EDITH COWAN UNIVERSITY

Edith Cowan University

2025 ATAR Revision Seminars

ATAR Mathematics Specialist

Curriculum Dot points

Examination and study tips

Revision notes Examination questions

Examination marker comments

Prepared and presented by

Question Number	Topic	Marks	Resource Free/Assumed
1	Complex Numbers	10	Free
2	Complex Numbers	5	Free
3	Complex Numbers	5	Free
4	Complex numbers	4	Free
5	Functions	4	Free
6	Functions	7	Free
7	Vectors Calculus	7	Free
8	Lines	6	Free
9	Lines/Spheres	7	Free
10	Systems of Equations	5	Free
11	Plane	5	**Assumed**
12	Vectors in Geometry	7	**Assumed**
13	Implicit Differentiation	9	Free
14	Integration	9	Free
15	Integration	4	Free
16	Applications of Differentiation	11	**Assumed**
17	Applications of Differentiation	6	**Assumed**
18	Parametric Equations	5	**Assumed**
19	SHM	5	**Assumed**
20	Slope Fields	6	Free
21	Logistic Equation	6	**Assumed**
22	SHM	7	**Assumed**
23	Sampling	4	**Assumed**
24	Sampling	4	**Assumed**
25	Separation of Variables	5	**Assumed**
26	Rectilinear Motion	7	**Assumed**
27	Complex Numbers	10	Free
28	Sampling	8	**Assumed**

Question 1 (10 marks)

Given $z = -3 + \sqrt{3}\, i$, express z in the form $r\ cis\ \theta$ $(r > 0, -\pi < \theta \leq \pi)$:

(i) z [2]

(ii) $5z$ [1]

(iii) z^2 [2]

(iv) $\bar{z}$ [1]

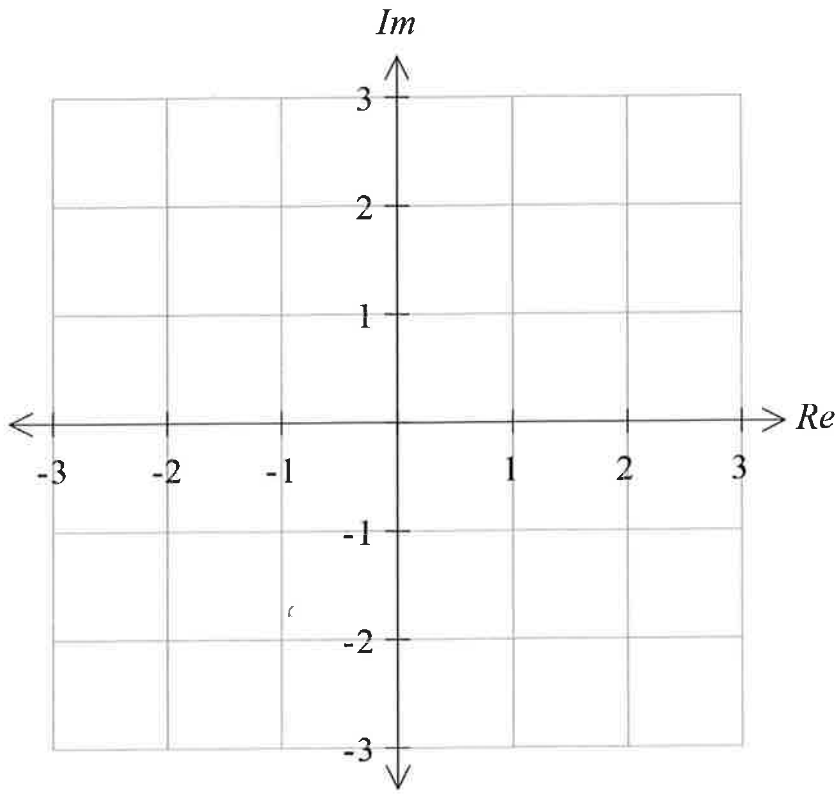
(v) $\frac{1}{z}$ [2]

(vi) $z + 6$ [2]

Question 2 (5 marks)

(i) Sketch, on the Argand diagram below, the set of points given by

$\{z : |z - i - 1|^2 = 2\}$. [3]



(ii) Determine the maximum value of $Re(z)$ for the values of z in the set defined in part (i). [2]

Question 3 (5 marks)

(i) Use de Moivre's theorem to find all three solutions of the equation $z^3 = 8$, expressing your answers in rectangular form. [3]

(ii) Hence write down all three solutions of the equation $(z + 1)^3 = 8$ in rectangular form. [2]

Question 4 (4 marks)

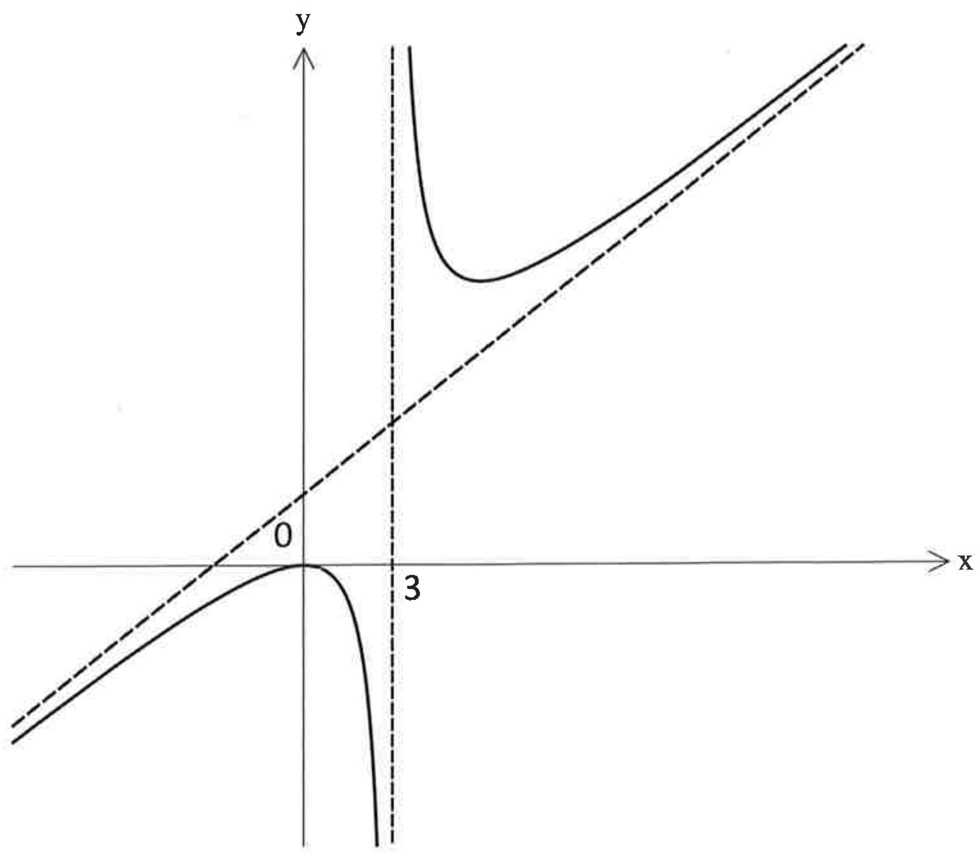
Given that $z = \sqrt{5}\, i$ is a solution of the equation

$$2z^3 - 3z^2 + 10z = 15 ,$$

find the other two solutions.

Question 5 (4 marks)

The graph of $y = \frac{x^2}{x-k}$ $(k > 0)$ is shown below:



(i) State the value of k. [1]

(ii) Determine the equation of the inclined asymptote. [3]

Question 6 (7 marks)

Given $f(x) = \sqrt{x-1}$

and $g(x) = |x|$,

(i) determine $(f \circ g)(x)$ and state its domain and range, [4]

(ii) determine the largest possible set of values $x > 0$ for which $(f \circ g)(x)$ is invertible, and give the formula for $(f \circ g)^{-1}(x)$. [3]

Question 7 (6 marks)

A particle moves with velocity $\boldsymbol{v}(t)$ at time t seconds given by

$$\boldsymbol{v}(t) \;=\; (-2\,sin\,2t)\,\boldsymbol{i} + (2\,cos\,2t)\,\boldsymbol{j}$$

and has initial position $3\boldsymbol{i} + 5\boldsymbol{j}$.

(i) Determine the object's acceleration $\boldsymbol{a}(t)$. [1]

(ii) Determine the object's position vector $\boldsymbol{r}(t)$. [3]

(iii) Find the Cartesian equation of the object's path. [2]

Question 8 (6 marks)

A line has equation $\boldsymbol{r} = \begin{pmatrix} 3 \\ -1 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} 4 \\ 2 \\ 1 \end{pmatrix}.$

(i) Determine the Cartesian equation of the line. [3]

(ii) Find the point on the line closest to $(0, 0, 0)$. [3]

Question 9 (7 marks)

Given the line $\boldsymbol{r} = (2, -2, 1) + \lambda(-1, 0, 3)$,

(i) find the co-ordinates of the points A and B on the line corresponding to $\lambda = -1$ and $\lambda = 1$ respectively, [1]

(ii) determine the vector equation of the sphere of which AB is a diameter, [2]

(iii) find a vector perpendicular to $(-1, 0, 3)$,
(Hint: let the vector be (a, b, c)) [2]

(iv) hence determine points C and D on the sphere in part (ii) such that CD is also a diameter and $CD \perp AB$. [2]

Question 10 (5 marks)

Consider the system of equations:

$$\begin{aligned} x - 2y + z &= 5 \\ ax + ay + z &= 1 \\ x + y + z &= b \end{aligned}$$

where a and b are constants.

Find all values of a and b for which the system has

(i) a unique solution, [2]

(ii) infinitely many solutions, [2]

(iii) no solution. [1]

Question 11 (5 marks) CA

Find a vector normal to the plane

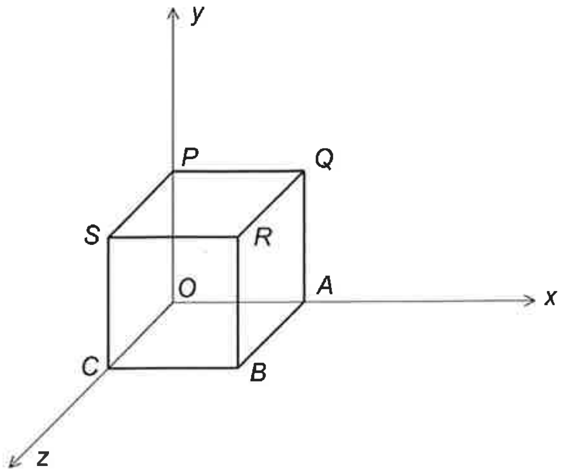
(i) whose Cartesian equation is $2x - 3y + 5z = 11$, [1]

(ii) whose vector equation is $\boldsymbol{r} = \begin{pmatrix} 4 - \lambda + \mu \\ -1 + 2\lambda - \mu \\ 6 + 3\lambda + 2\mu \end{pmatrix}$ [2]

(iii) containing the points whose position vectors are $\boldsymbol{i}$, $\boldsymbol{j}$ and $\boldsymbol{k}$. [2]

Question 12 (7 marks) CA

OABCPQRS is a unit cube, as shown in the diagram:



(i) In terms of the basis vectors ***i***, ***j*** and ***k***, write down the position vectors of the points *O*, *R*, *A* and *S*. [2]

(ii) Hence show that the diagonals *OR* and *AS* bisect each other. [3]

(iii) Find the acute angle of intersection between the diagonals *OR* and *AS*, accurate to 0.01°. [2]

Question 13 [2, 3, 4 marks]

(a) Given that $p = \cos(xy)$, find an expression for $\frac{dp}{dx}$ in terms of x and y.

Hint: Let $t = xy$ and use chain rule.

(b) Given that $y = x^{\sin y}$, find an expression for $\frac{dy}{dx}$ in terms of x and y.

(c) A curve is defined implicitly by $xy^2 = 3x^2 - 2x\sqrt{y}$. Find the gradient of the tangent to the curve where $y = 4$ and $x > 6$.

Question 14 [3, 3, 3 marks]

Find the indefinite integrals:

(a) $\int 2\sin^3 2x \, dx$

(b) $\int \frac{2x-3}{x^2+4x-5} \, dx$

(c) $\int \frac{e^{-2x} - e^{2x}}{e^{2x} + e^{-2x}} \, dx$

Question 15 [4 marks]

Evaluate $\int_{1}^{\frac{5}{2}} 3x\sqrt{2x-1}\, dx$

Question 16 [3, 4, 4 marks] CA

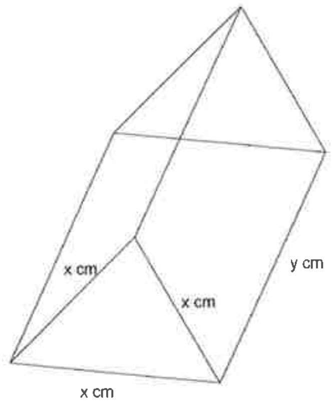
(a) Find the volume generated when the region enclosed by the ellipse $\frac{x^2}{3}+\frac{y^2}{4}=1$ and the line $y=1$ above the x-axis is rotated 360^0 about the y-axis.

(b) Water is pour into an inverted cone at a rate of 5 cm^3s^{-1}. If the height of the cone is twice the radius of its base, what would be the rate of increase in the depth of the water level measured from the vertex at the instant when the depth of the water is 10 cm?

(c) A 6m ladder is resting against a vertical wall. If the base of the ladder is sliding outwards (away from the wall) at a constant rate of $0.02\ ms^{-1}$, what would be the rate of change of the height of the ladder at the instant when the ladder makes an angle of 20^0 with the wall?

Question 17 [2, 2, 2 marks] CA

The diagram shows a triangular prism with equilateral triangles at both ends.



If the volume of the prism is 2400 cm^3,

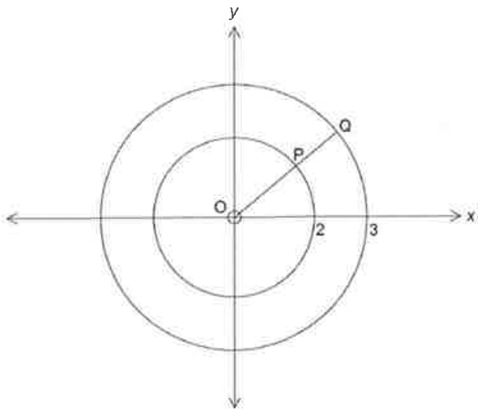
(a) establish a relationship between x and y

(b) show that the total surface area A is given by, $A = \frac{\sqrt{3}}{2}x^2 + \left(\frac{9600\sqrt{3}}{x}\right)$

(c) use the incremental formula to find the approximate change in the total surface area if x is increased from 4 cm to 4.01 cm.

Question 18 [2, 3 marks] CA

The diagram shows a Cartesian plane with two concentric circles with radius 2 and 3 units.



P and Q are points that move on the circle with radius of 2 units and 3 units respectively. O, P and Q are collinear at any time. Point T moves inside the region between the two circles in such a way that its x – coordinate is the same as Q's and its y-coordinate is the same as P's. Assume that the line OPQ makes an angle of α radians with the positive x – axis.

(a) Find the parametric equations of the locus of T in terms of α.

(b) Find the gradient of the tangent to the locus of T when $\alpha = \frac{\pi}{3}$.

Question 19 (5 marks) CA

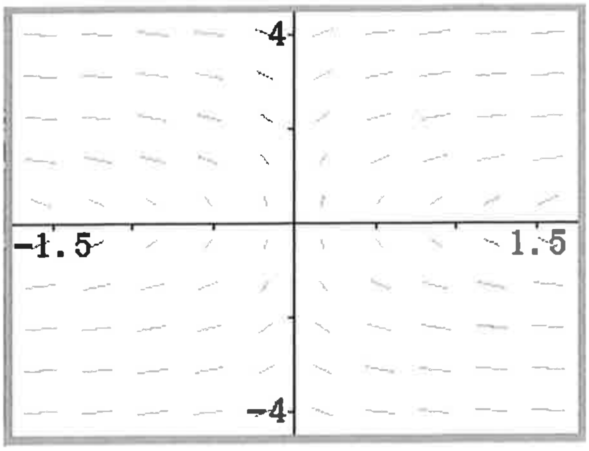
An object suspended on the end of a spring oscillates in Simple Harmonic Motion about its mean position with a frequency of 4 cycles per second and an amplitude of 5 cm.

(i) What is the exact speed of the object as it passes through its mean position? [3]

(v) What is the exact distance travelled by the object in 1 second? [2]

Question 20 (6 marks)

The first-order differential equation, $\frac{dy}{dx} = \frac{1}{xy}$, has a slope field shown in the diagram below.



(a) Sketch the particular solution which passes through the points (1, 2) and (1, -2). [2]

(b) Determine the equation of the curve sketched in part (b). [4]

Question 21 (6 marks) CA

A biologist applies the logistic model to the growth of bacteria in an experiment. She models the population $P(t)$ after t minutes according to the differential equation

$$\frac{dP}{dt} = \frac{P}{1000}\left(3 - \frac{P}{10}\right)$$

where P is in millions. The initial population is 100 000.

(a) What is the growth rate of the bacteria when the population reaches 1 million? [2]

(b) What is the population after one hour? [3]

(c) What is the limiting population? [1]

Question 22 (7 marks) CA

A weight on the end of a spring is oscillating. It's displacement, x metres, from the mean position at time t seconds is given by $x = 3\sin(kt + \frac{\pi}{3})$, where $k > 0$.

(a) Show that the weight is moving with simple harmonic motion. [2]

(b) Determine the value of k given that the weight has an acceleration of $a = (-15x)\ m/s^2$. [2]

(c) Determine the distance travelled by the weight during the third second. [3]

Question 23 (4 marks) CA

If X is a Binomial random variable with probability of success p in each of n trials, then:

$$\bar{X} = np$$

and $$s = \sqrt{np(1-p)}\,.$$

A fair die is rolled 12 times. Let X = the number of sixes rolled.

(a) Determine $\bar{X}$ and s exactly. [2]

Suppose that the above experiment is carried out 40 times (each time with 12 rolls of the die).

Let Y = the average number of sixes rolled per trial over the 40 trials.

(b) Use an appropriate normal distribution to find the probability (accurate to 4 decimal places) that Y is less than 1.8. [2]

Question 24 (4 marks) CA

In a dairy, a machine produces blocks of butter purported to weigh 500 g.

The owners suspect that the mean weight of the blocks is not actually 500 g and measure a random sample of 300 blocks, yielding a mean of 498 g and a standard deviation of 2.4 g.

(a) Determine a 95% confidence interval for the true mean weight (correct to 2 decimal places). [2]

(b) How many blocks should be in a sample in order to be 95% confident that the true mean is within 0.100 g of the sample mean? [2]

Question 25 (5 marks) CA

Consider the curve that passes through the origin and satisfies

$$\frac{dy}{dx} = e^{x+y} \quad .$$

(a) Prove that $e^x - 2 = -e^{-y}$. [3]

(b) Hence show that $x < ln2$ for all points on the curve. [2]

Question 26 (7 marks) CA

A particle moves with rectilinear motion such that $\frac{d^2x}{dt^2} = 3x$. Initially the particle has a velocity of 6 m/s and a displacement of 3 m. The velocity and displacement of the particle are always greater than 0.

(a) Determine the velocity of the particle, v, in terms of x. [3]

(b) Determine x when $t = 4$, correct to the nearest metre. [4]

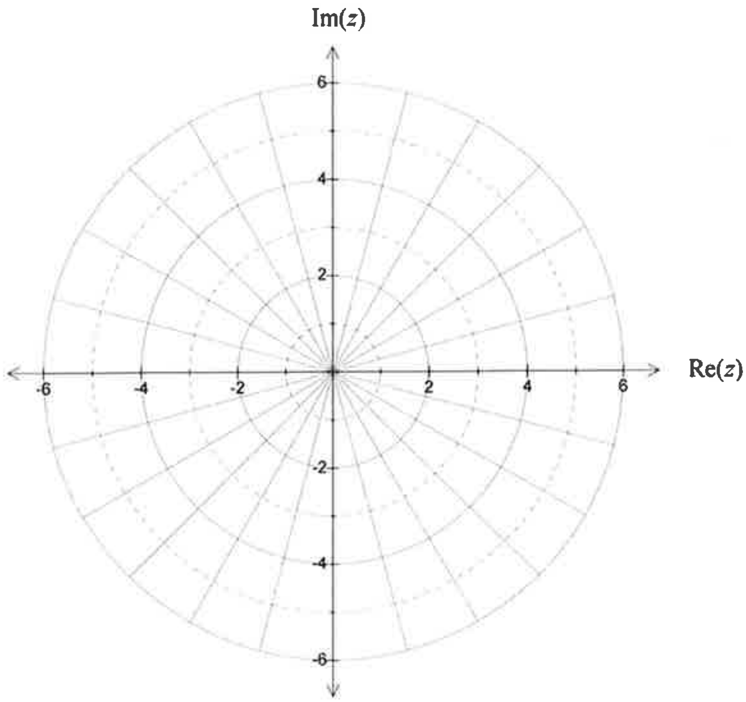
Question 27 (10 marks)

(a) By sketching appropriate diagrams, show that there are exactly two solutions to the simultaneous complex equations: $|z - 1 - i| = 1$ and $arg(z) = \frac{\pi}{4}$.

State these solutions in exact Cartesian form. [5]

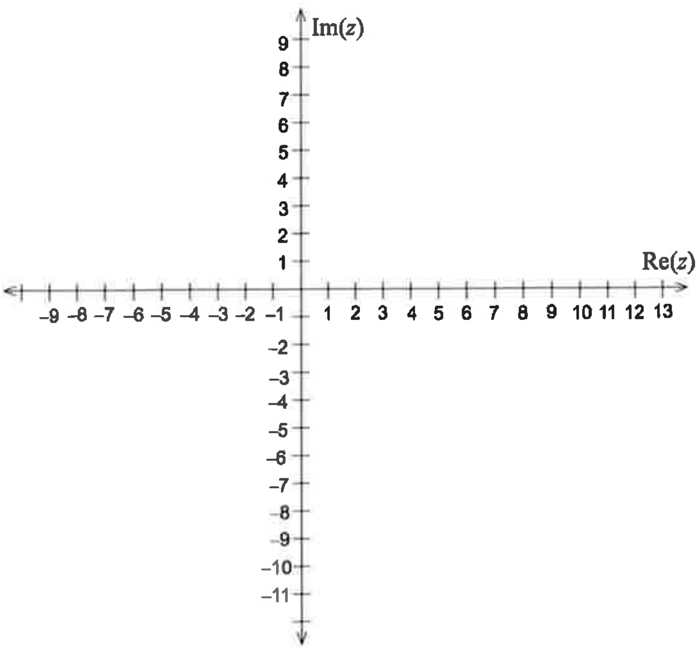
(b) Sketch the following set of points on the argand plane provided: [3]

$\{z: 1 \leq Im\, z \leq 3, \frac{\pi}{3} \leq arg\, z \leq \frac{2\pi}{3}\}$



(c) Sketch the following set of points on the argand plane: [2]

$\{z: |z - 3 - 3i| = |z + 3 + 3i|\}$



Question 28 (8 marks) CA

Bob takes a random sample of 180 'Best Brand' tyres and calculates their circumferences. The sample mean is 150 cm and the sample standard deviation is 2.5 cm.

(a) Using Bob's sample, obtain a 90% confidence interval for the population mean of the circumference of 'Best Brand tyres. [4]

(b) Greg takes a sample of 75 'Best Brand tyres and calculates their circumferences. The sample standard deviation is 3.0 cm and a confidence interval for the population mean of the circumference of 'Best Brand' tyres is found to be $149.5\ cm \leq \mu \leq 151\ cm$. Determine the confidence level of the interval correct to 3 significant figures. [4]

2019 ATAR.

Section Two: Calculator-assumed **65% (85 Marks)**

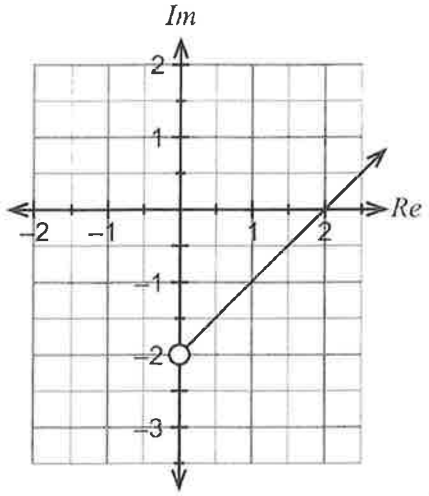
This section has **10** questions. Answer all questions. Write your answers in the spaces provided.

Supplementary pages for planning/continuing your answers to questions are provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.

Working time: 100 minutes.

Question 10 **(5 marks)**

The sketch of the locus of a complex number $z = x + iy$ is shown below.

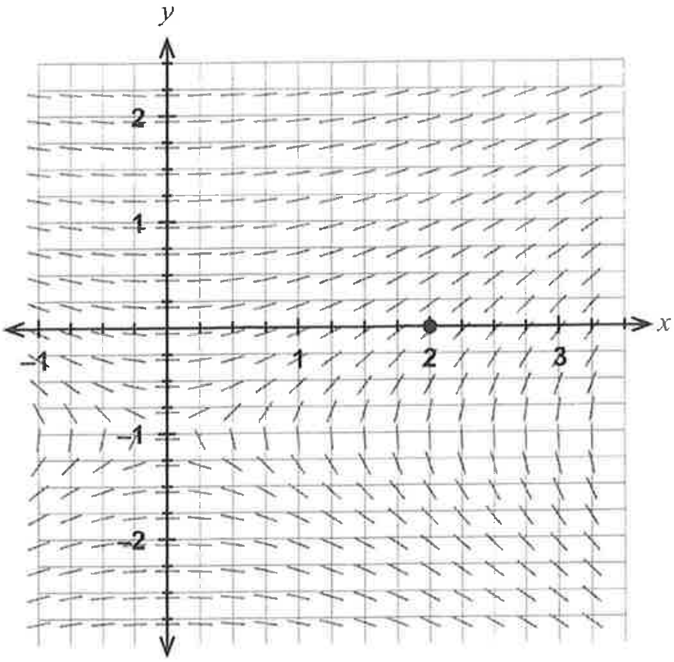


(a) Given that the equation for the above locus is written as $Arg(z - z_0) = k\pi$, determine the value of the constants z_0 and k. (2 marks)

(b) Determine the minimum value for $|z - i|$ as an exact value. (3 marks)

Question 11 **(6 marks)**

The slope field given by $\frac{dy}{dx} = \frac{x}{2y+2}$ is shown in the diagram below.



(a) Calculate the value of the slope field at the point $(2,0)$. (1 mark)

(b) On the diagram above, draw the solution curve that contains the point $(2,0)$. (2 marks)

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(c) Determine the equation for the solution curve that contains the point $(2,0)$. (3 marks)

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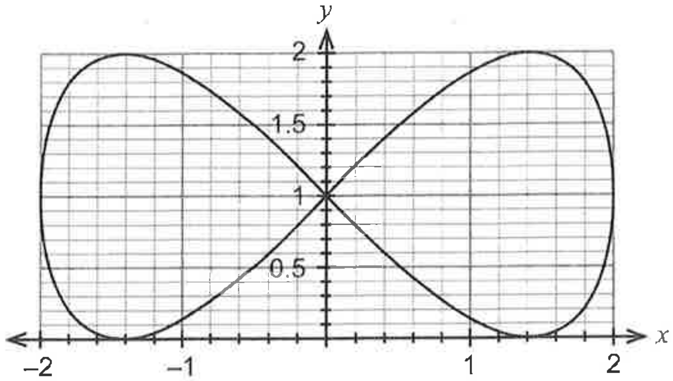
Question 13 **(10 marks)**

The path of a particle is shown below. This particle moves so that its position vector $\underset{\sim}{r}(t)$ is given by $\underset{\sim}{r}(t) = \begin{pmatrix} -2\cos\left(\frac{t}{2}\right) \\ 1 - \sin(t) \end{pmatrix}$ metres, where t is the number of seconds the particle has been in motion.



(a) Determine the starting position of the particle and mark this as point A on the diagram above. (1 mark)

(b) Determine the initial velocity of the particle and illustrate this on the diagram above. (3 marks)

(c) Write the expression, in terms of trigonometric functions, for the distance the particle would travel in completing one circuit of the given path. Do **not** evaluate this expression. (3 marks)

(d) Determine the Cartesian equation for the path of the particle. (3 marks)

Question 14 **(10 marks)**

Trucks carrying iron ore for the Croc Rock mining company arrive at a weighing station. The service time T per truck is defined to be the time elapsed from the moment a truck enters the station zone, including the time to be positioned and then weighed, up to the time it leaves the zone.

It is known that the population mean $\mu(T) = 80$ seconds and the population standard deviation $\sigma(T) = 20$ seconds.

At the Croc Rock weighing station, 100 trucks are weighed.

(a) State the (approximate) distribution of the sample mean service time per truck for the 100 trucks. (3 marks)

(b) What is the probability that the sample mean service time will be more than 83 seconds? (2 marks)

Suppose that more than 100 trucks were weighed at the Croc Rock weighing station.

(c) How would this affect your answer to part (b)? Explain without recalculation. (2 marks)

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It is desired that the probability that the sample mean service time will be between 80 seconds and 82 seconds is greater than 40%.

(d) Determine the minimum number of trucks that will need to be weighed. (3 marks)

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Question 15 **(9 marks)**

A random sample of n commuters in Melbourne in August 2018 found that the average time to commute to work was 40 minutes. Repeated sampling of the mean indicated that the standard deviation of the sample mean was 3 minutes.

(a) Determine a 90% confidence interval for the population mean commuting time μ to work, correct to 0.01 minutes. (3 marks)

Another random sample of $2n$ commuters in November 2018 found that the average time to commute to work was 45 minutes. Assume that both the August and November samples were drawn from the same population.

(b) What is the standard deviation of the sample mean for the November sample, correct to 0.01 minutes? (2 marks)

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Suppose that the August and November samples are combined to form a sample with $3n$ commuters. Consider 90% confidence intervals for the following samples for the purpose of determining the population mean commuting time μ.

90% confidence interval	Sample	Size
A	August	n
N	November	$2n$
C	Combined	$3n$

(c) Which of the three confidence intervals, A, N or C, will provide the greatest precision in determining the population mean μ? Justify your answer. (2 marks)

(d) Which of the three confidence intervals, A, N or C, contains the true value of the population mean μ? Justify your answer. (2 marks)

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Question 16 **(12 marks)**

Plane Π_1 has Cartesian equation $z = 2x + y + 4$.

(a) Determine a vector that is normal to plane Π_1. (2 marks)

Line L has equation $\underset{\sim}{r} = \begin{pmatrix} 2 \\ 0 \\ 3 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix}$.

(b) Determine the point of intersection between line L and plane Π_1. (3 marks)

Plane Π_2 contains line L and is perpendicular to plane Π_1.

(c) Determine the vector equation for plane Π_2. (4 marks)

Sphere S has vector equation $|\underset{\sim}{r} - (3\underset{\sim}{i} + \underset{\sim}{j} + 4\underset{\sim}{k})| = \sqrt{35}$.

(d) Determine whether line L is a tangent to sphere S. Justify your answer. (3 marks)

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Question 17 **(8 marks)**

In Australia, the killing of humpback whales was banned in 1963.

At the end of 2018, 45 years later, the population P of migrating humpback whales off the coast of Western Australia was estimated at 30 000, i.e. $P(45) = 30\ 000$.

(a) Assuming that the population of humpback whales had been increasing at an instantaneous rate equal to 10% of the population, estimate the number of humpback whales at the end of 1963. (3 marks)

To model the growth in the population from the end of 2018, a marine biologist suggests that the rate of growth be modelled by the equation below.

$$\frac{dP}{dt} = 0.1P - \frac{P^2}{700\ 000}$$

The biologist re-defines $P(0) = 30\ 000$, i.e. t = number of years from the end of 2018.

(b) If $P(t)$ is written in the form $P(t) = \dfrac{a}{1 + be^{-ct}}$, determine the values of the constants a, b and c. (2 marks)

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(c) Hence determine the year during which the population of humpback whales off the coast of Western Australia will reach double that estimated at the end of 2018. (2 marks)

(d) State the major difference in the variation in the population $P(t)$ using the model in part (b) compared with that in part (a). (1 mark)

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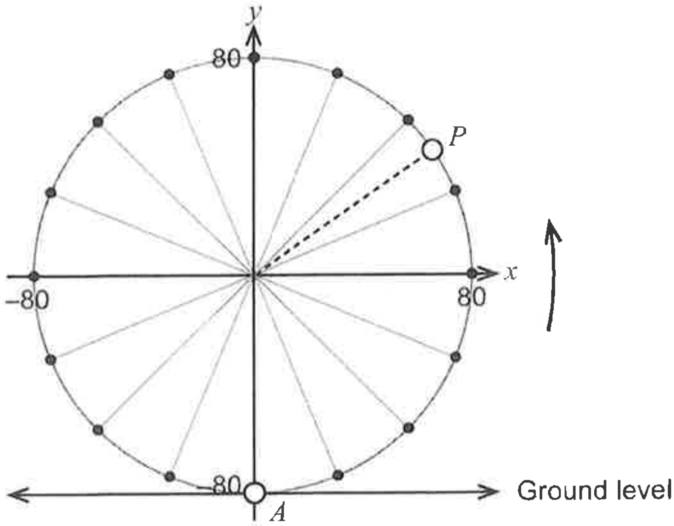
Question 18 **(11 marks)**

A ferris wheel has a radius of 80 metres and rotates in an anticlockwise direction at a rate of one revolution every 72 seconds. The ferris wheel has 16 cars that are equally spaced around the wheel as shown in the diagram.

A coordinate system is set up so that the centre of the ferris wheel is at the origin and the ground level has equation $y = -80$. Passengers begin their ride when a car is at position A $(0, -80)$.

Consider a passenger in a car at position P.

Let t = the number of seconds the ride has been in progress from position A.
θ = the angle in radians that the car has rotated from position A.
y = the height of a car above the centre of the ferris wheel (metres).



(a) Show that $\dfrac{d\theta}{dt} = \dfrac{\pi}{36}$ radians per second. (1 mark)

(b) Given that $y(\theta) = 80\sin(\theta + \alpha)$, explain why $\alpha = -\dfrac{\pi}{2}$. (1 mark)

(c) Determine how quickly a passenger is moving upward when they are 100 metres above the ground, correct to the nearest 0.01 metres per second. (4 marks)

(d) Show that function $y(t)$ satisfies the condition for simple harmonic motion. (2 marks)

A different passenger happens to be in a car that is two cars ahead of a particular car on the ferris wheel.

(e) At what speed, correct to the nearest 0.01 metres per second, is the trailing passenger moving upward when the other passenger is moving downward at exactly the same speed? (3 marks)