$f(x)$	1	0.9096	0.6546	

(b) Use the inscribed rectangles of width 1 unit, shown on the axes above, to underestimate the area between $f(x)$ and the x-axis from $x = 0$ to $x = 3$. [2]

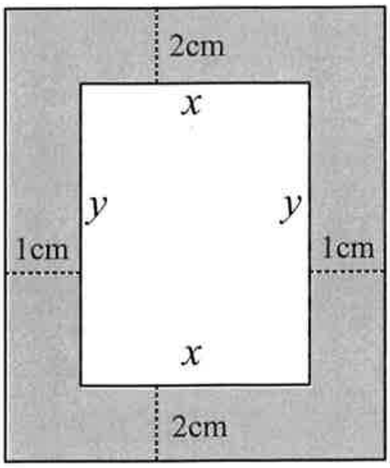
(c) Use circumscribed rectangles of width 1unit to overestimate the area between $f(x)$ and the x-axis from $x = 0$ to $x = 3$. [2]

(d) By averaging out your results from (b) and (c), determine a better estimate for the area between $f(x)$ and the x-axis from $x = 0$ to $x = 3$. [1]

(e) Using calculus, determine the area of $f(x)$ and the x axis from 0 to 3. [3]

Question 30 (11 marks)

A picture frame (shaded below) is being designed with a $2cm$ border at the top and bottom and a $1cm$ border at the sides, with the area for the picture being fixed at $200cm^2$.



(a) Show that the area of the picture frame, A, can be modelled by the equation

$$A = \frac{400}{x} + 4x + 8$$ [3]

(b) Determine the values of x and y that will minimise A, using $\frac{d^2A}{dx^2}$ to prove it is a minimum.

[5]

(c) Using the incremental formula, approximate the change in A, when x changes from $2cm$ to $1.99cm$.

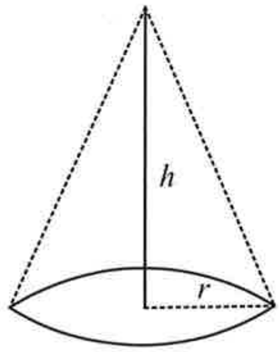
[3]

Question 31 (7 marks) CA

A tent in the shape of a cone is to be pitched. A bamboo frame is needed for the circumference of the base and the height of the cone. 8 metres of bamboo to be used for the framework, represented by the solid lines in the diagram below.

(a) Show that the volume V, of the tent in terms of its radius r, is given by [2]

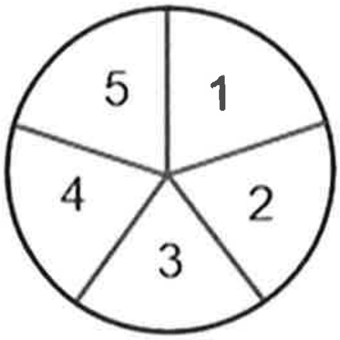
$$V = \frac{8}{3}\pi r^2 - \frac{2}{3}\pi^2 r^3$$



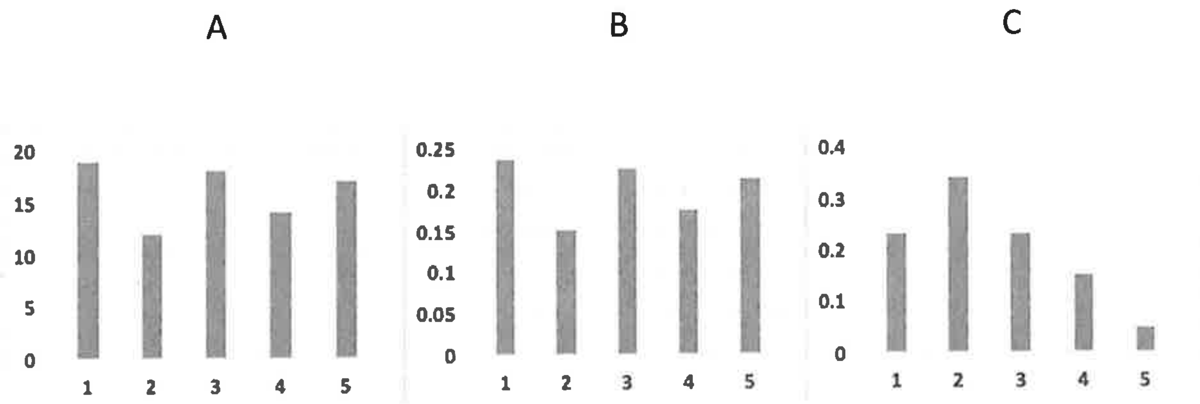
(b) Determine the radius of the tent that will maximise the volume, leaving your answer in terms of π. State this maximum volume and prove it is indeed a maximum. [5]

Question 32 (9 marks) CA

The spinner pictured below is such that each outcome (1, 2, 3, 4 and 5) is equally likely. An experiment consists of spinning the spinner 80 times.



(a) In the experiment, the frequencies of each outcome were noted. Which histogram below is most likely to show this data, if the outcome is listed on the x-axis and the relative frequency is listed on the y-axis? Explain your answer. [2]



(b) If X is the number of times that an odd number is spun:

(i) describe the distribution of X, stating its parameters. [2]

(ii) determine the probability that X is greater than 40. [1]

(c) The experiment involving 80 spins of the spinner is repeated 760 times. In how many of the 760 experiments, would you expect the proportion of odd numbers to be between 0.35 and 0.65 ? [4]

2019 ATAR

Section Two: Calculator-assumed 65% (99 Marks)

This section has **11** questions. Answer **all** questions. Write your answers in the spaces provided.

Supplementary pages for planning/continuing your answers to questions are provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.

Working time: 100 minutes.

Question 8 (7 marks)

Big Foods is a large supermarket company. The manager of Big Foods wants to estimate the proportion of households that do the majority of their grocery shopping in their stores.

A junior staff member at Big Foods conducted a survey of 250 randomly-selected households and found that 56 did the majority of their grocery shopping at a Big Foods store.

(a) (i) Calculate the sample proportion of households who did the majority of their grocery shopping at Big Foods. (1 mark)

(ii) Determine the 95% confidence interval for the proportion of households who do the majority of their grocery shopping at Big Foods. Give your answer to four decimal places. (3 marks)

(iii) What is the margin of error of the 95% confidence interval? Give your answer to four decimal places. (1 mark)

An independent research company conducted a large-scale survey of household supermarket preferences and estimated that the true proportion of households that conduct most of their grocery shopping at Big Foods was 0.17 (assume that this is indeed the true proportion).

(b) With reference to your answer to part (a)(ii), does this result suggest that the junior staff member at Big Foods made a mistake? (2 marks)

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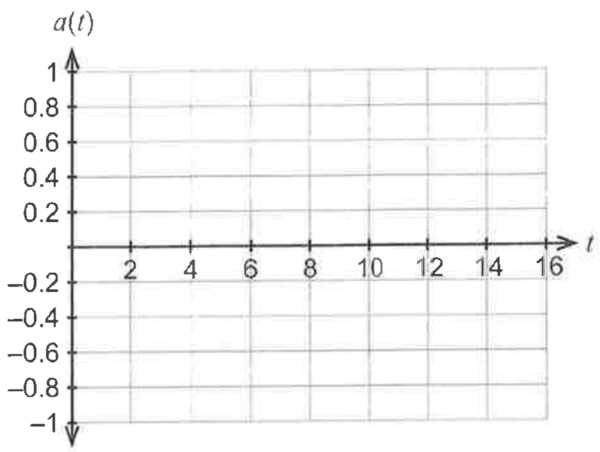
Question 9 **(8 marks)**

It takes an elevator 16 seconds to ascend from the ground floor of a building to the sixth floor. The velocity of the elevator during its ascent is given by

$$v(t) = \frac{9\pi}{16}\sin\left(\frac{\pi t}{16}\right) \text{ m/s.}$$

The velocity, v, is measured in metres per second, while the time, t, is measured in seconds.

(a) Determine the acceleration of the elevator during its ascent and provide a sketch of the acceleration function for $0 \le t \le 16$. (2 marks)



(b) With reference to your answer from part (a), explain what is happening to the velocity of the elevator in the interval $0 < t < 8$ and in the interval $8 < t < 16$. (3 marks)

(c) Suppose that the ground floor has displacement $x = 0$ m. Determine the displacement function of the elevator and hence determine the height above the ground floor of the sixth floor. (3 marks)

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Question 11 **(8 marks)**

A pizza company runs a marketing campaign based on the delivery times of its pizzas. The company claims that it will deliver a pizza in a radius of 5 km within 30 minutes of ordering or it is free. The manager estimates that the actual time, T, from order to delivery is normally distributed with mean 25 minutes and standard deviation 2 minutes.

(a) What it the probability that a pizza is delivered free? (1 mark)

(b) On a busy Saturday evening, a total of 50 pizzas are ordered. What is the probability that more than three are delivered free? (2 marks)

The company wants to reduce the proportion of pizzas that are delivered free to 0.1%.

(c) The manager suggests this can be achieved by increasing the advertised delivery time. What should the advertised delivery time be? (2 marks)

After some additional training the company was able to maintain the advertised delivery time as 30 minutes but reduce the proportion of pizzas delivered free to 0.1%.

(d) Assuming that the original mean of 25 minutes is maintained, what is the new standard deviation of delivery times? (3 marks)

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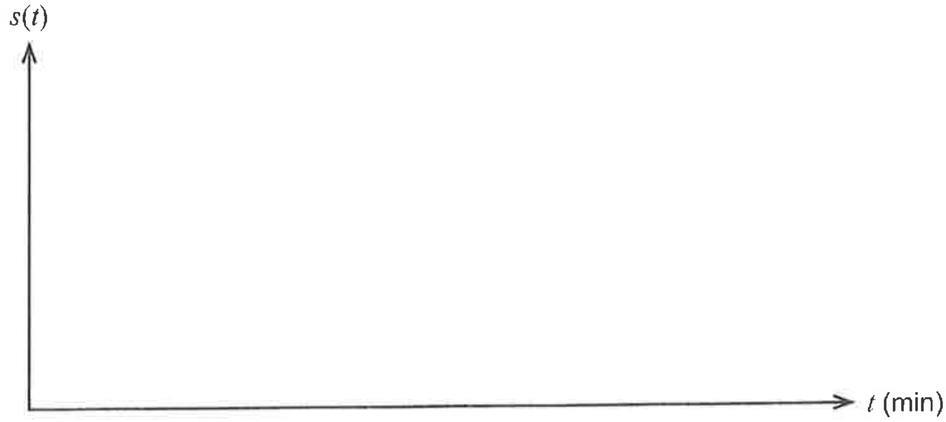
Question 12 **(6 marks)**

Part of Josie's workout at her gym involves a 10 minute run on a treadmill. The treadmill's program makes her run at a constant 12.3 km/h for the first 2 minutes and then her speed, $s(t)$, is determined by the equation below, where t is the time in minutes after she began running.

$$s(t) = 10 - \frac{\ln(t - 1.99)}{t} \text{ km/h}$$

(a) Sketch the graph of her speed during this run versus time on the axes below. (3 marks)



(b) At what time(s) is Josie's speed 10 km/h? (1 mark)

(c) At what time(s) during her run is Josie's acceleration zero? (2 marks)

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Question 13 **(8 marks)**

The proportion of working adults who miss breakfast on week days is estimated to be 40%. A study takes a random sample of 400 working adults.

(a) For this sample:

(i) What is the (approximate) distribution of the sample proportion of workers who miss breakfast? (2 marks)

(ii) What is the probability that the sample proportion of workers who miss breakfast is greater than 44%? (2 marks)

Tom takes a random sample of 400 adults. He obtained his sample by selecting the first 400 workers he met in a busy mall in Perth city during lunch time.

(b) Discuss briefly **two** possible sources of bias in Tom's sample. (2 marks)

Amir suggests that a better sampling scheme is to obtain a random sample of 400 voters and contact them by telephone.

(c) (i) Outline **one** source of bias in Amir's sampling scheme. (1 mark)

(ii) Which of Tom's or Amir's sampling scheme is better? Provide a reason for your choice. (1 mark)

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Question 14 **(7 marks)**

(a) What is the minimum sample size required to estimate a population proportion to within 0.01 with 95% confidence. (3 marks)

(b) Identify **two** factors that affect the width of a confidence interval for a population proportion and describe the effect of each. (4 marks)

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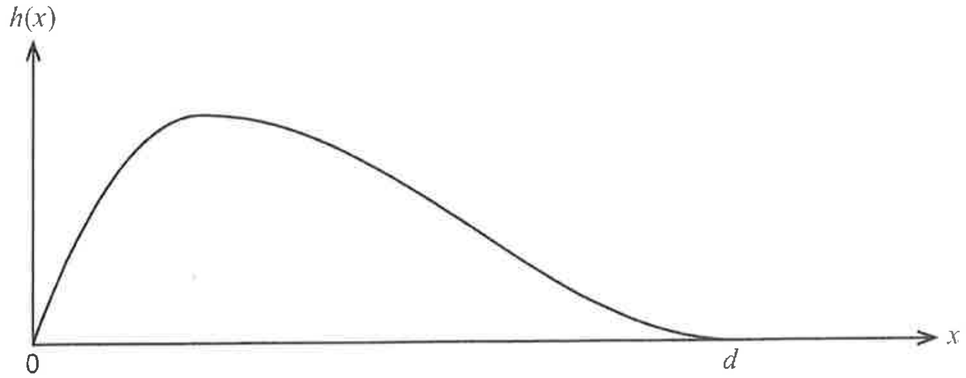
Question 15 **(14 marks)**

A wall in a new Western Australian hotel is to feature a rolling, wave-shaped window. Engineers have modelled the top edge of the wave shape by joining together two functions,

$$h_1(x) = 4 - 4(x-1)^2, \quad 0 \le x \le 1 \text{ and}$$

$$h_2(x) = a(\cos(x-1)+1), \quad 1 < x \le d \quad a, d \text{ constants.}$$

The functions give the height, h, above ground level of the top edge of the window measured in metres. The origin is defined as the leftmost point of the window which is at ground level and x is the horizontal distance to the right of the origin measured in metres. The graph of the two functions is shown below.



(a) Determine the value of the constant a in the function $h_2(x) = a(\cos(x-1)+1)$. (3 marks)

(b) Determine the length of the bottom edge of the window. (2 marks)

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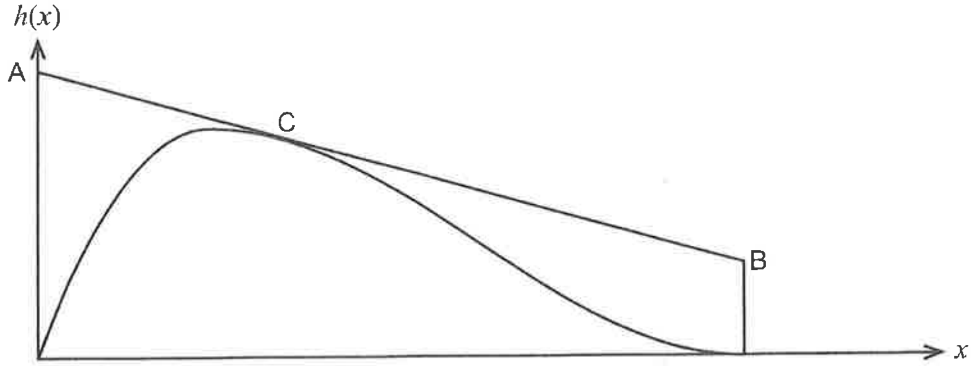
(c) Determine the volume of glass required for the window if it has a uniform thickness of 3 cm. (5 marks)

The top edge of the wall, shown as the line AB below, is to just touch the window at the point C shown below. Point A is 1.39 m above the point B.



(d) How high is point C above the ground? (4 marks)

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Question 18 **(9 marks)**

A building has five alarms configured in such a way that the system functions if at least two of the alarms work. The probability that an alarm fails overnight is 0.05. Let the random variable X denote the number of alarms that fail overnight.

(a) State the distribution of X. (2 marks)

(b) What assumptions are required for the distribution in part (a) to be valid? (2 marks)

(c) What is the probability that the alarm system fails overnight? (2 marks)

One of the alarms is removed in the evening for maintenance and is not replaced.

(d) What is the probability that the alarm system still works in the morning? (3 marks)

End of questions

DO NOT WRITE IN THIS AREA AS IT WILL BE CUT OFF