2025/108476

Government of **Western Australia**
School Curriculum and Standards Authority

MATHEMATICS APPLICATIONS

Calculator-assumed

ATAR course examination 2025

Marking key

Marking keys are an explicit statement about what the examining panel expect of candidates when they respond to particular examination items. They help ensure a consistent interpretation of the criteria that guide the awarding of marks.

Section Two: Calculator-assumed **65% (100 Marks)**

Question 6 **(13 marks)**

Josh lives in Sydney and Simon lives in Perth. They each decide to drive to Alice Springs to meet before driving together to Darwin. Simon and Josh will both leave for Alice Springs on Saturday, 2 January.

Due to traffic in Sydney, Josh can only drive 250 km on the first day. After this he plans to drive 3 km more each day than the previous day.

Let n represent the number of days spent driving.

(a) Write a recursive rule to model the distance Josh drives each day. (2 marks)

Solution
$T_{n+1} = T_n + 3, \quad T_1 = 250$
Specific behaviours
✓ states correct rule ✓ states correct value for T_1

(b) Calculate how far Josh is away from Sydney by the end of day four. (1 mark)

Solution
250 + 253 + 256 + 259 = 1018 kilometres
Specific behaviours
✓ calculates correct distance

The distance Simon drives each day can be represented by the rule $T_n = 220(1.06)^{n-1}$.

(c) Interpret what the numbers 220 and 1.06 represent in the context of the question. (2 marks)

Solution
The 220 represents the distance travelled in kilometres by Simon by the end of day one. 1.06 represents a 6% increase in distance travelled each day.
Specific behaviours
✓ correctly interprets what the 220 represents in the context of the question ✓ correctly interprets what the 1.06 represents in the context of the question

(d) Determine the day and date when Simon drove 330.8 km. (3 marks)

Solution
$330.8 = 220(1.06)^{n-1}$ $n = 8$ or states $T_8 = 330.8$ Therefore Simon travels 330.8 km on day 8 which is Saturday, 9 January.
Specific behaviours
✓ solves correctly for n or states $T_8 = 330.8$ ✓ states correct day ✓ states correct date

The approximate distance from Sydney to Alice Springs is 2770 km and from Perth to Alice Springs is 2550 km.

(e) Determine who is the first to arrive in Alice Springs. Justify your answer. (3 marks)

Solution
Josh →total distance after 10 days = 2635 kilometres total distance after 11 days = 2915 kilometres Therefore Josh will arrive on the 11th day. Simon →total distance after 9 days = 2528.09 kilometres total distance after 10 days = 2899.77 kilometres Therefore Simon will arrive on the 10th day. Simon will arrive in Alice springs first.
Specific behaviours
✓ determines Simon's correct day of arrival ✓ determines Josh's correct day of arrival ✓ concludes who will arrive first with justification based on the day of arrival

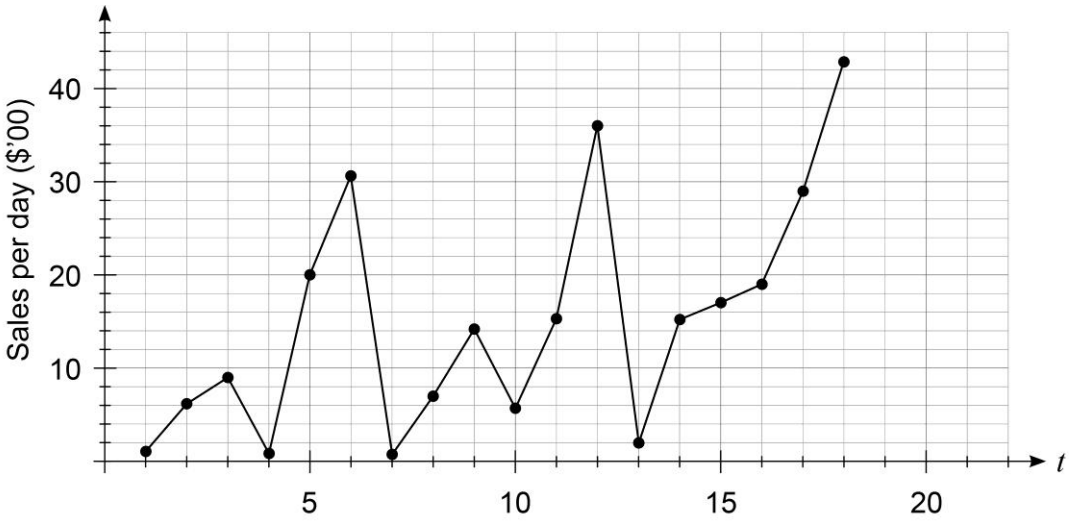
After Josh and Simon meet in Alice Springs, they drive together to Darwin. The distance they travel each day can be represented by the rule $T_{n+1} = aT_n + b$.

(f) Given they travel 380 km on day one, 309 km on day two and 269.95 km on day three, determine the value of a and b. (2 marks)

Solution
$309 = a(380) + b$ $269.95 = a(309) + b$ $a = 0.55,\ b = 100$
Specific behaviours
✓ determines the correct value for a ✓ determines the correct value for b

Question 7 **(13 marks)**

A trial pop-up Christmas gift wrapping store has opened in a shopping centre. To determine whether the shopping centre will continue the pop-up store in future years, data on the sales per day, in hundreds of dollars, have been recorded. The graph below shows the time series data of the sales for the Christmas gift wrapping store since the beginning of December.



(a) Identify the moving average that would be the most suitable for the data displayed in the graph above. (2 marks)

Solution
six-point centred moving average
Specific behaviours
✓ states correct moving average ✓ states centred moving average

A more detailed view of the data (correct to two decimal places) is given in the table below.

Week	**Day**	**Time (t)**	**Seasonal mean**	**Sales per day as a percentage of the seasonal mean**	**Deseasonalised data (d)**
1	Tuesday	1	11.30	9.73	12.99
	Wednesday	2		54.87	10.27
	Thursday	3		79.65	10.02
	Friday	4		7.96	1.90
	Saturday	5		177.00	13.87
	Sunday	6		270.80	12.25
2	Tuesday	7	13.13	6.09	9.45
	Wednesday	8		53.30	11.59
	Thursday	9		108.12	15.81
	Friday	10		42.64	11.85
	Saturday	11		116.50	**A**
	Sunday	12		273.35	14.37
3	Tuesday	13	20.83	9.60	23.61
	Wednesday	14		72.96	25.17
	Thursday	15		81.60	18.93
	Friday	16		91.20	40.19
	Saturday	17		139.20	20.11
	Sunday	18		205.44	17.13

(b) Complete the seasonal index table below. (2 marks)

Day	Tuesday	Wednesday	Thursday	Friday	Saturday	Sunday
Daily index	**8.47%**	**60.38%**	89.79%	47.27%	144.23%	249.86%

Solution
Tuesday – 8.47% Wednesday – 60.38%
Specific behaviours
✓ determines the correct daily index for Tuesday ✓ determines the correct daily index for Wednesday

(c) Using the table from part (b), complete the following sentence: (2 marks)

On average, Thursday's sales are _____ % _____ the trend value.

Solution
'10.21%' 'below' or '89.79%' 'of'
Specific behaviours
✓ calculates the correct percentage ✓ correctly states 'below' or 'of'

(d) Given the sales for Saturday in week 2 were \$1530, determine the value of **A** in the table on page 6. (2 marks)

Solution
$A = 15.3 \div 1.4423$ $= 10.61$
Specific behaviours
✓ uses the correct seasonal index ✓ determines correct value of A

The equation of the least-squares line using the deseasonalised data is $\hat{d} = 1.03t + 5.79$.

(e) Use the equation of the least-squares line to predict the sales that will be made on Thursday of week 4. (3 marks)

Solution
Thursday week 4: $t = 21$ $\hat{d} = 1.03(21) + 5.79 = 27.42$ $27.42 \times 0.8979 = 24.62$ Therefore the sales for Thursday of week 4 will be \$2462.
Specific behaviours
✓ determines the correct deseasonalised value ✓ multiplies by the correct seasonal index ✓ states correct value for the sales

Question 7 (continued)

(f) Comment on this prediction. (2 marks)

Solution
The prediction would not be valid due to extrapolation.
Specific behaviours
✓ states prediction is not valid ✓ states due to extrapolation

Question 8 **(13 marks)**

Peter works at a hairdressing salon. He hopes to purchase his own hairdressing salon in three years' time. He has already saved $12 500 and is going to open an investment account with this money.

Peter considers the following investment options.

Option	Annual interest rate (%)	Number of compounding periods each year	Balance after three years ($)
A	2.75	12	**13 573.70**
B	**2.80**	365	13 595.32

(a) Complete the table above. (2 marks)

Solution
see table above
Specific behaviours
✓ correctly calculates the balance after three years for option A ✓ correctly determines the annual interest rate for option B

Peter selects a different investment option which pays an annual interest rate of 3.2%, compounded quarterly. He also decides to deposit $350 into his account at the end of each quarter.

(b) State a recurrence relation that will model the balance of his investment at the end of each quarter. (2 marks)

Solution
$T_{n+1} = 1.008T_n + 350, \quad T_0 = 12\ 500$
Specific behaviours
✓ uses correct interest rate (quarterly) ✓ states correct rule

(c) Determine the value of Peter's investment after three years. (2 marks)

Solution
$T_{12} = 18\ 144.05$ Peter's investment is $18 144.05 after three years. or uses finance app with N = 12, I = 3.2, PV = −12 500, PMT = −350, P/Y = C/Y = 4 gives FV = 18 144.05152
Specific behaviours
✓ uses correct value for the number of instalment periods ✓ states correct value of investment

Question 8 (continued)

Peter's goal was to have at least \$20 000 in his account after three years.

(d) Determine the equal quarterly deposits he should have made to reach his goal. (2 marks)

Solution
uses the finance app with N = 12, I = 3.2, PV = −12 500, FV = 20 000, P/Y = C/Y = 4 gives PMT = −497.97… Therefore quarterly deposits needed to be \$497.98.
Specific behaviours
✓ correctly states the parameters: payments periods/year (P/Y), compounding periods/year (C/Y), present value (PV), future value (FV) ✓ correctly states remaining parameters and gives correct quarterly deposits

After saving for three years, Peter finds a hairdressing salon he wants to buy. Even though he has been saving, he still needs to borrow \$45 000 to purchase the salon.

He has secured a loan for this amount, with the balance owing at the end of each month modelled by the recurrence relation $T_{n+1} = 1.004T_n - 500, \quad T_0 = 45\,000.$

(e) Calculate the annual interest rate being charged. (1 mark)

Solution
0.004 × 12 = 0.048 = 4.8%
Specific behaviours
✓ calculates the correct annual interest rate

(f) Assuming the conditions of the loan remain the same, determine

(i) how many months it will take Peter to pay off the loan. (1 mark)

Solution
using the recurrence relation: $T_{111} = 395.96, T_{112} = -102.45$ Therefore it will take 112 months to pay off the loan. or uses the finance app with I = 4.8, PV = 45000, PMT = −500, FV = 0, P/Y = C/Y = 12 gives N = 111.79 Therefore it will take 112 months to pay off the loan.
Specific behaviours
✓ states correct number of months

(ii) the total amount paid by Peter over the duration of the loan. (3 marks)

Solution
using the recurrence relation: $T_{111} = 395.96,\ T_{112} = -102.45$ $112 \times 500 - 102.45 = 55\ 897.55$ Therefore total amount paid = \$55 897.55 or $T_{111} = 395.96,\ 111 \times 500 = 55\ 500$ $1.004 \times 395.96 = 397.54$ $55\ 500 + 397.54 = 55\ 897.54$ Therefore total amount paid = \$55 897.54 or uses amortization with PM1 = 1, PM2 = 112, I = 4.8, PV = 45 000, PMT = −500, P/Y = C/Y = 12 gives BAL = −102.45, ΣINT = −10 897.55, ΣPRN = −45 102.45 Therefore total amount paid = 45 102.45 + 10 897.55 − 102.45 = \$55 897.55
Specific behaviours
✓ correctly calculates either 112 × 500 or 111 × 500 ✓ correctly compares T_{111} to T_{112} or calculates interest on T_{111} ✓ states correct total amount paid

Question 9 **(15 marks)**

A statistician is analysing whether chess players perform better when they stay in a hotel with a higher rating. Hotels are rated by online reviews on a scale from 3 to 5, with 3 being the worst and 5 being the best rating that can be achieved. Chess players are rated by an Elo score ranging from 1000 to 2800. The higher the Elo score the better the performance by the chess player.

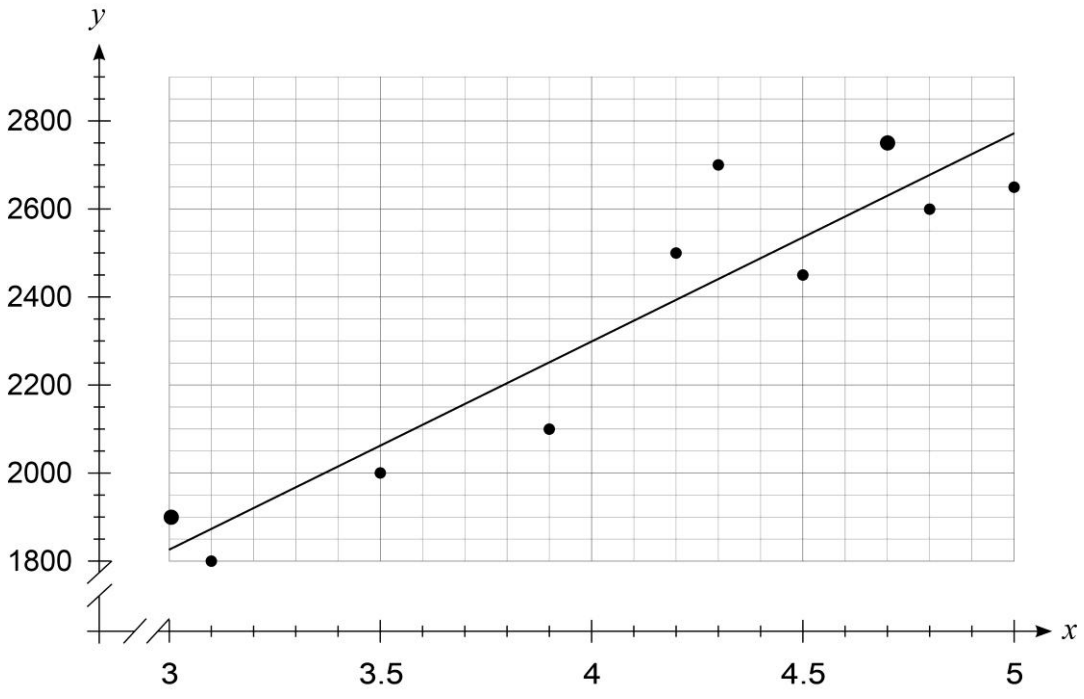
The table below shows the hotel rating and the Elo score for ten chess players.

Hotel rating (x)	5	4.3	3.9	3.5	4.2	4.8	3.1	4.5	4.7	3
Elo rating (y)	2650	2700	2100	2000	2500	2600	1800	2450	2750	1900

(a) Identify the response variable for these data. (1 mark)

Solution
Elo rating
Specific behaviours
✓ states correct response variable

(b) On the graph below, plot the last two data points from the table. (1 mark)



Solution
see graph above
Specific behaviours
✓ correctly plots both points

(c) Determine the equation of the least-squares line and correlation coefficient for these data. (2 marks)

Solution
$\hat{y} = 472.098x + 409.397$ and r = 0.929
Specific behaviours
✓ determines correct equation ✓ determines correct correlation coefficient

(d) Draw the least-squares line on the graph shown in part (b) on page 10. (1 mark)

Solution
see graph in part (b)
Specific behaviours
✓ correctly draws least-squares line

(e) Interpret the gradient of the least-squares line in the context of the question. (2 marks)

Solution
For every one increase of hotel rating, the Elo rating increases by 472.1.
Specific behaviours
✓ refers to the correct variables in the context of the question ✓ states correct increase of 472.1

(f) (i) Use your equation, from part (c) on page 10, to predict a professional chess player's Elo rating if they stay in a hotel with a rating of 4.5. (1 mark)

Solution
$\hat{y} = 472.098(4.5) + 409.397$ $\hat{y} = 2533.84$
Specific behaviours
✓ determines correct Elo rating prediction

(ii) Explain why your prediction is different from the value given in the table on page 10. (1 mark)

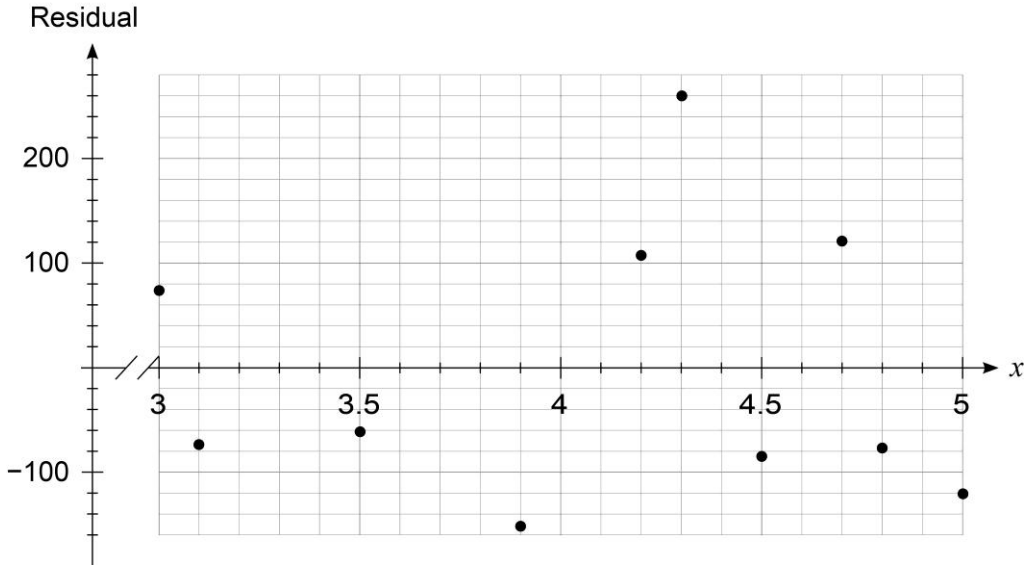
Solution
The value in the table is the true result, whereas the prediction comes from the least-squares line.
Specific behaviours
✓ gives the correct explanation

(iii) Outline **one** reason as to why extrapolation in the context of this question would make no sense. (1 mark)

Solution
Hotel ratings are only from 3 to 5. Therefore, anything higher or lower would not be possible.
Specific behaviours
✓ gives an appropriate reason

Question 9 (continued)

The residual plot for these data is shown below.



(g) Assess whether a linear model is appropriate for these data. Justify your answer. (2 marks)

Solution
A linear model is appropriate because there is no pattern in the residual plot and the correlation coefficient is strong.
Specific behaviours
✓ states a linear model is appropriate ✓ provides appropriate justification

(h) State **one** reason that might contribute to a chess player's Elo rating, other than the hotel rating. (1 mark)

Solution
Answers could include: • level of opponent • physical or mental health on day of competition • level of preparation prior to competition
Specific behaviours
✓ states one suitable reason
Accept other relevant answers.

If a chess player has an Elo rating above 2400, they are classified as an International Master.

(i) For those chess players who are classified as an International Master, determine what percentage of the variation in Elo rating can be explained by the variation in hotel rating for these data. (2 marks)

Solution
$r^2 = 0.11779$ Therefore 11.78%
Specific behaviours
✓ determines correct value of r^2 ✓ states the correct percentage

Question 10 **(13 marks)**

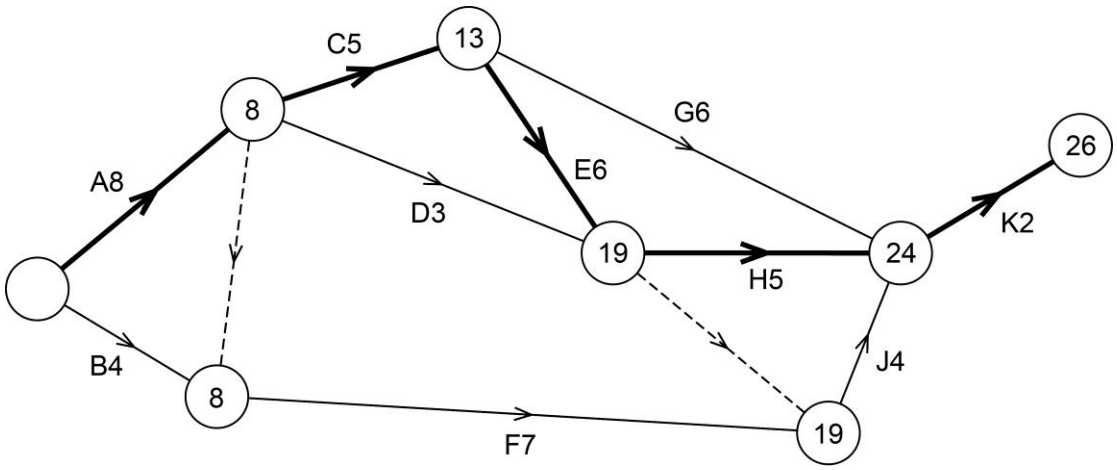
A restaurant is due to be renovated. The owner has identified 10 activities that need to be carried out to complete the renovation project. The network below shows these activities, labelled with capital letters, and the time to complete them in days.

(a) Identify the immediate predecessor/s for activity J. (2 marks)

Solution
F, D, E
Specific behaviours
✓ identifies F as an immediate predecessor ✓ identifies D and E as immediate predecessors (before the dummy link)

(b) Determine the critical path and minimum completion time for the project. Show working on the network. (3 marks)

Solution



Critical path is ACEHK. Minimum completion time is 26 days.

Specific behaviours
✓ shows systematic working on network ✓ determines critical path ✓ determines minimum completion time

(c) Calculate the latest starting time (LST) for activity G. (1 mark)

Solution
day 18
Specific behaviours
✓ states correct latest starting time

Question 10 (continued)

(d) (i) Determine the float time for activity F. (1 mark)

Solution
five days
Specific behaviours
✓ states correct number of days

(ii) Explain why some activities have a float time of zero days. (2 marks)

Solution
They are on the critical path, so any change to their times will affect the minimum completion times.
Specific behaviours
✓ states activities are on the critical path ✓ states change to times will affect the minimum completion time

The restaurant owner has decided to employ additional workers, allowing activities A, E and H to be reduced by two days each. He claims that the minimum completion time will therefore be reduced by six days.

(e) (i) Determine if the restaurant owner's claim is correct. Justify your answer. (2 marks)

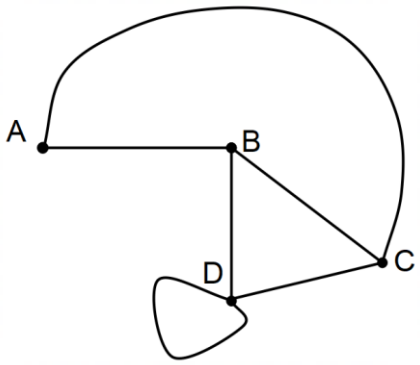
Solution
Minimum completion time is now 21 days (reduction of five days). Therefore the owner is not correct.
Specific behaviours
✓ states correct time reduction or new completion time ✓ states restaurant owner's claim is not correct based on justification

(ii) On the basis of your answer to part (e)(i), did the critical path change? If so, state the new critical path. If not, explain why. (2 marks)

Solution
The critical path has changed. The new critical path is ACEJK.
Specific behaviours
✓ states that the critical path has changed ✓ correctly states new critical path

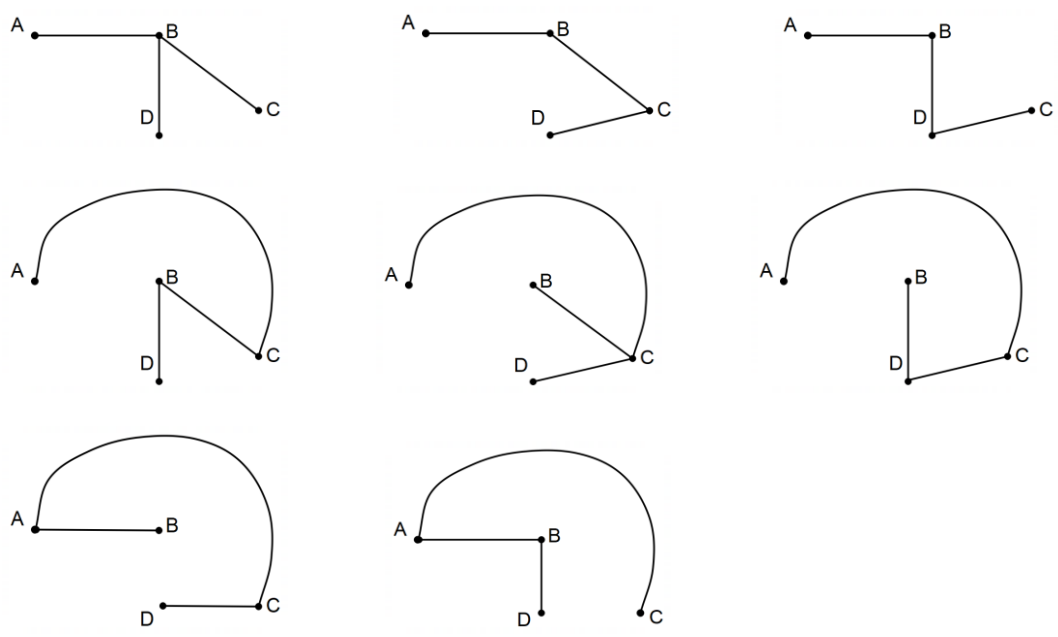
Question 11 **(8 marks)**

The connected graph below shows a rural area where the vertices represent towns, and each edge represents the roads joining each town.



The council plan to install bicycle paths alongside some of the existing roads, to connect the towns to each other.

(a) Draw **four** spanning trees, in the table below, to show how the council could do this. (2 marks)

Solution
Any four spanning trees: 
Specific behaviours
✓ draws two or three correct spanning trees (subgraphs of the connected graph that are trees) ✓ draws four correct spanning trees (subgraphs of the connected graph that are trees)

Question 11 (continued)

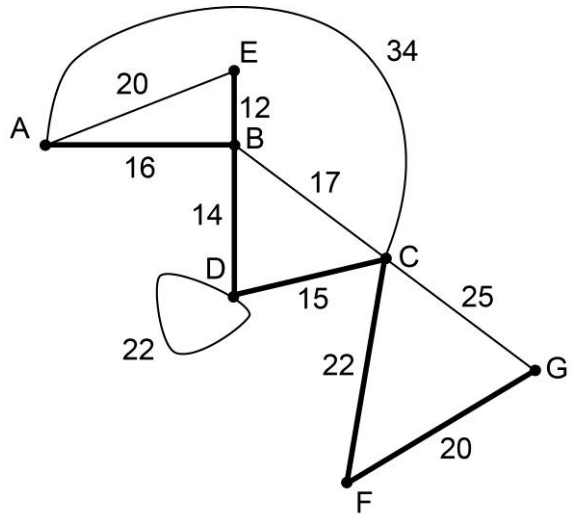
The council would like to include two more towns in their plan to install bicycle paths that will connect the towns to each other.

(b) Determine the minimum number of bicycle paths (edges) they would need to install, if there was a total of six connected towns. (1 mark)

Solution
five edges
Specific behaviours
✓ states correct number of edges

The council decides to include seven towns in their plan. The distances, in kilometres, between the towns are shown on the edges of the network below. The council wants to install the minimum total length of bicycle paths alongside the existing roads between towns.

(c) (i) Highlight, on the network above, the bicycle paths that should be installed. (2 marks)

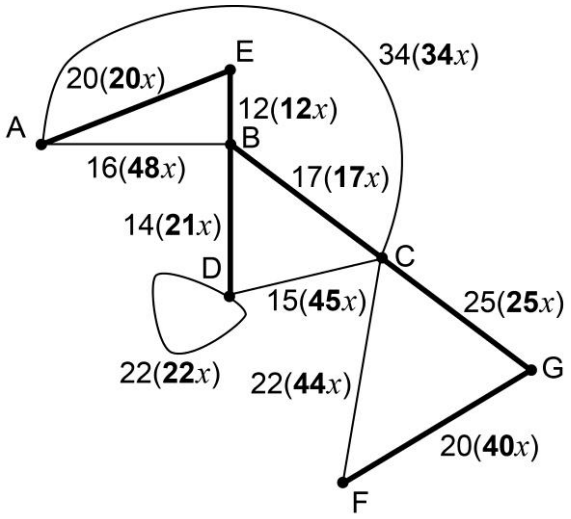
Solution

Specific behaviours
✓ correctly draws at least four edges ✓ correctly draws minimum spanning tree

(ii) Calculate the minimum length of bicycle paths required. (1 mark)

Solution
Minimum length = 16 + 12 + 14 + 15 + 22 + 20 = 99 km
Specific behaviours
✓ states correct minimum length

The contractors employed to install the bicycle paths will charge a base rate (\$/km). However, due to differences in terrain, path BD will cost 1.5 times the base rate per km, paths CF and FG will cost double the base rate per km, and paths AB and CD will cost three times the base rate per km. The council wants to keep the total cost to a minimum.

(d) Highlight, on the network below, the bicycle paths that should be installed and determine the total length of bicycle paths required. (2 marks)

Solution
If x is the base rate (\$/km):    Total length = 20 + 12 + 17 + 14 + 25 + 20 = 108 km
Specific behaviours
✓ draws minimum spanning tree correctly ✓ states correct total length of bicycle paths

Question 12 **(13 marks)**

A retiring mathematics teacher donates $4000 to the school where she has worked for many years to pay for a prize to be awarded to a student at the school's annual prize-giving ceremony.

The school principal sets up an annuity with this money, receiving an interest rate of 0.3% compounded monthly and using $250 at the end of each year to purchase the prize.

(a) (i) Calculate the nominal annual interest rate. (1 mark)

Solution
0.3 × 12 = 3.6%
Specific behaviours
✓ states correct rate

(ii) Determine a recursive rule to model the balance of the annuity at the end of each year. (2 marks)

Solution
Effective annual rate of interest = 3.66% (CAS 2 d.p.) $T_{n+1} = 1.0366T_n - 250, \quad T_0 = 4000$
Specific behaviours
✓ determines effective annual rate of interest ✓ states correct rule

(b) Determine

(i) how much money will be left in the annuity after five years. (1 mark)

Solution
$T_5 = 3442.669$ i.e. $3442.67 or uses finance app with N = 5, I = 3.6, PV = −4000, PMT = 250, P/Y = 1, C/Y = 12 gives FV = 3442.6686 i.e. $3442.67
Specific behaviours
✓ correctly determines the amount left after 5 years

(ii) the number of years the school will be able to award this prize using this annuity. (2 marks)

Solution
$T_{24} = 123.30,\ T_{25} = -122.18$ Therefore the school will be able to award the prize for 24 years. or uses finance app with I = 3.6, PV = −4000, PMT = 250, FV = 0, P/Y =1, C/Y = 12 gives N = 24.507… Therefore the school will be able to award the prize for 24 years.
Specific behaviours
✓ correctly calculates 24th and 25th terms ✓ correctly concludes it is 24 years or ✓ correctly states the parameters: amount paid each period (PMT), present value (PV) ✓ correctly states remaining parameters and gives correct conclusion

The school principal is considering changing this investment to a perpetuity after ten years so there will always be money available to award this prize. The financial institution at that time will offer them an annual interest rate of 4.2% compounded monthly.

The school principal states that the new minimum value of the annual prize should be $130.

(c) Show that the yearly perpetuity amount received by the school will be insufficient to purchase the annual prize. (4 marks)

Solution
$T_{10} = 2775.60$ $2775.60 = 2775.60\left(1 + \frac{0.042}{12}\right)^{12} - x$ $x = 118.85$ Therefore yearly amount = $118.85 or uses finance app with I = 4.2, PV = −2775.60, N = any positive value, FV = 2775.60, P/Y = 1, C/Y = 12, gives PMT = 118.846…, i.e. $118.85 Therefore there is not enough money for the yearly prize (less than $130).
Specific behaviours
✓ correctly determines value of annuity after 10 years ✓ sets up correct equation ✓ calculates correct yearly payment ✓ states correct conclusion about amount of money for the yearly prize or ✓ correctly determines value of annuity after 10 years ✓ correctly states parameters: payment periods/year(P/Y), compounding periods/year(C/Y), present value(PV) and future value(FV) have opposite signs ✓ calculates correct yearly payment ✓ states correct conclusion about amount of money for the yearly prize

Question 12 (continued)

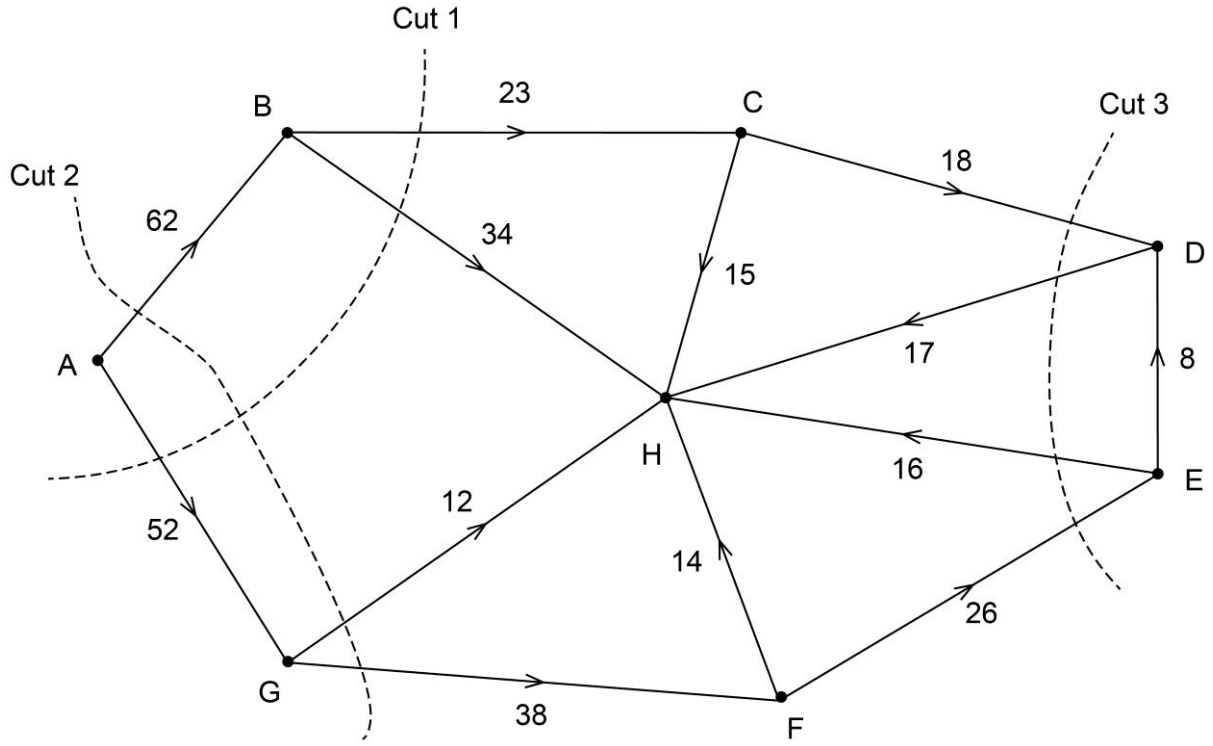
(d) Determine the largest number of years the school principal can maintain the annuity before changing to a perpetuity and receive enough to cover the annual prize of $130.

(3 marks)

Solution
$T_9 = 2918.78$, $T_8 = 3056.89$ uses finance app I = 4.2, PV = −2918.78, N = any positive value, FV = 2918.78, P/Y = 1, C/Y = 12, gives PMT = 124.98, i.e. $124.98, which is not enough uses finance app I = 4.2, PV = −3056.89, N = any positive value, FV = 3056.89, P/Y = 1, C/Y = 12, gives PMT = 130.89, i.e. $130.89, which is enough Therefore the school principal can maintain the annuity for eight years.
Specific behaviours
✓ correctly determines value of annuity after nine and eight years ✓ calculates correct yearly payments (PMT) ✓ states that eight years is the largest number of years the school principal can maintain the annuity

Question 13 **(12 marks)**

In a factory, water is pumped from a holding tank and circulated through machines for cooling purposes. It is then collected at a cooling tank. This is shown in the network below with the numbers indicating how many litres per minute can travel along the connecting pipes.



(a) Identify the source and the sink. (2 marks)

Solution
The source is A and the sink is H.
Specific behaviours
✓ correctly identifies the source ✓ correctly identifies the sink

(b) Determine how many machines are in this cooling circuit. (1 mark)

Solution
six machines
Specific behaviours
✓ correctly identifies six machines

(c) (i) Determine the value of Cut 1 and Cut 2. (2 marks)

Solution
Cut 1 = 109 and Cut 2 = 112
Specific behaviours
✓ correctly determines the value of Cut 1 ✓ correctly determines the value of Cut 2

Question 13 (continued)

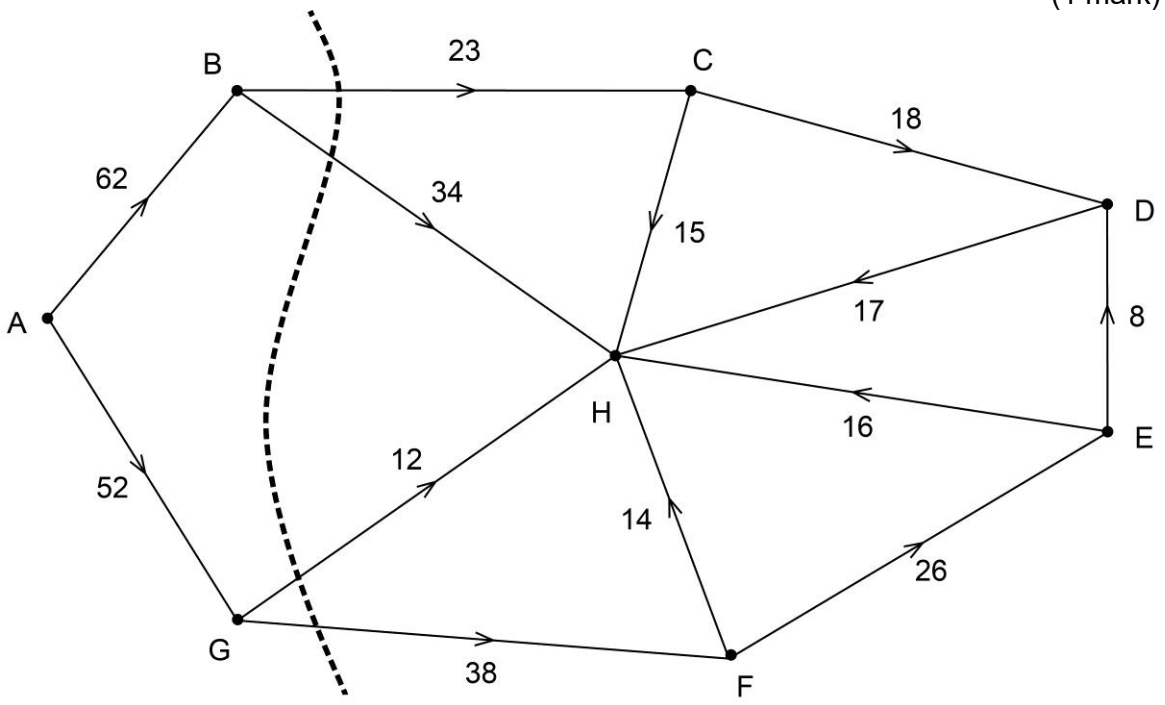
(ii) On the basis of your answer to part (c)(i), determine what can be interpreted about the maximum flow through the network. (1 mark)

Solution
The maximum flow is ≤ 109 litres per minute
Specific behaviours
✓ states correct interpretation

(d) By listing each path used, determine the maximum flow through the network each hour. (4 marks)

Solution
ABH = 34 ABCH = 15 ABCDH = 8 AGH = 12 AGFH = 14 AGFEH = 16 AGFEDH = 8 Maximum flow = 107 litres per minute = 6420 litres per hour
Specific behaviours
✓ lists five correct paths and lengths ✓ lists all correct paths and lengths ✓ determines correct total in litres per minute ✓ determines correct total in litres per hour

(e) Draw the minimum cut that corresponds to the maximum flow on the network below. (1 mark)



Solution
see network above
Specific behaviours
✓ draws correct cut
Accept other correct answers.

(f) Explain why Cut 3 is an invalid cut. (1 mark)

Solution
Cut 3 does not disconnect the sink from the source.
Specific behaviours
✓ gives correct explanation

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