2025/108469

Government of **Western Australia**
School Curriculum and Standards Authority

MATHEMATICS APPLICATIONS

Calculator-free

ATAR course examination 2025

Marking key

Marking keys are an explicit statement about what the examining panel expect of candidates when they respond to particular examination items. They help ensure a consistent interpretation of the criteria that guide the awarding of marks.

Section One: Calculator-free **35% (51 Marks)**

Question 1 **(10 marks)**

When people become ill, their body responds with a change to their white blood cell count (WBCC) measured in ’000s cells per microlitre (cells/μL).

The table below shows a person’s white blood cell count after contracting an illness.
Let n represent the number of hours after contracting an illness.

Time (n hours)	1	2	3	4	5	6
WBCC ’000s (cells/μL)	8.4	8.7	9	**9.3**	**9.6**	**9.9**

(a) The person’s white blood cell count is increasing at a constant rate each hour. Show that this rate is 300 cells/μL. (1 mark)

Solution
8700 – 8400 = 300 or 9000 – 8700 = 300 Therefore the constant rate is 300 cells/μL.
Specific behaviours
✓ correctly shows that the constant rate is 300 cells/μL

(b) Complete the table above. (1 mark)

Solution
see table above
Specific behaviours
✓ determines all correct entries

(c) Determine a simplified rule for the n^{th} term to model the person’s white blood cell count after contracting an illness. (2 marks)

Solution
$T_n = 8.4 + (n-1)(0.3)$ $T_n = 8.1 + 0.3n$
Specific behaviours
✓ uses the rule for an arithmetic sequence and substitutes correct a and d values ✓ correctly simplifies the rule

(d) Determine the white blood cell count in the person's body after 12 hours, if the white blood cell count maintains the same rate of increase. (2 marks)

Solution
$T_{12} = 8.1 + 0.3(12)$ $T_{12} = 11.7$ Therefore 11 700 cells/μL.
Specific behaviours
✓ correctly determines the twelfth term ✓ states correct white blood cell count in cells/μL

After an illness is diagnosed, a person receives the appropriate medication to aid their recovery. An indirect effect of the medication will be an overall reduction in the person's white blood cell count.

A person becomes ill and is given medication. Their white blood cell count ('000s cells/μL) follows the recursive rule $T_m = -0.5T_{m-1} + 20, \quad T_0 = 20$, where m is the number of hours after the person is given the medication.

(e) (i) Determine the white blood cell count in this person at the end of the second hour. (2 marks)

Solution
$T_1 = (-0.5)(20) + 20 = 10$ $T_2 = (-0.5)(10) + 20 = 15$ Therefore the white blood cell count in the body is 15 000 cells/μL at the end of the second hour.
Specific behaviours
✓ determines correct value for T_1 ✓ states correct white blood cell count in cells/μL

(ii) Determine if the given medication, in the long run, can ensure this person's white blood cell count does not fall below 13 000 cells per microlitre. (2 marks)

Solution
$x = -0.5x + 20 \Rightarrow x = \frac{20}{1.5} = \frac{200}{15} = \frac{40}{3} = 13\frac{1}{3} \approx 13.3$ i.e. approximately 13 300 cells/μL, which is greater than 13 000 Therefore the given medication will ensure the person's white blood cell count does not fall below 13 000 cells per microlitre.
Specific behaviours
✓ determines correct steady-state value ✓ correctly justifies medication will ensure the person's white blood cell count does not fall below 13 000 cells per microlitre

Question 2 **(11 marks)**

John surveyed 300 people to collect data on the amount of money spent on a T-shirt (T-shirt price) and the age of the person the T-shirt was purchased for. The information is displayed in the table below. John concluded there is a possible relationship between the T-shirt price and the age of the recipient.

		T-shirt price ($)			
		10–19	**20–39**	**40–59**	**Total**
Age (years)	**4–10**	21	**12**	3	**36**
	11–17	**21**	48	39	108
	18–24	15	54	**87**	156
	Total	**57**	114	129	300

(a) Complete the two-way table above. (2 marks)

Solution
see table above
Specific behaviours
✓ determines two correct values ✓ determines all correct values

To see if John's conclusion is reasonable, the following analyses were conducted.

(b) Assuming age is the explanatory variable,

(i) show that the percentage of T-shirts purchased for 4–10-year-olds with a price of 40–59 dollars is 8.3 correct to one decimal place. (1 mark)

Solution
$\frac{3}{36} = \frac{1}{12}$ $\frac{1}{12} \times 100 = \frac{25}{3} \approx 8.3$
Specific behaviours
✓ shows how to correctly calculate the percentage

(ii) complete the two-way percentaged table below. All percentages, apart from the totals, are correct to one decimal place. (2 marks)

		T-shirt price ($)			
		10–19	**20–39**	**40–59**	**Total**
Age (years)	**4–10**	**58.4**	33.3	**8.3**	**100**
	11–17	19.4	**44.5**	36.1	**100**
	18–24	**9.6**	34.6	55.8	100

Solution
see table above
Specific behaviours
✓ determines four correct values ✓ determines all correct values

(c) Assuming T-shirt price is the explanatory variable, complete the two-way percentaged table below. All percentages, apart from the totals, are correct to one decimal place. (2 marks)

		T-shirt price ($)		
		10–19	**20–39**	**40–59**
Age (years)	**4–10**	36.8	10.5	**2.4**
	11–17	**36.8**	**42.1**	30.2
	18–24	**26.4**	47.4	67.4
	Total	**100**	100	**100**

Solution
see table above
Specific behaviours
✓ determines four correct values ✓ determines all correct values

(d) (i) State an association that can be observed from the two-way percentaged table in part (b)(ii) on page 6. (1 mark)

Solution
As the age of the person for whom the T-shirt is being purchased increases, the percentage of people purchasing the 40–59 dollar T-shirts increases. or As the age of the person for whom the T-shirt is being purchased increases, the percentage of people purchasing the 10–19 dollar T-shirts decreases.
Specific behaviours
✓ states correct association

(ii) State an association that can be observed from the two-way percentaged table in part (c). (1 mark)

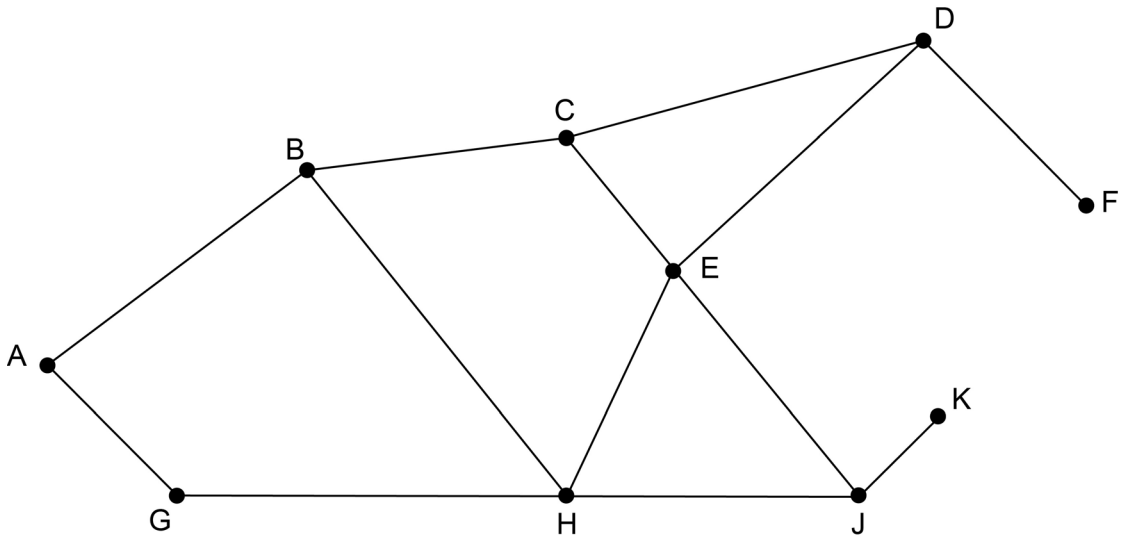
Solution
As the T-shirt price increases, the percentage of T-shirts being purchased for 4–10-year-olds decreases. or As the T-shirt price increases, the percentage of T-shirts being purchased for 18–24-year-olds increases.
Specific behaviours
✓ states correct association

(e) On the basis of your answers to part (d), explain whether John's conclusion is reasonable. (2 marks)

Solution
Since there are associations in the row and column percentages, John's conclusion, that there is a possible relationship, seems reasonable.
Specific behaviours
✓ comments on the associations for rows and columns ✓ correctly states that John's conclusion, that there is a possible relationship, seems reasonable

Question 3 **(9 marks)**

A bus service provides passengers with a convenient way of travelling around the city. The graph below shows the roads (edges) used by buses when travelling between bus stops (vertices).



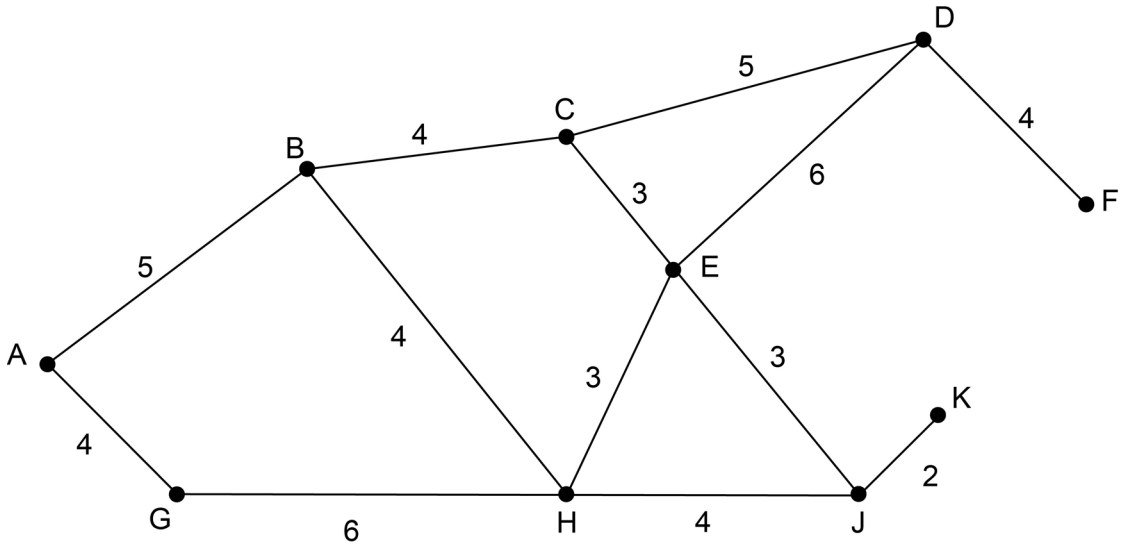
(a) Explain why this is a simple graph. (2 marks)

Solution
It is a simple graph as it does not contain any loops or multiple edges.
Specific behaviours
✓ correctly states there are no loops ✓ correctly states there are no multiple edges

(b) Select **three** terms from those listed below that could be used to accurately describe the graph shown above. Circle the correct terms. (3 marks)

Solution
complete (connected) directed (planar) Eulerian semi-Eulerian Hamiltonian (semi-Hamiltonian)
Specific behaviours
✓ correctly circles one term and no more than two incorrect ✓ correctly circles two terms and no more than one incorrect ✓ correctly circles three terms only, no other terms are circled

The average times, in minutes, taken for buses to travel between the bus stops are shown on the edges of the graph below.

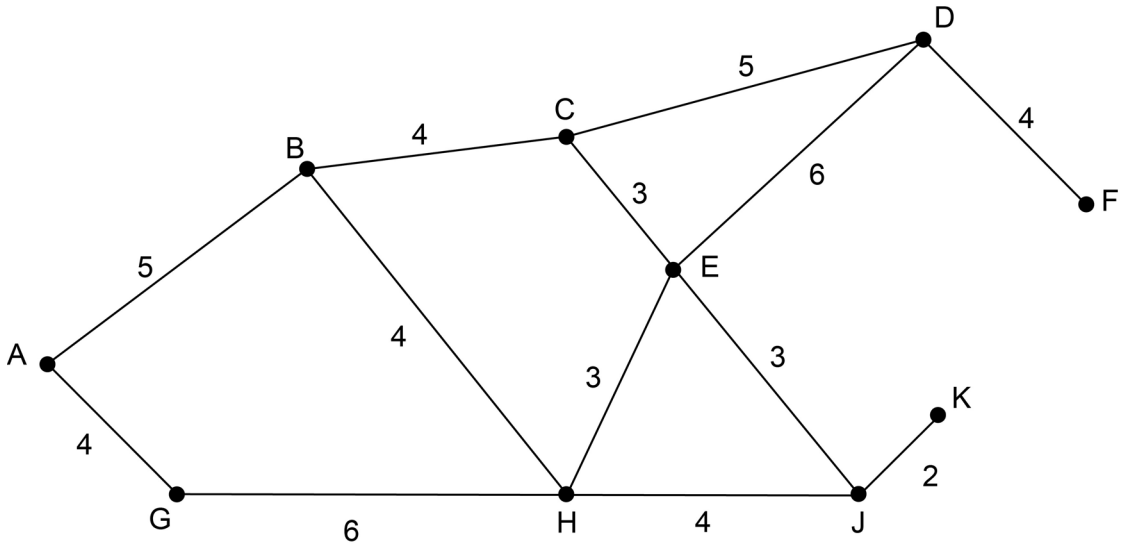


A tourist visiting the city wants to travel from stop A to stop K in the shortest possible time.

(c) Determine the shortest possible time for this journey and the bus route they should take.
(2 marks)

Solution
The shortest time is 15 minutes. Route is ABHJK.
Specific behaviours
✓ correctly determines the shortest time ✓ correctly determines the bus route with the shortest time

The tourist leaves from stop K, and travels to stop B. On the way, they visit an attraction at stop F, where they stay for 45 minutes.



(d) Determine the shortest possible time for this journey and the bus route they should take.
(2 marks)

Solution
The shortest time is 73 minutes (28 + 45 minutes). Route is KJEDFDCB.
Specific behaviours
✓ correctly determines the shortest time ✓ correctly determines the bus route with the shortest time

Question 4 **(8 marks)**

A company employs four sales representatives, who are based in different cities and regularly travel to promote the company's products. The next promotional campaign will require all four sales representatives to each travel to a different city. The cost, in hundreds of dollars, for each sales representative to travel to each of the four cities, is shown in the table below.

		Sales representative			
		Jordan	**Kylie**	**Li**	**Mikal**
Travel cost to the city ($'00)	**Adelaide**	24	16	18	19
	Brisbane	15	13	14	22
	Perth	16	13	10	17
	Sydney	13	16	15	15

The company director wants to assign the sales representatives in a way that minimises the total cost to the company.

(a) Show use of the Hungarian algorithm to determine the minimum cost. (4 marks)

Solution

$$\begin{bmatrix} 24 & 16 & 18 & 19 \\ 15 & 13 & 14 & 22 \\ 16 & 13 & 10 & 17 \\ 13 & 16 & 15 & 15 \end{bmatrix}$$

$$\begin{bmatrix} 8 & 0 & 2 & 3 \\ 2 & 0 & 1 & 9 \\ 6 & 3 & 0 & 7 \\ 0 & 3 & 2 & 2 \end{bmatrix}$$

$$\begin{bmatrix} 8 & 0 & 2 & 1 \\ 2 & 0 & 1 & 7 \\ \cancel{6} & \cancel{3} & \cancel{0} & \cancel{5} \\ \cancel{0} & \cancel{3} & \cancel{2} & \cancel{0} \end{bmatrix}$$

$$\begin{bmatrix} 7 & 0 & 1 & \textcircled{0} \\ 1 & \textcircled{0} & 0 & 6 \\ 6 & 4 & \textcircled{0} & 5 \\ \textcircled{0} & 4 & 2 & 0 \end{bmatrix}$$

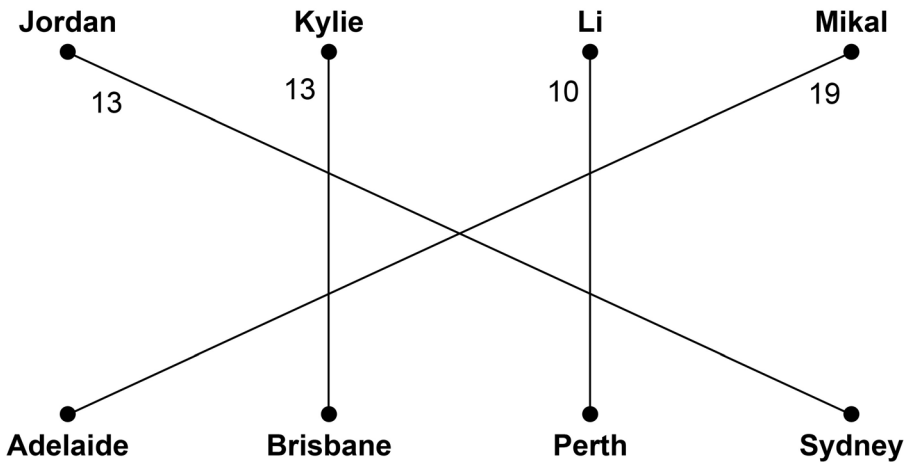
Minimum cost: 13 + 13 + 10 + 19 = 55, i.e. $5500

Specific behaviours

- ✓ subtracts lowest score in each row from all scores in the row
- ✓ subtracts lowest score in each column from all scores in the column
- ✓ only three lines needed to cover zeros, so subtracts 1 from uncovered numbers and adds 1 to those at intersection points
- ✓ determines correct minimum cost

(b) Draw the weighted bipartite graph, showing the assignment of sales representatives that minimises the total cost. (2 marks)

Solution



Specific behaviours

✓ shows correct assignments
✓ shows correct costs on each assignment

Mikal has asked to travel to Brisbane for the next promotional campaign, to visit family while he is there. The company director has approved the request.

(c) Complete the table below to show the updated assignment of sales representatives and state the associated minimum cost to the company. (2 marks)

Solution

Sales representative	**Jordan**	**Kylie**	**Li**	**Mikal**
City	**Sydney**	**Adelaide**	**Perth**	Brisbane

Minimum cost: 10 + 16 + 13 + 22 = 61, i.e. $6100

Specific behaviours

✓ correctly determines updated assignment
✓ correctly calculates updated minimum cost

Question 5 **(13 marks)**

The average fuel consumption, c (L/100 km), of cars manufactured since 1975 was recorded, with n being the number of years since 1975. The results are shown in the scatterplot below.

For these data, the equation of the least-squares line is $\hat{c} = -0.18n + 16.7$ and the correlation coefficient (r_{nc}) is −0.88.

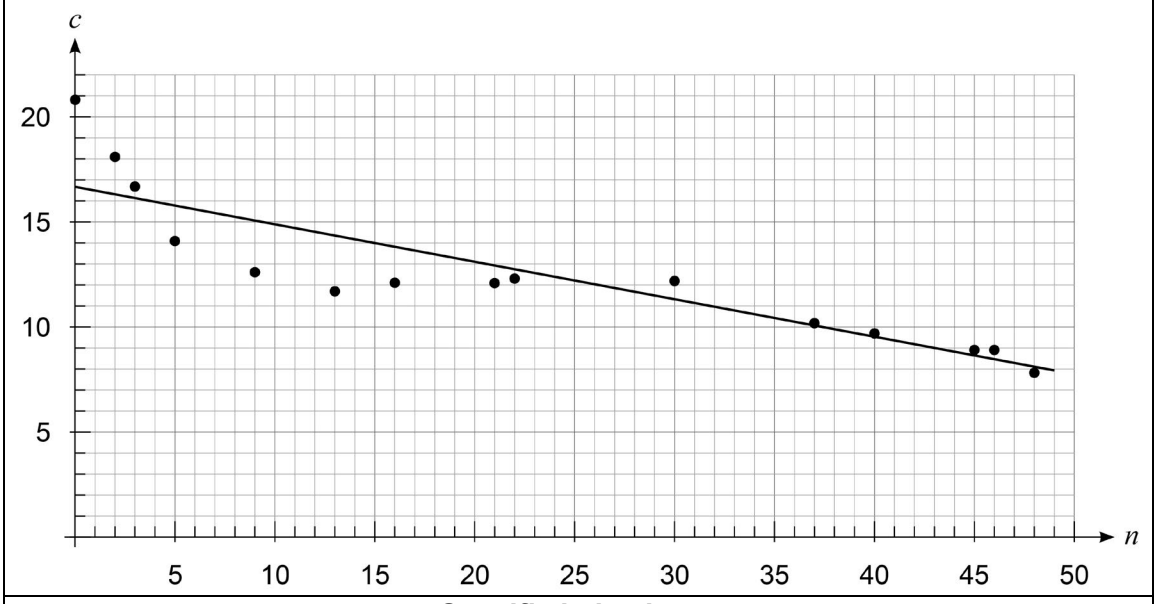
(a) Use the equation of the least-squares line to predict the average fuel consumption for a car manufactured in the year 2000. (2 marks)

Solution
$\hat{c} = -0.18(25) + 16.7$ $\hat{c} = -4.5 + 16.7 = 12.2$ L / 100 km
Specific behaviours
✓ correctly substitutes n = 25 into equation ✓ calculates correct answer

(b) Comment on your prediction in part (a). (2 marks)

Solution
The prediction is valid as the point is interpolated with a strong correlation coefficient.
Specific behaviours
✓ correctly states it is valid ✓ correctly states interpolation with strong correlation coefficient

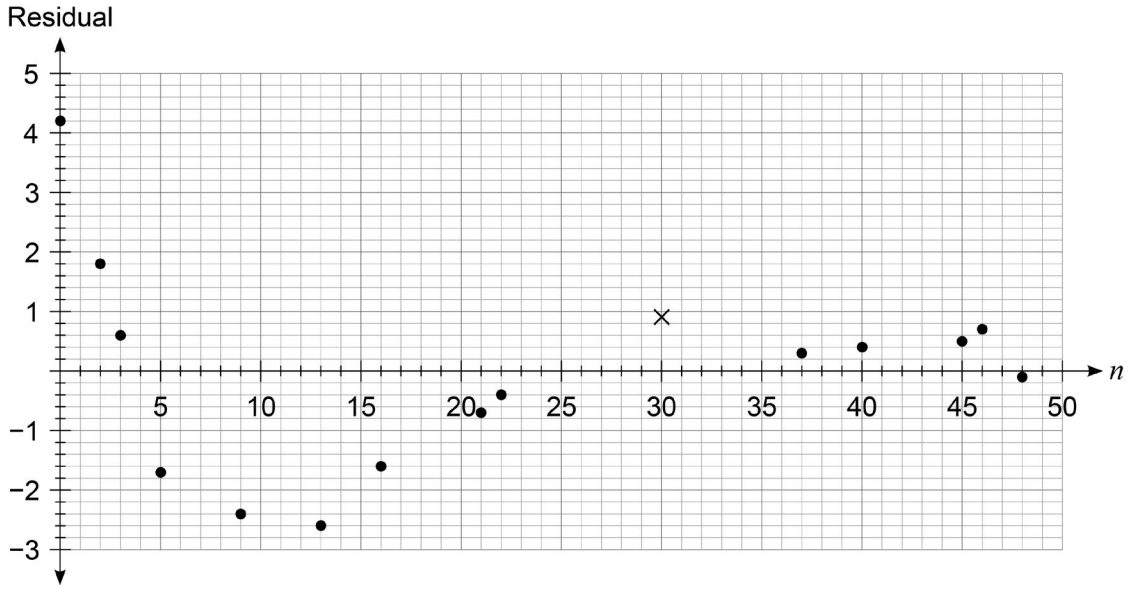
(c) Draw the least-squares line for these data on the scatterplot above. (2 marks)

Solution

Specific behaviours
✓ uses correct vertical intercept ✓ draws correct least-squares line

The incomplete residual plot for these data is drawn below. The residual for 2005 is missing.

The average fuel consumption for cars manufactured in 2005 was 12.2 L/100 km.

(d) Determine the residual for 2005 and plot it on the residual plot above. (3 marks)

Solution
$\hat{c} = -0.18(30) + 16.7$ $\hat{c} = -5.4 + 16.7 = 11.3$ Residual $= 12.2 - 11.3 = 0.9$ 
Specific behaviours
✓ calculates correct predicted value ✓ determines correct residual ✓ plots residual correctly

(e) Assess the appropriateness of fitting a linear model to these data. (2 marks)

Solution
Fitting a linear model to these data is not appropriate as there is a pattern evident in the residuals.
Specific behaviours
✓ states a linear model is not appropriate ✓ correctly states a pattern is evident in the residual plot (not randomly scattered)

Additional records from the year 1995 show an average fuel consumption of 10.2 L/100 km.

(f) If this data was added to the scatterplot on page 12, determine if the correlation coefficient would become stronger, weaker or stay the same. Justify your response. (2 marks)

Solution
Weaker, as this point would be further away from the least-squares line (relationship not as strong).
Specific behaviours
✓ states that the correlation coefficient would become weaker ✓ gives a valid reason

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