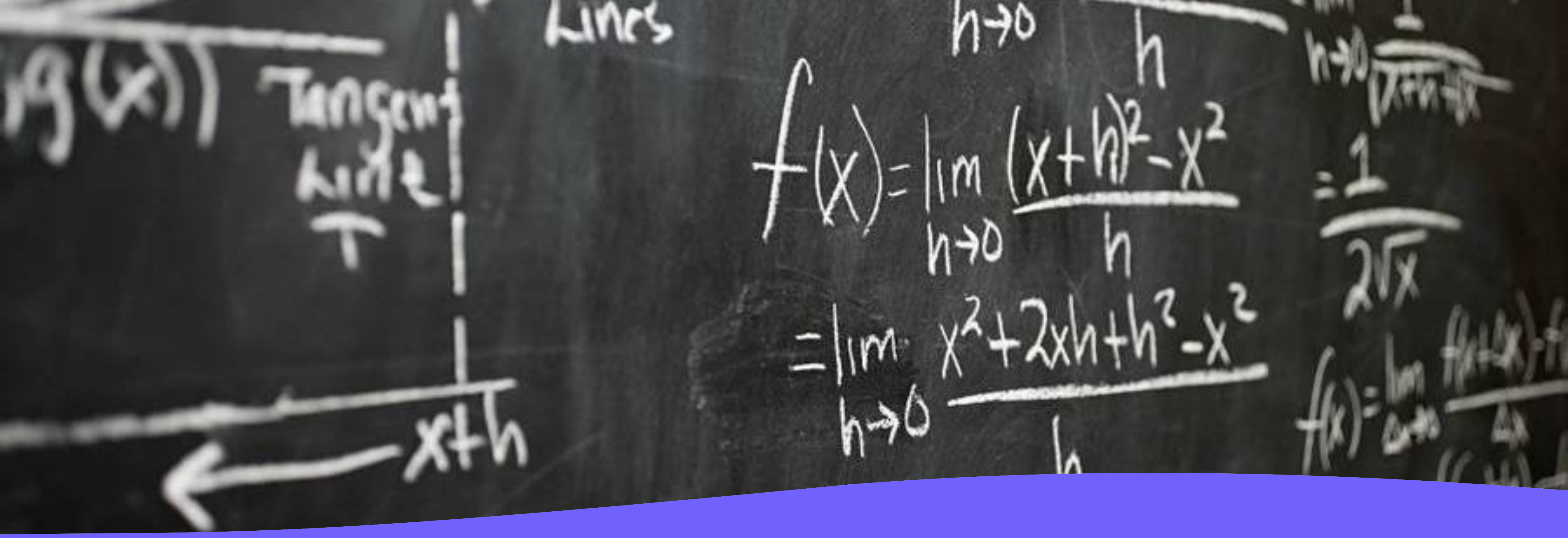
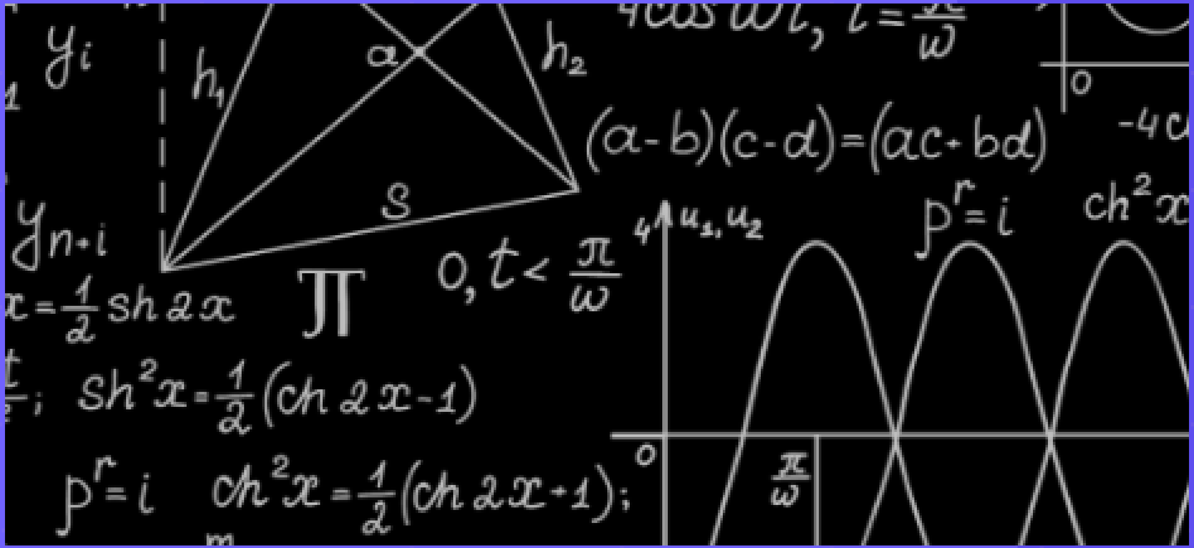



# Maths Methods 2026

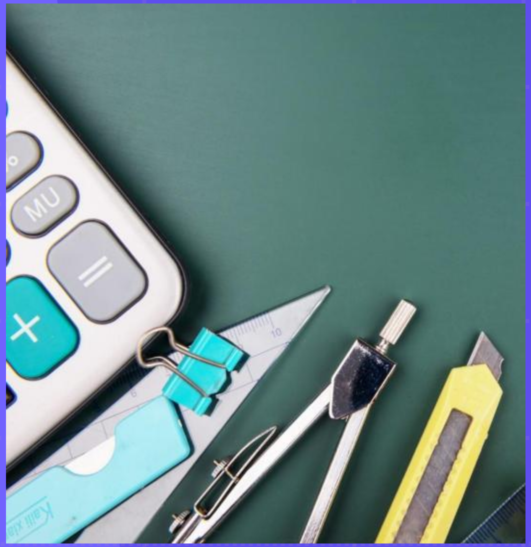
Chris Simpson

# Welcome



# Who am I?

- I have been teaching since 2001…. Before you were born (Wow now I feel old!)
- I have taught at a range of schools – Bullsbrook SHS, Sacred Heart College, Nagle Catholic College, Servite College, Aranmore Catholic College, Laverton DHS, Prendiville Catholic College.
- I have been Head of Maths in three schools (Nagle, Servite and Aranmore)
- I have since those days been in leadership roles in schools (Director of College Operations, Deputy Principal)
- For the last two years I have been teaching from home and working with Catholic Education in the ViSN program teaching to students in schools where a Specialist class can't be run.
- I will continue to teach with ViSN next year as well.



# Agenda

| | Syllabus Points | Time(min) |
|---|---|---|
| Introductions and welcome | | 15 |
| Unit 3 - Topic 1 – Differentiation | 3.1.1 – 3.1.16 | 20 |
| Unit 3 - Topic 2 – Integration | 3.2.1 - 3.2.22 | 20 |
| Break time | | 10 |
| Unit 3 - Topic 3 – Discrete Random Variables | 3.3.1 – 3.3.16 | 20 |
| Unit 4 - Topic 1 – Logarithms | 4.3.1 – 4.3.14 | 20 |
| Break time | | 10 |

# Agenda

| | Syllabus Points | Time(min) |
|---|---|---|
| Unit 4 - Topic 2 – CRV and ND | 4.2.1 – 4.2.7 | 20 |
| Unit 4 - Topic 3 – Int Est for Proportions | 4.3.1 - 4.3.10 | 20 |
| Break time | | 10 |
| | | |
| WACE Exam - the details/tips | | 10 |
| WACE questions | | 40 |
| What's next/finishing up | | 15 |

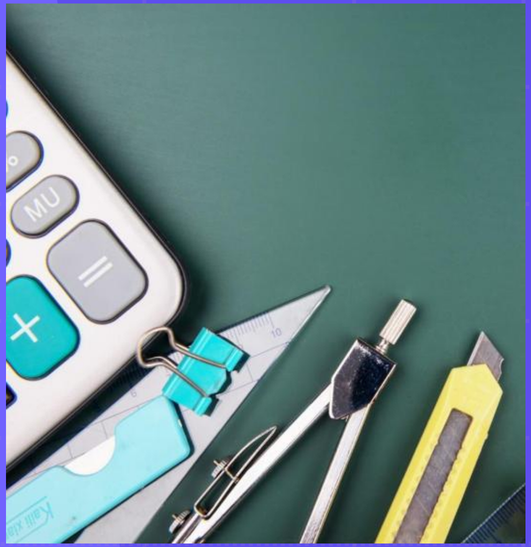
# What is the course?

The only document you really need now, is the syllabus.

Not the textbook, not the teacher's notes, not the videos online, not WACE Vault!

The examiners were given the syllabus, maybe some suggestions/areas to emphasise, the previous year's exam reports and told "go write an exam for the course".

They didn't consult your teacher, textbooks, WACE Vault, Snapchat, etc



The course is the syllabus

# Glossary? This isn’t English!

But we (well you….) and the examiners need to be talking the same language.  To get around differing interpretation of words, there is a glossary with each subject’s syllabus. It is at the back of the syllabus (in a dark place you might never have seen!)

## Appendix 2 – Glossary

This glossary is provided to enable a common understanding of the key terms in this syllabus.

### Unit 3

**Further differentiation and applications**

**Composition of functions**

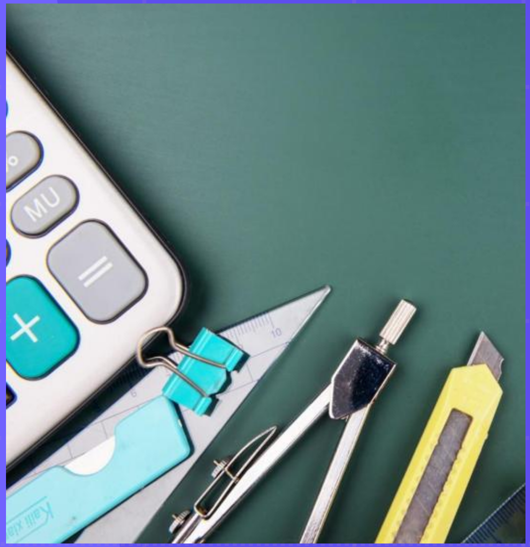
If $y = g(x)$ and $z = f(y)$ for functions $f$ and $g$, then $z$ is a composite function of $x$. We write $z = f \circ g(x) = f(g(x))$. For example, $z = \sqrt{x^2 + 3}$ expresses $z$ as a composite of the functions $f(y) = \sqrt{y}$ and $g(x) = x^2 + 3$.

**Chain rule**

The chain rule relates the derivative of the composite of two functions to the functions and their derivatives.

If $h(x) = f \circ g(x)$ then $(f \circ g)'(x) = f'\big(g(x)\big)g'(x)$,

and in Leibniz notation: $\frac{dz}{dx} = \frac{dz}{dy}\frac{dy}{dx}$

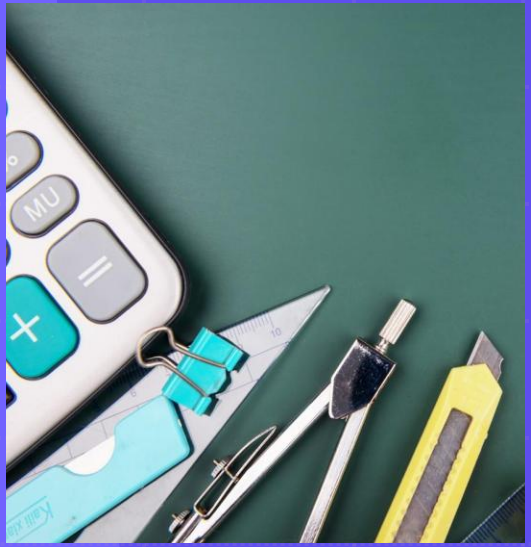


# What's ahead?

We will cover each topic in the following manner

- Starting with a syllabus review
- Some teaching staying true to the syllabus points
- An exam question or two for you to do
- Questions from you?

Ready to go?



# Unit 3 Topic 1

# Differentiation

# 3.1.1 – 3.1.16

# Syllabus points

**Exponential functions**

3.1.1 estimate the limit of $\frac{a^h-1}{h}$ as $h \to 0$, using technology, for various values of $a > 0$

3.1.2 identify that $e$ is the unique number $a$ for which the above limit is 1

3.1.3 establish and use the formula $\frac{d}{dx}(e^x) = e^x$

3.1.4 use exponential functions of the form $Ae^{kx}$ and their derivatives to solve practical problems

**Trigonometric functions**

3.1.5 establish the formulas $\frac{d}{dx}(\sin x) = \cos x$ and $\frac{d}{dx}(\cos x) = -\sin x$ by graphical treatment, numerical estimations of the limits, and informal proofs based on geometric constructions

3.1.6 use trigonometric functions and their derivatives to solve practical problems

# Syllabus points

**Differentiation rules**

3.1.7 examine and use the product and quotient rules

3.1.8 examine the notion of composition of functions and use the chain rule for determining the derivatives of composite functions

3.1.9 apply the product, quotient and chain rule to differentiate functions **such as** $xe^x$, $\tan x$, $\frac{1}{x^n}$, $x\sin x$, $e^{-x}\sin x$ and $f(ax-b)$

# Syllabus points

**The second derivative and applications of differentiation**

3.1.10 use the increments formula: $\delta y \approx \frac{dy}{dx} \times \delta x$ to estimate the change in the dependent variable $y$ resulting from changes in the independent variable $x$

3.1.11 apply the concept of the second derivative as the rate of change of the first derivative function

3.1.12 identify acceleration as the second derivative of position with respect to time

3.1.13 examine the concepts of concavity and points of inflection and their relationship with the second derivative

3.1.14 apply the second derivative test for determining local maxima and minima

3.1.15 sketch the graph of a function using first and second derivatives to locate stationary points and points of inflection

3.1.16 solve optimisation problems from a wide variety of fields using first and second derivatives

# Euler's Number

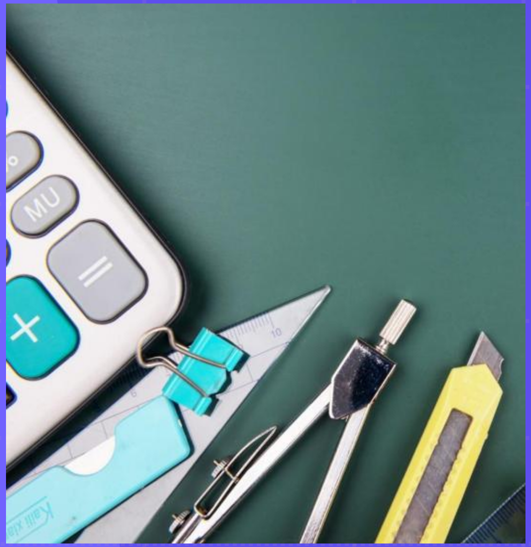
e = 2.71828…….

Usually defined as

$$\lim_{n \to \infty} \left(1 + \frac{1}{n}\right)^n$$

If we tabulate the expression, we see that as n $\to \infty$, the expression approaches a number we call 'e' or Euler's number.

| $n$ | $\left(1 + \frac{1}{n}\right)^n$ |
|---|---|
| **1** | 2 |
| **2** | 2.25 |
| **5** | 2.48832 |
| **10** | 2.59374 |
| **100** | 2.70481 |
| **1000** | 2.71692 |
| **10000** | 2.71827 |
| **1000000** | 2.71828 |



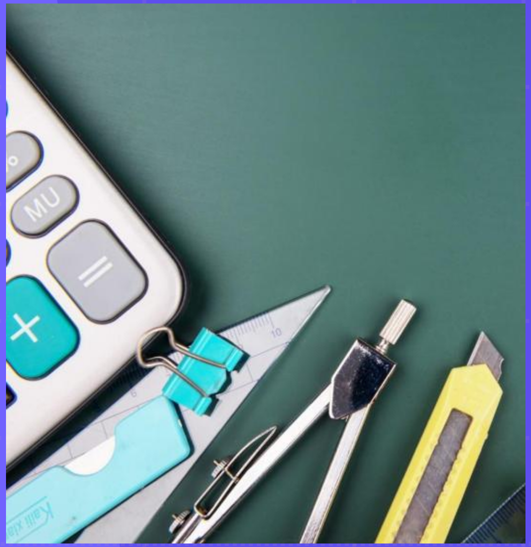
# Euler's Number

Similarly, if we use the expression from the syllabus

$$\lim_{t \to 0} \left( \frac{a^t - 1}{t} \right)$$

If we tabulate this expression, we see that as t $\to \infty$ and a = e, the expression approaches 1

| $a$ | $\left( \frac{a^t - 1}{t} \right)$ |
|---|---|
| **2** | 0.69315 |
| **2.5** | 0.91629 |
| **2.7** | 0.99325 |
| **2.71** | 0.99695 |
| **2.72** | 1.00063 |
| **2.718** | 0.99990 |
| **2.7183** | 1.00001 |
| **2.71828** | 1.00000 |



# Euler’s Number

Differentiating $y = e^x$ gives $\frac{dy}{dx} = e^x$ , meaning that the value of the derivative (the function’s slope) is equal to the value of the function.

We can differentiate $y = e^x$ by first principles and show this to be the case, but this is highly unlikely to be needed in an exam.

## The derivative of $y = e^x$ using first principles

$$f'(x) = \lim_{h\to0} \frac{f(x+h)-f(x)}{h}$$

$$f'(x) = \lim_{h\to0} \frac{e^{x+h} - e^x}{h}$$

$$f'(x) = \lim_{h\to0} \frac{e^x e^h - e^x}{h}$$

$$f'(x) = \lim_{h\to0} \frac{e^x(e^h - 1)}{h}$$

$$f'(x) = \boxed{\lim_{h\to0} e^x} \boxed{\lim_{h\to0} \frac{e^h - 1}{h}}$$

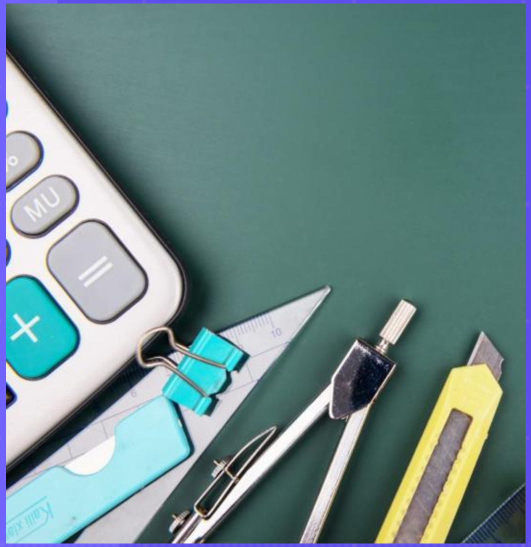
$$\lim_{h\to0} e^x = e^x \qquad \lim_{x\to0} \frac{a^x - 1}{x} = \ln(a)$$

$$f'(x) = e^x \ \ln(e)$$

$$f'(x) = e^x$$







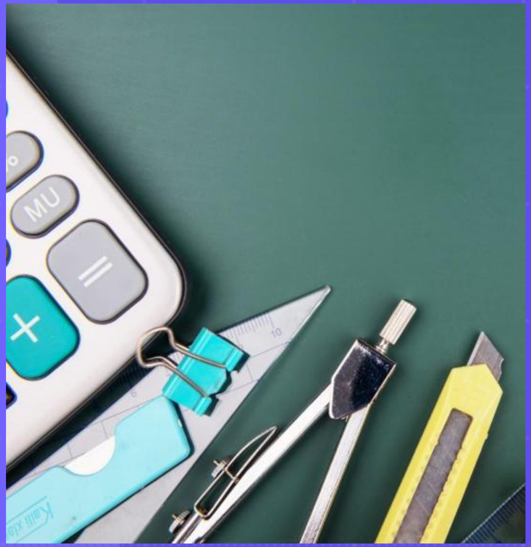
# Growth and decay

We now use the previous concept (differentiating $y = e^{x}$ ) to apply to situations where there is growth and decay.

Given $A = A_0 e^{kt}$ it follows that $\frac{dA}{dt} = k \times A_0 e^{kt}$ which can be written as $\frac{dA}{dt} = k \times A$

It then follows the reverse must be true as well, given $\frac{dA}{dt} = k \times A$ then $A = A_0 e^{kt}$

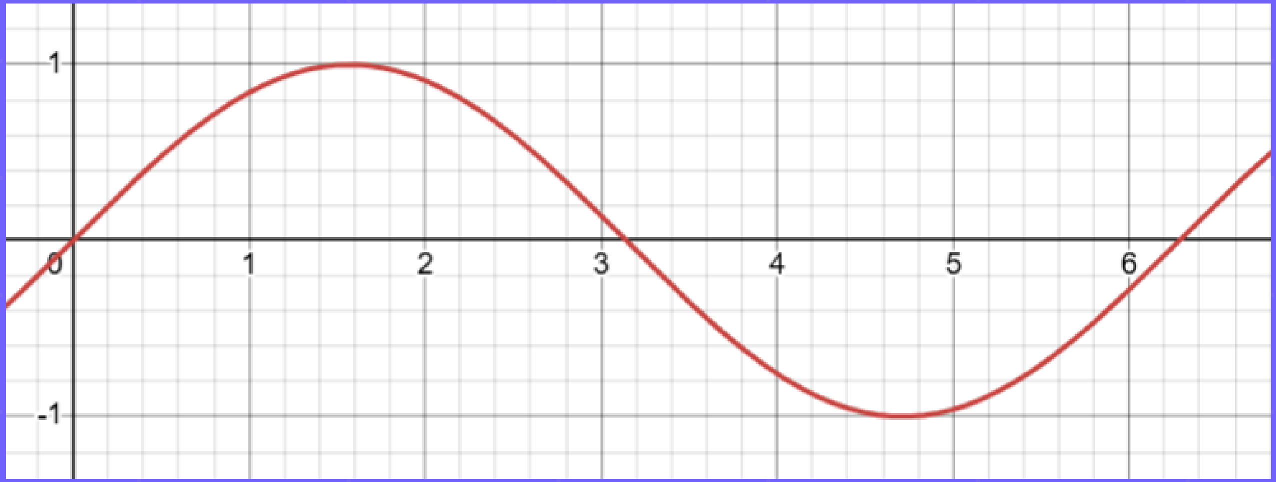
| | |
|---|---|
| Exponential growth and decay | $\frac{dP}{dt} = kP \Leftrightarrow P = P_0 e^{kt}$ |

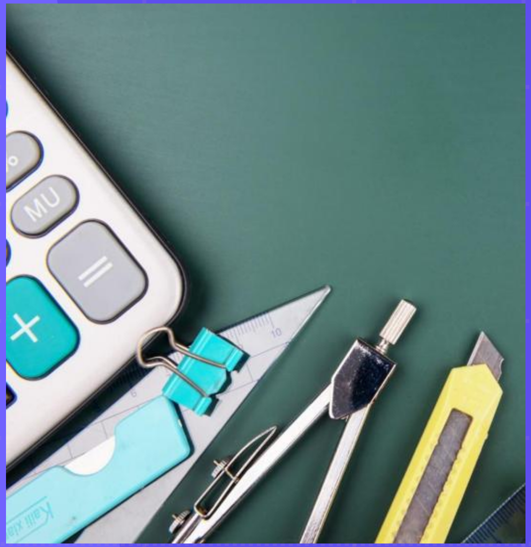


# Differentiation of trig functions

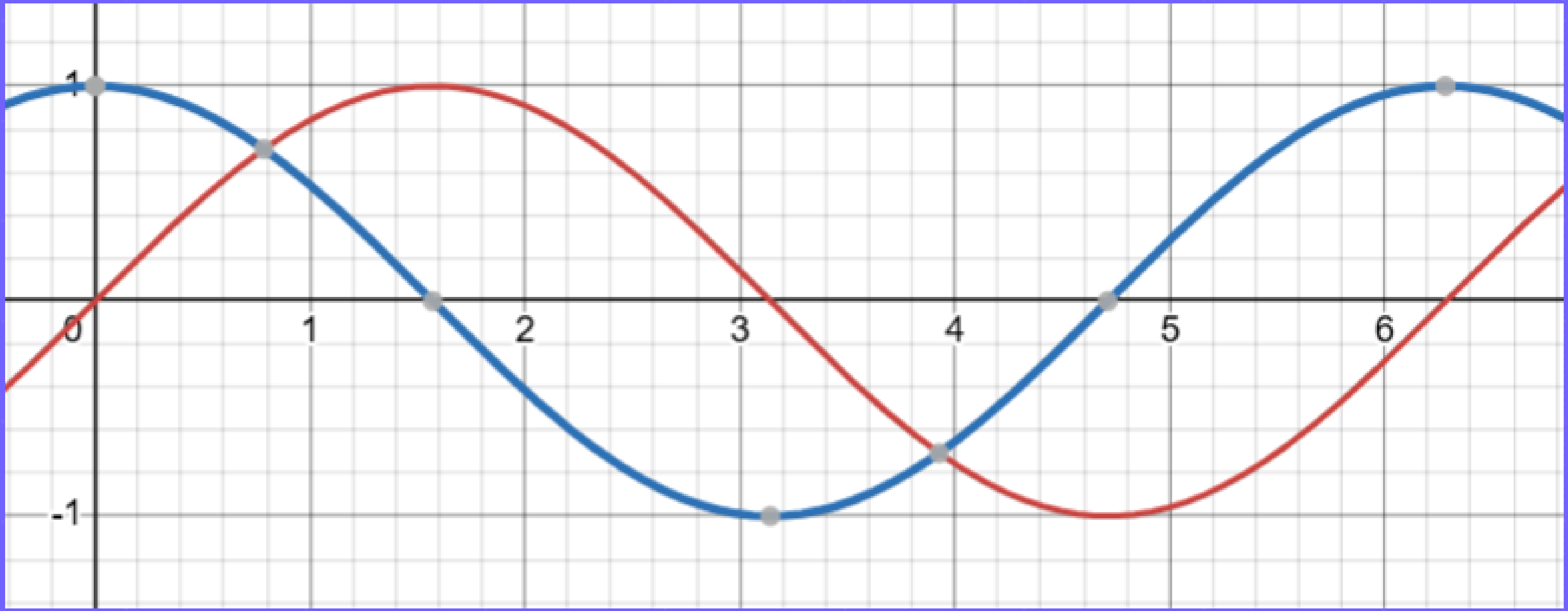
Sine and Cosine are cyclical function.  That is, they repeat after some time (their period).

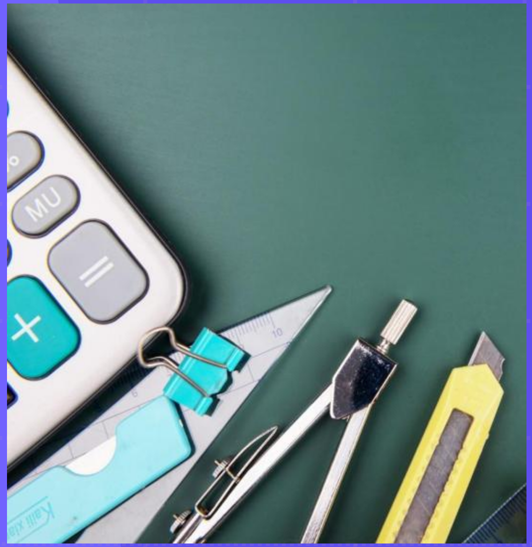
Take sine, it has a maximum of 1, a minimum of -1.  Its gradient can also have a maximum of 1 (which happens between the minimum and maximum points) and a minimum of -1 (which happens between the maximum and minimum points)



# Differentiation of trig functions



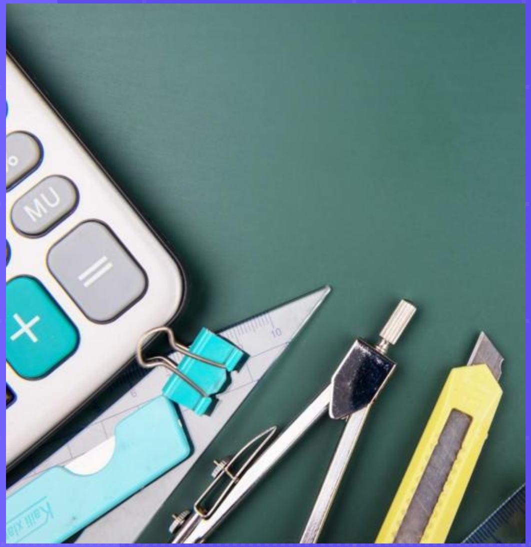
# Differentiation of trig functions

And so, we can conclude

$$\frac{d}{dx}(\sin x) = \cos x \quad \text{and} \quad \frac{d}{dx}(\cos x) = -\sin x$$

And it follows that if we continue to differentiate

$$\frac{d}{dx}(\sin x) = \cos x \qquad \frac{d}{dx}(\cos x) = -\sin x \qquad \frac{d}{dx}(-\sin x) = -\cos x \qquad \frac{d}{dx}(-\cos x) = \sin x$$

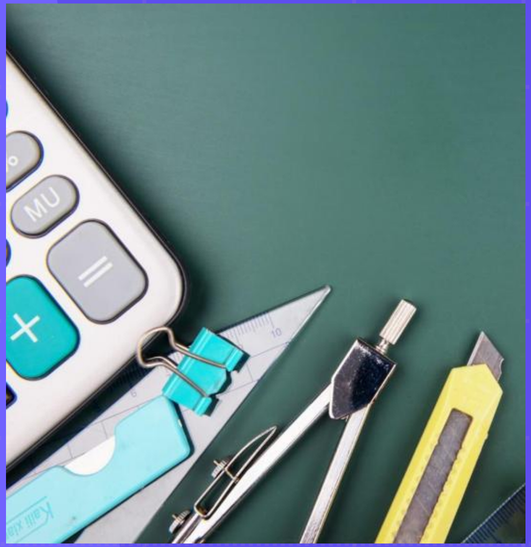


This can be shown  by first principles as well, but this is unlikely to be needed in the examination

# Differentiation of trig functions

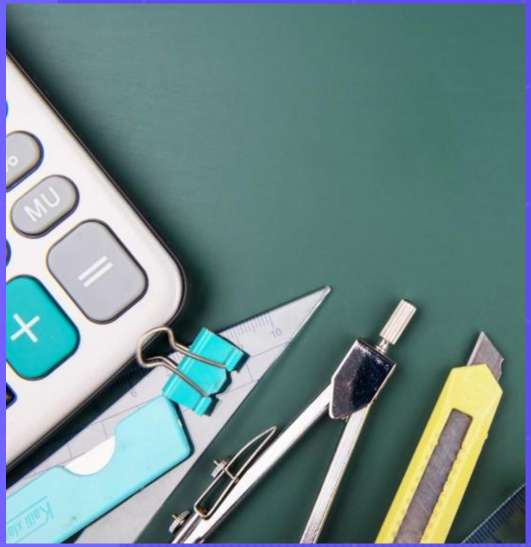
This can be shown by several methods including by first principles as well, but the ability to show this in an exam is unlikely to be needed.

3.1.5 establish the formulas $\frac{d}{dx}(\sin x) = \cos x$ and $\frac{d}{dx}(\cos x) = -\sin x$ by graphical treatment, numerical estimations of the limits, and informal proofs based on geometric constructions



# Differentiation of trig functions

| | |
|---|---|
| $\frac{d}{dx}\sin(ax-b) = a\cos(ax-b)$ | $\int \sin(ax-b)\,dx = -\frac{1}{a}\cos(ax-b) + c$ |
| $\frac{d}{dx}\cos(ax-b) = -a\sin(ax-b)$ | $\int \cos(ax-b)\,dx = \frac{1}{a}\sin(ax-b) + c$ |



# Differentiation of polynomial functions

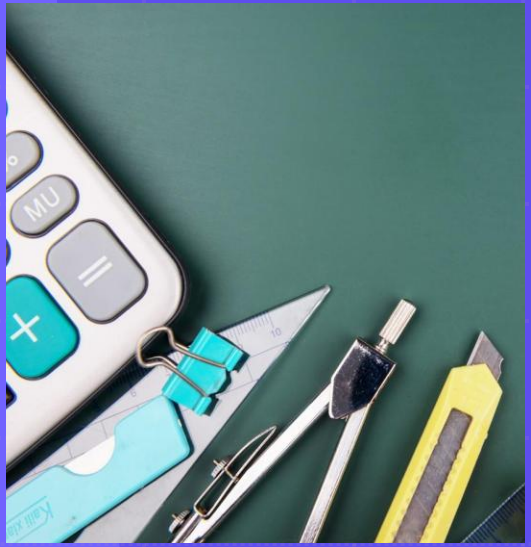
The styles mentioned in the syllabus are the product rule, quotient rule and chain rule.

**Differentiation rules**

3.1.7 examine and use the product and quotient rules

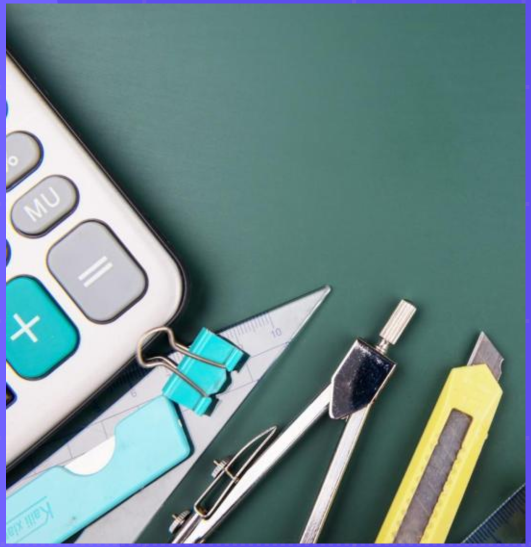
3.1.8 examine the notion of composition of functions and use the chain rule for determining the derivatives of composite functions

3.1.9 apply the product, quotient and chain rule to differentiate functions such as $xe^x$, $\tan x$, $\frac{1}{x^n}$ , $x \sin x$, $e^{-x} \sin x$ and $f(ax - b)$



# Differentiation of polynomial functions

| | | | |
|---|---|---|---|
| Product rule | If $y = uv$<br>then<br>$\frac{d}{dx}(uv) = v\frac{du}{dx} + u\frac{dv}{dx}$ | or | If $y = f(x)\,g(x)$<br>then<br>$y' = f'(x)\,g(x) + f(x)\,g'(x)$ |
| Quotient rule | If $y = \frac{u}{v}$<br>then<br>$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$ | or | If $y = \frac{f(x)}{g(x)}$<br>then<br>$y' = \frac{f'(x)\,g(x) - f(x)\,g'(x)}{(g(x))^2}$ |
| Chain rule | If $y = f(u)$ and $u = g(x)$<br>then<br>$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$ | or | If $y = f(g(x))$<br>then<br>$y' = f'(g(x))\,g'(x)$ |



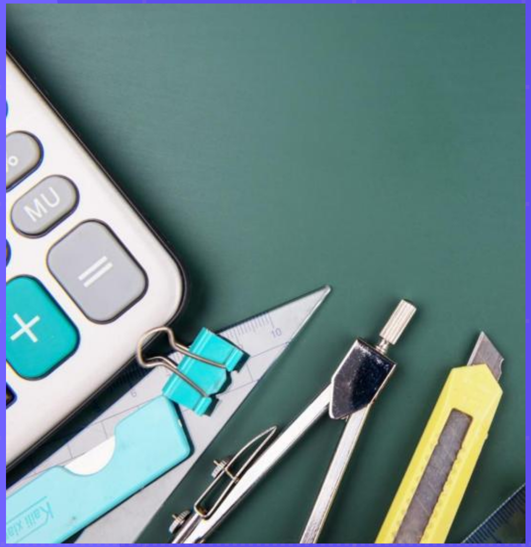
# Incremental change

At values close to the point of the derivative, we can estimate the change in a function.

The derivative gives us the approximation to the gradient at a point, since we can not generate a value for the derivative at a single point.

Remember derivative by first principles is always written as the limit as h → 0

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

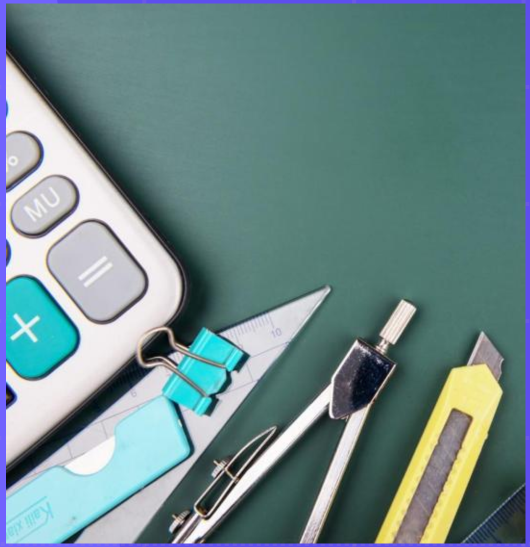


# Incremental change

Hence, we can write

| | |
|---|---|
| Increments formula | $\delta y \approx \frac{dy}{dx} \times \delta x$ |

You may have seen it written as $\frac{\delta y}{\delta x} \approx \frac{dy}{dx}$



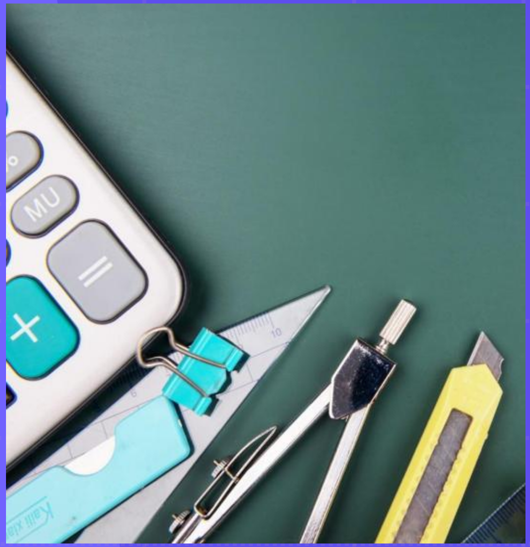
# Percentage change

To manage calculations which require you to work with percentage small change, you divide both sides by y and write the percentage change of y as $\frac{\delta y}{y}$ and x as $\frac{\delta x}{x}$.

$\frac{\delta y}{y}$ will form immediately you divide the LHS by y

$\frac{\delta x}{x}$ will form when you divide the RHS by y, and substitute the expression you have for y in terms of x.  Some of the expression will cancel out with the derivative, some will be left to form $\frac{\delta x}{x}$

$$\left(\boldsymbol{\frac{\delta y}{y}} \times \mathbf{100}\right)\% \approx \left(\frac{\frac{dy}{dx} \times \delta x}{y} \times 100\right)\%$$



# Percentage change

Example if y $= 3x^2$ determine the percentage increase in y when x is increased by 3%

That is $\frac{\delta x}{x} = 0.03$, simply our 3% converted to a percentage.

Now $\delta y \approx \frac{dy}{dx} \text{ x } \delta x$

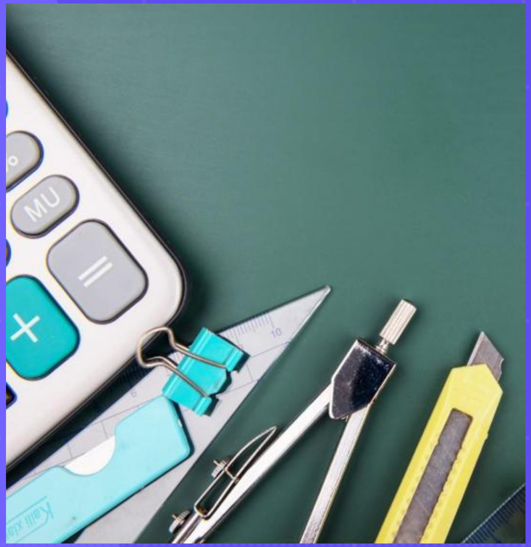
$$\frac{\delta y}{y} \approx \frac{\frac{dy}{dx} \text{x } \delta x}{y}$$

$$\frac{\delta y}{y} \approx \frac{6x\ \delta x}{3x^2}$$

$$\frac{\delta y}{y} \approx \frac{2\ \delta x}{x}$$

$$\frac{\delta y}{y} \approx 2(0.03)$$

$$\frac{\delta y}{y} \approx 0.06$$



# Marginal rates of change

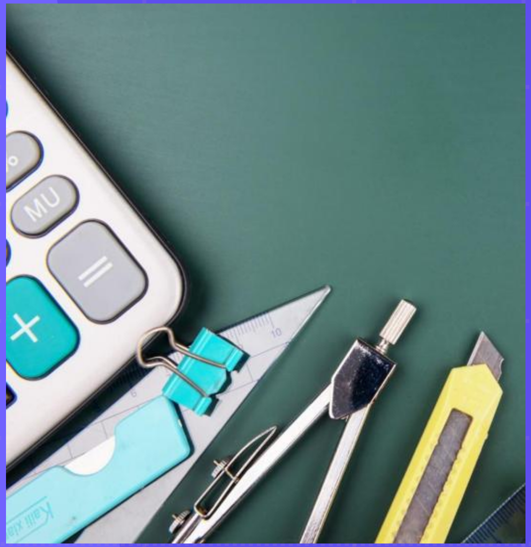
Marginal rates of change are just a fancy way of saying, what would change if you add one more of that item?

Let x = number of units

Cost function → C(x), Revenue function → R(x), Profit function → P(x)

And it follows

$$P(x) = R(x) - C(x)$$



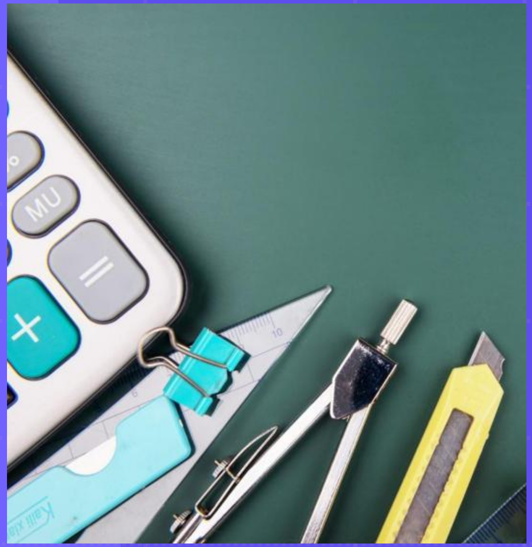
# Marginal rates of change

The marginal cost is therefore equal to the gradient at the point, since the gradient is a measure of the slope/gradient. If $\Delta x = 1$, that is the horizontal change then the corresponding vertical change is equal to $dy/dx$ at that point.

The marginal cost $\mathrm{C}'(\mathrm{x})$ gives the approximate cost of producing one more item.

The marginal revenue $\mathrm{R}'(\mathrm{x})$ gives the approximate revenue from producing one more item.

The marginal profit $\mathrm{P}'(\mathrm{x})$ gives the approximate profit of producing (and selling) one more item.

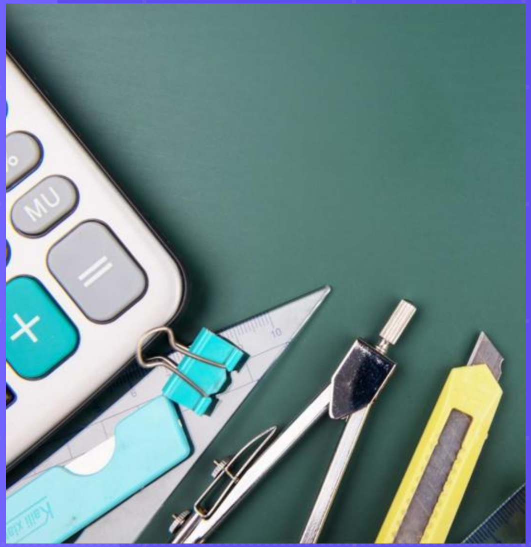


# Second Derivative

Differentiating a function twice gives us the second derivative, or a function describing how the slope of the graph is changing

We write it as $\frac{d^2y}{dx^2}$ or $y$“ or $f''(x)$

Let’s look at this one $\frac{d^2y}{dx^2}$ this looks odd, but it is really $\frac{d}{dx}\frac{d}{dx}\ y.$ We are only taking the derivative with respect of x, so there is never a $y^2$ in there.



# Displacement, Velocity, Acceleration

These three functions are highly interrelated for any particle in motion.

Displacement is usually written as s(t) or x(t)

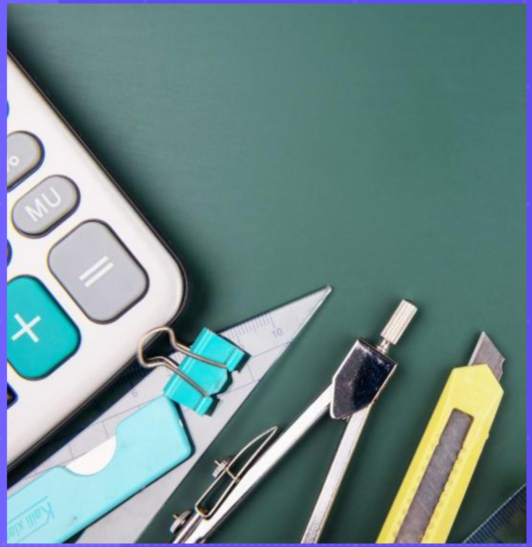
Velocity is usually written as v(t) or $\frac{ds}{dt}$ or $\frac{dx}{dt}$

Acceleration is usually written as a(t) or $\frac{d^2s}{dt^2}$ or $\frac{d^2x}{dt^2}$ or $\frac{dv}{dt}$

Remember when differentiating

Displacement → Velocity → Acceleration

When integrating it will be the reverse (see next section)

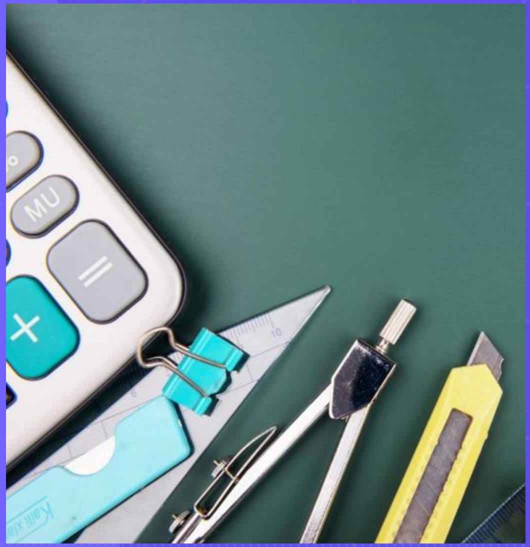


# Graph sketching - basics

Very common exam question… almost guaranteed!

When given a curve to sketch look for the following features.

- What is the basic shape of the graph, (cubic, parabola, hyperbola etc)
- Find any intercepts
- Find limits at infinity
- Check for asymptotes
- Identify any turning points
- Find points of inflection

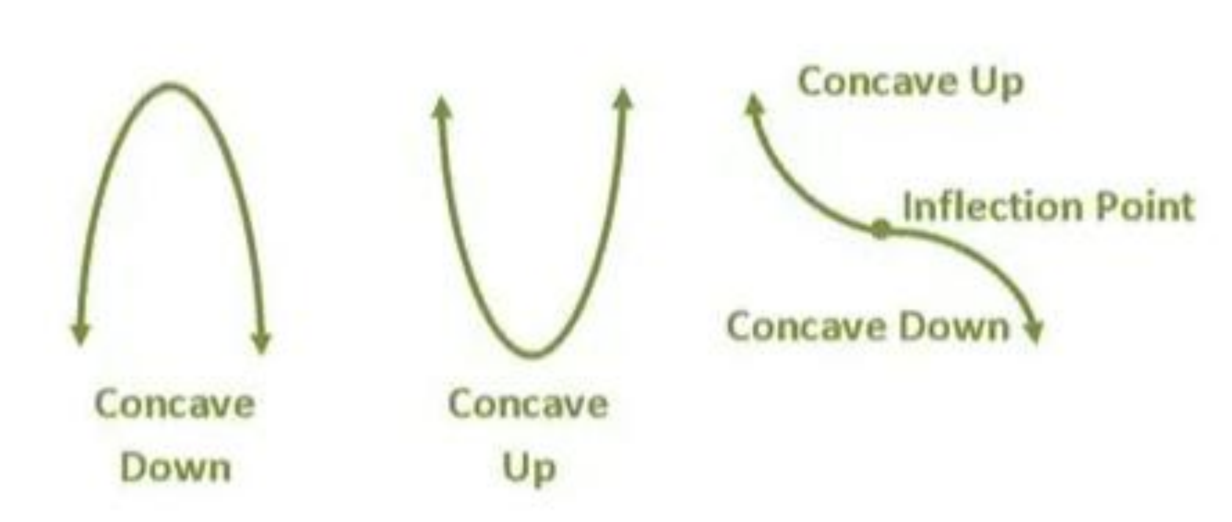


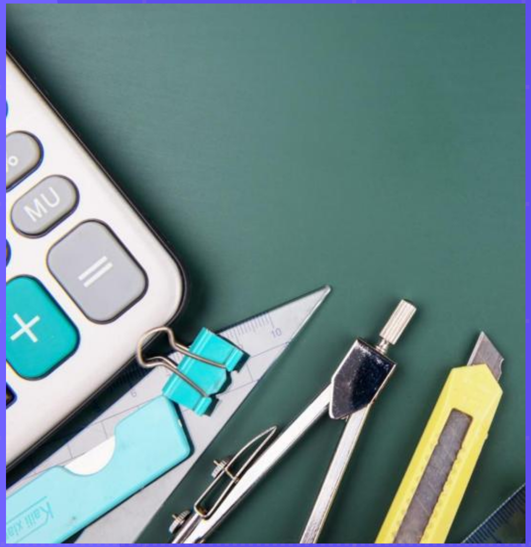
# Graph sketching - concavity

A minimum point is concave up (lucky horseshoe ☺)

A maximum point is concave down (unlucky horseshoe ☹ )

A point of inflection has a derivative of zero but is not either of the above



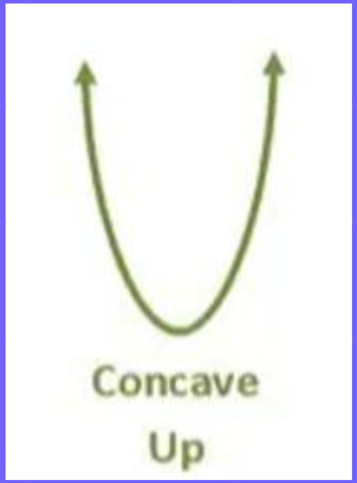



# Graph sketching - concavity

A minimum point is concave up (lucky horseshoe ☺)

x = a, y′ = 0, y″ > 0

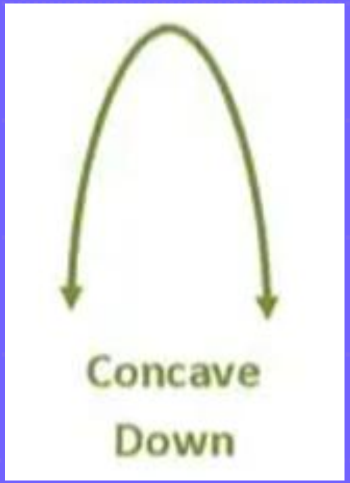
| $x$ | $a^-$ | $a$ | $a^+$ |
|---|---|---|---|
| $y'$ | $-$<br>\ | 0<br>$-$ | $+$<br>/ |

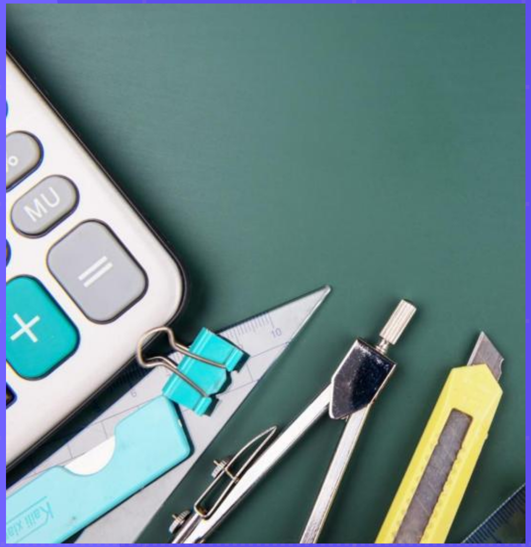



A maximum point is concave down (unlucky horseshoe ☹ )

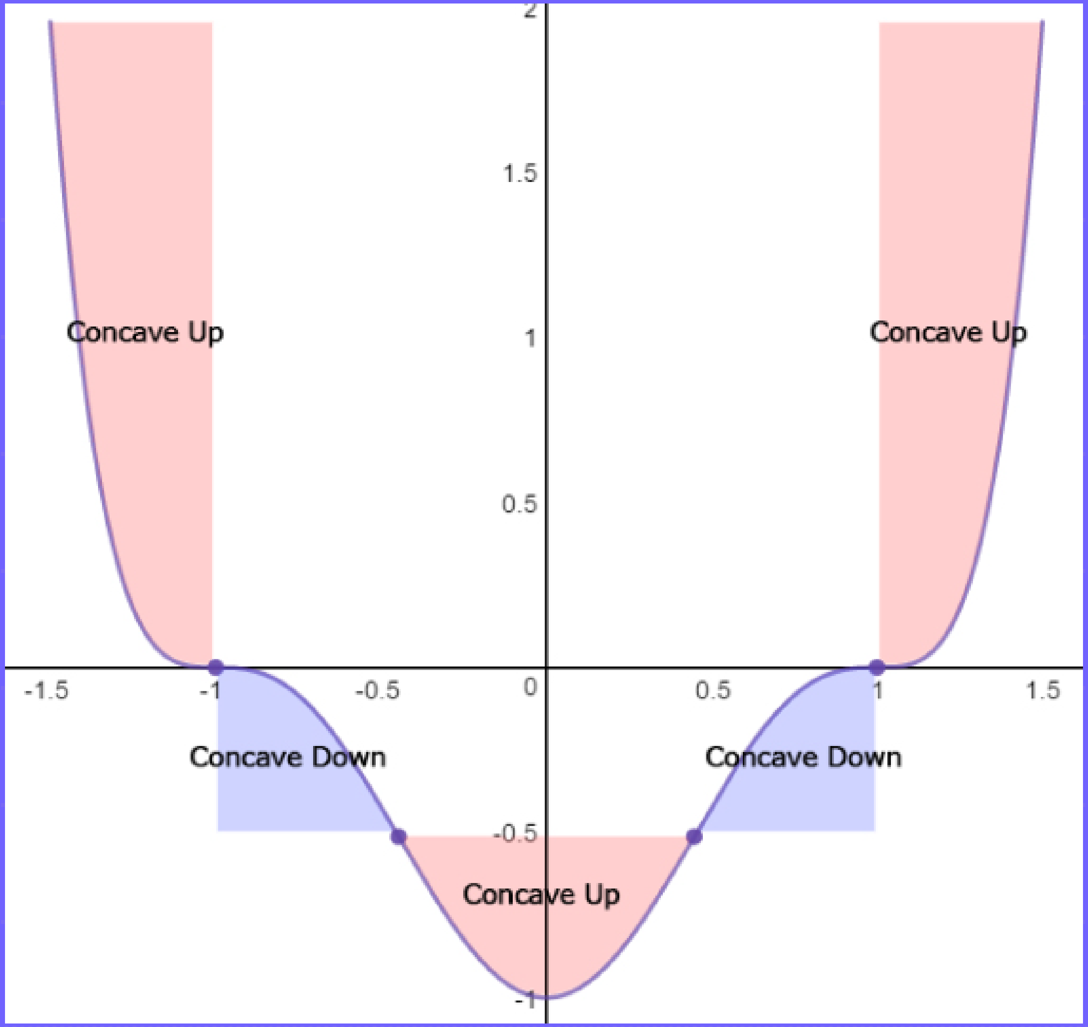
x = a, y′ = 0, y″ < 0

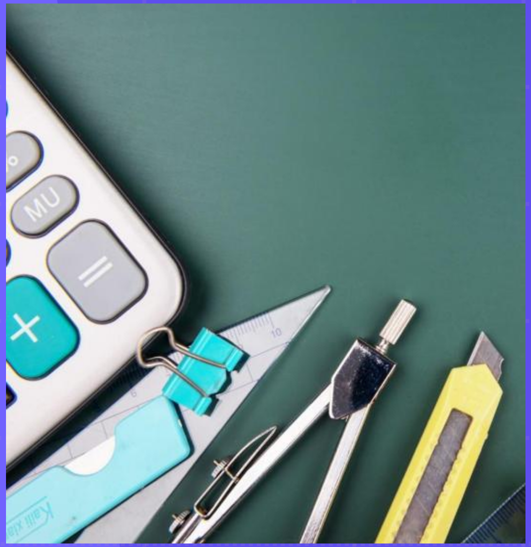
| $x$ | $a^-$ | $a$ | $a^+$ |
|---|---|---|---|
| $y'$ | $+$<br>/ | 0<br>$-$ | $-$<br>\ |



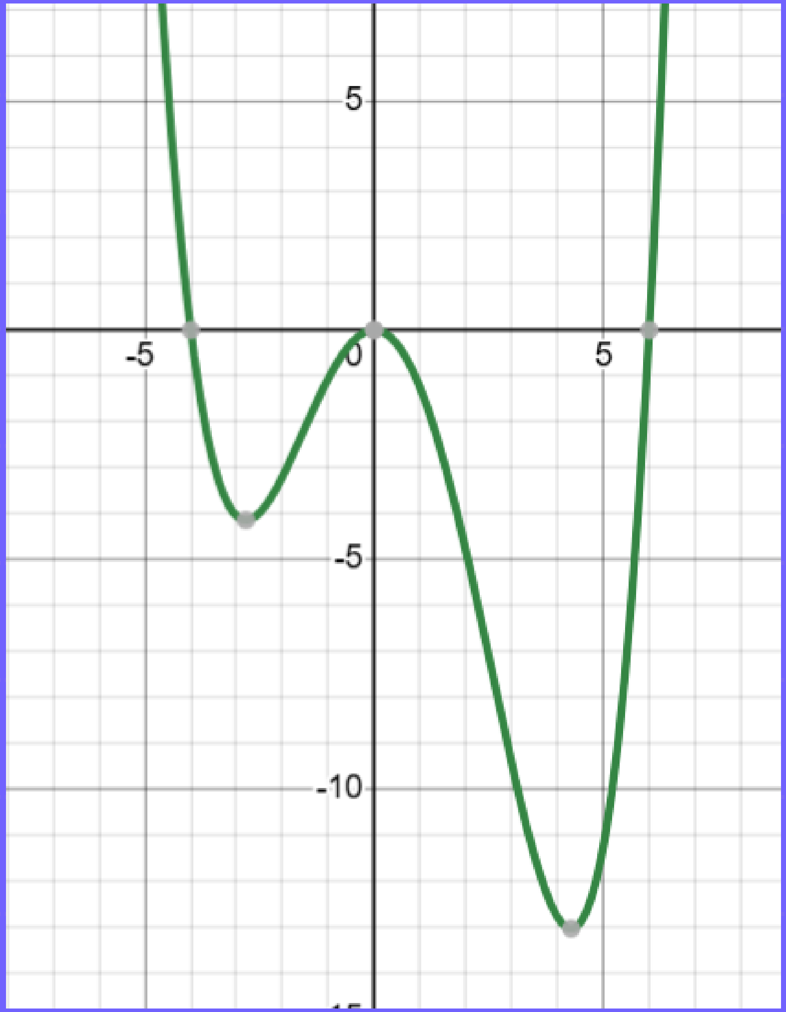



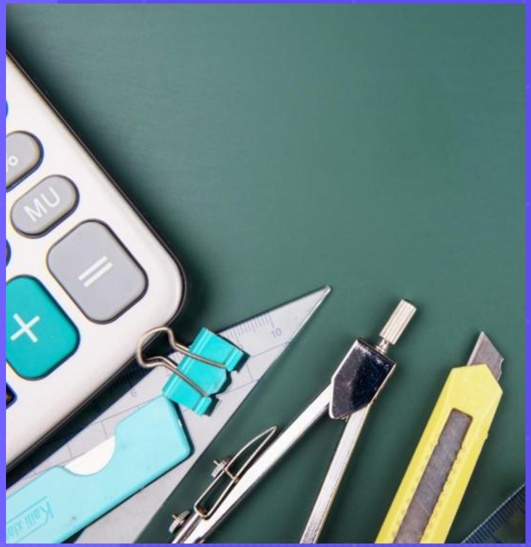
# Graph sketching - concavity



# Graph sketching - concavity

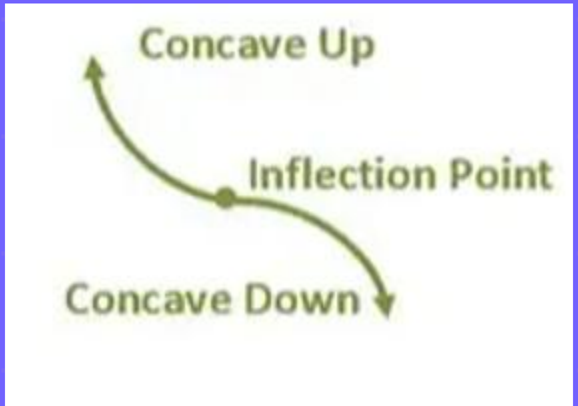


# Graph sketching – horizontal POI

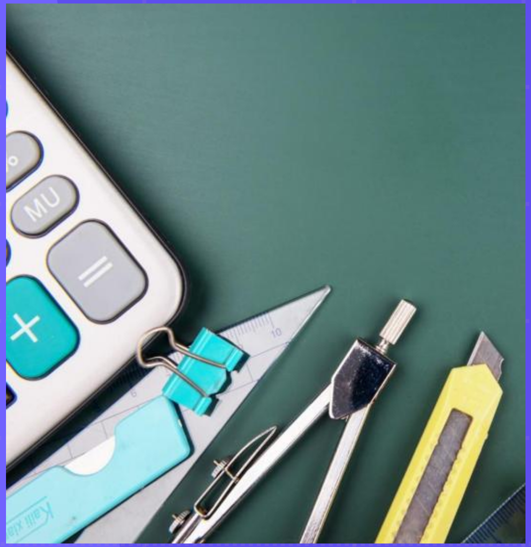
Another feature is a horizontal point of inflection where x = a, y′ = 0 and!! y″ = 0

In these cases, y″($a^+$) and y″($a^-$) have the opposite signs



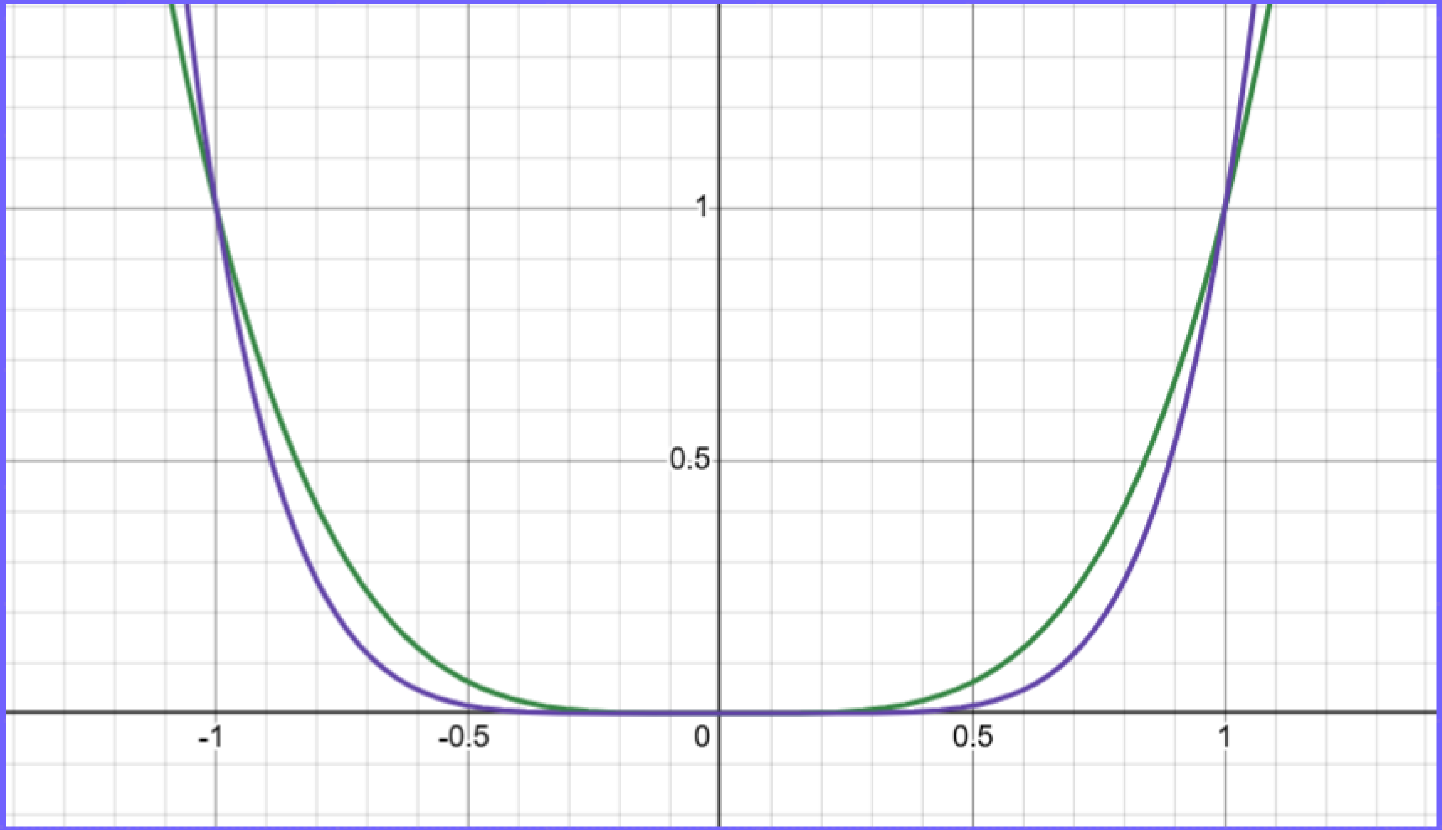

| $x$ | $a^-$ | $a$ | $a^+$ |
|---|---|---|---|
| $y'$ | $\pm$ | 0 | $\pm$ |
| $y''$ | $\pm$ | 0 | $\mp$ |

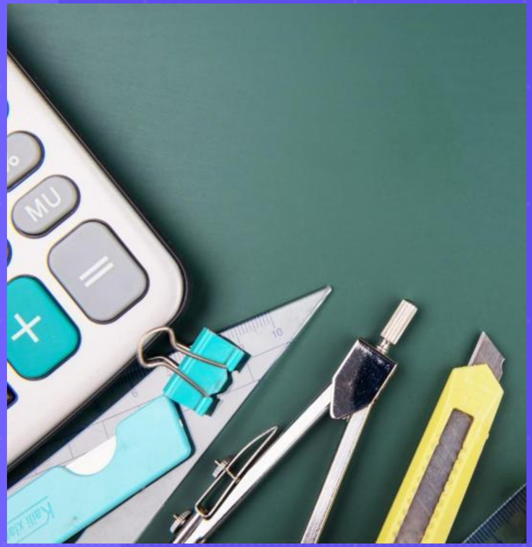
If y′ = 0 and y″ = 0, do not assume a horizontal point of inflection. Example, if you look at $y = x^{2n}$, this will suggest a horizontal point of inflection, but it will be a turning point



# Graph sketching - horizontal POI

If y′ = 0 and y″ = 0, do not assume a horizontal point of inflection. Example, if you look at $y = x^{2n}$, this will suggest a horizontal point of inflection, but it will be a turning point. A sign test will confirm the turning point.



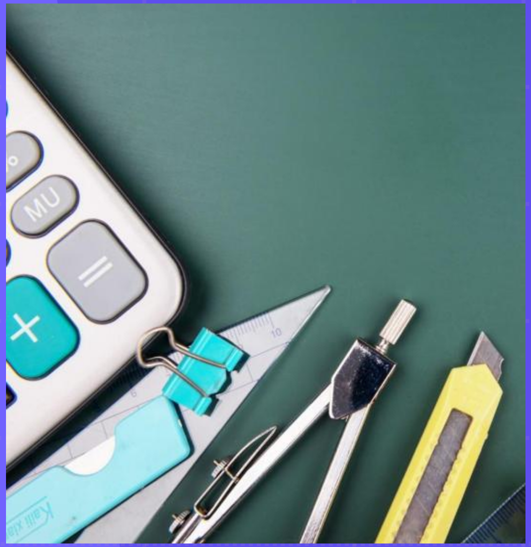



# Optimisation

A very likely exam question!

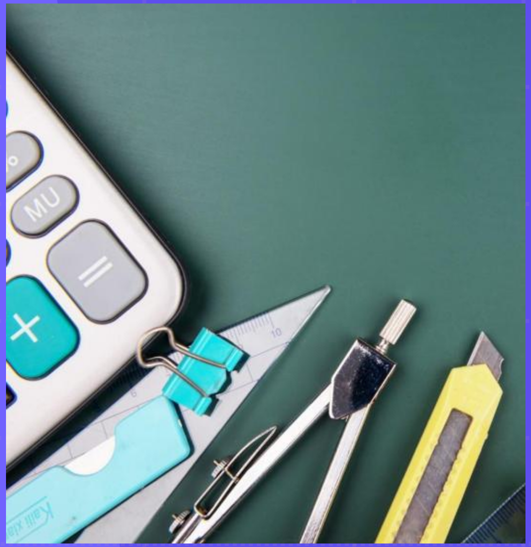
There is no specific teaching on these, and it is just applying everything we have just seen into practical cases.  Sounds easy right!

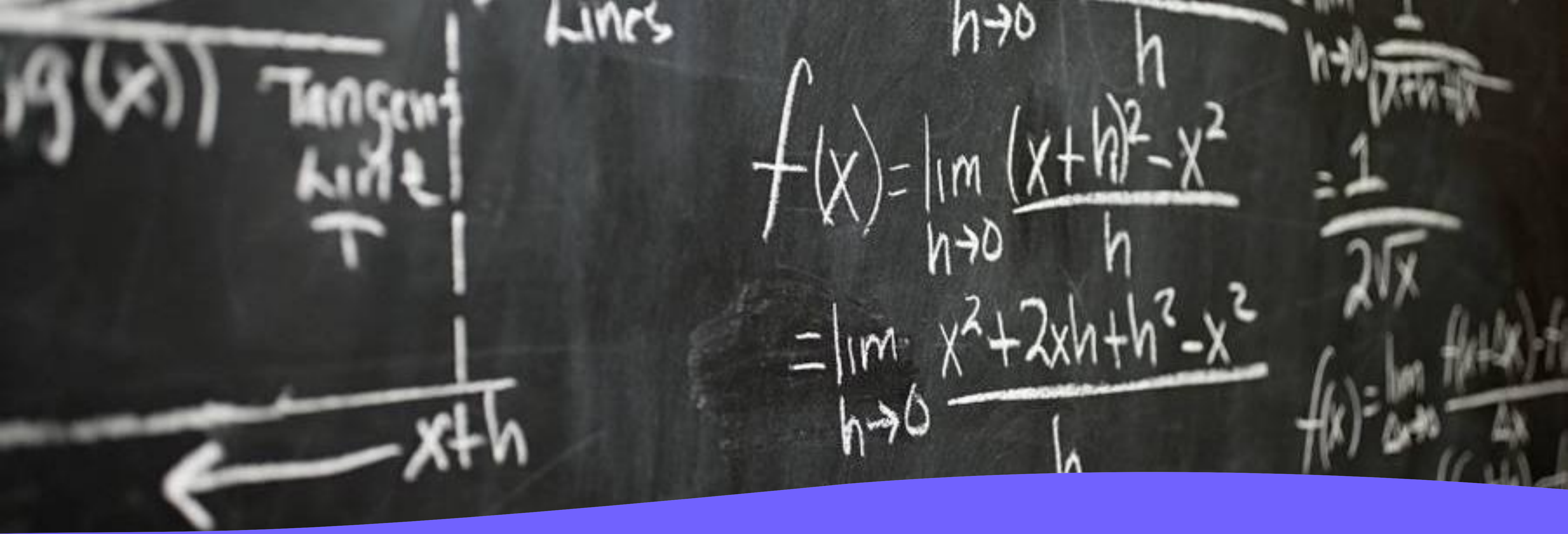
Some hints on how to approach these questions



# Optimisation

- Draw a diagram if possible
- Define the variables you will use
- Define the function to be optimised in terms of these variables
- Construct a relationship between the variables to express the function in terms of one variable (often in terms of time)
- Use your mathematics to determine stationary points and the nature of turning points
- Check end points of the allowable domain to see if the global maximum or minimum could be a suitable answer
- State conclusion in words



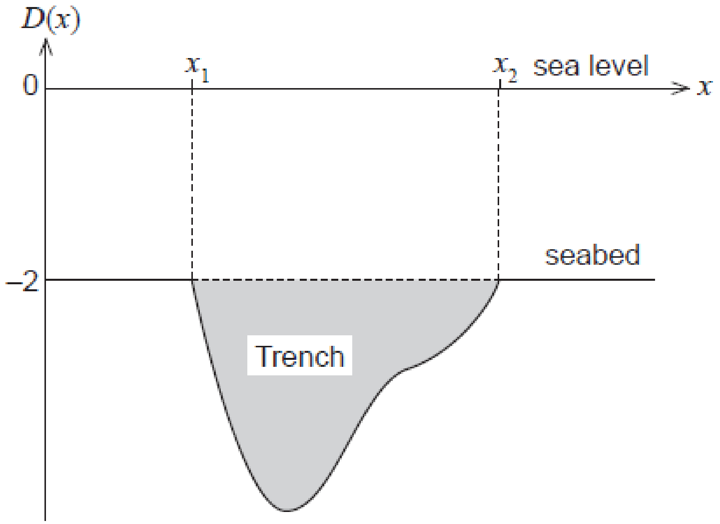



# Questions?

# WACE 2022 Question 7 (C)

**Question 7** **(10 marks)**

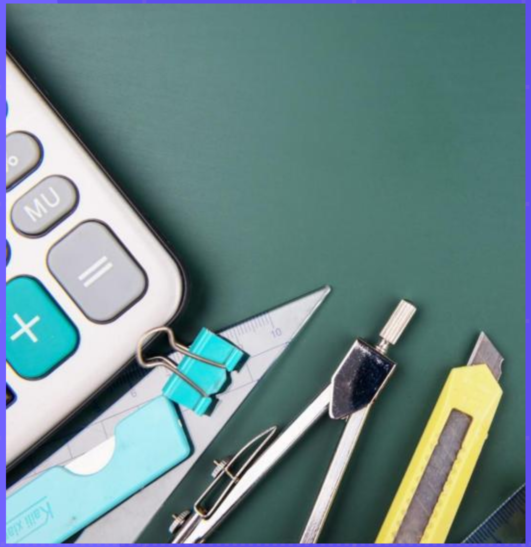
A team of oceanographers surveyed the depth of the ocean in a region populated by a particular endangered fish species. They discovered a large trench extending below the otherwise flat seabed as shown in the figure below.




The displacement, in kilometres, from sea level to the ocean floor is given by

$$D(x) = \begin{cases} (x-4)^2 + \cos(2x - 3\pi) - 5, & x_1 \le x \le x_2 \\ -2, & \text{otherwise} \end{cases}$$

where $x$ (measured in kilometres) is the east–west horizontal displacement relative to a reference marker at sea level.

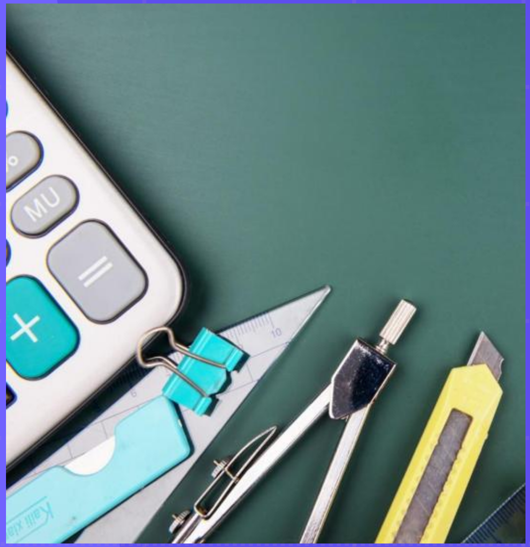


# WACE 2022 Question 7 (C)

(a) With reference to the figure above:

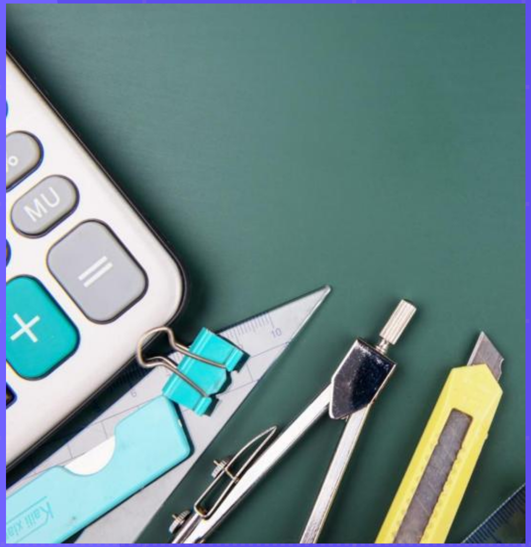
(i) determine the values of $x_1$ and $x_2$. (2 marks)

(ii) use calculus to determine the cross-sectional area of the trench shaded in the figure above. (3 marks)



# WACE 2022 Question 7 (C)

(b) Using calculus, determine the maximum distance of the trench below sea level. (5 marks)



# WACE 2022 Question  15 (C)

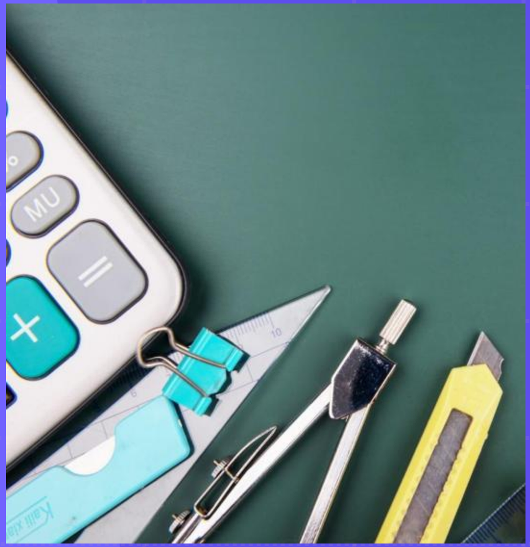
**Question 15** **(5 marks)**

An object moves from the point $(0, 0)$ along the curve $y = \sqrt{3}\,\sin(x)$. The distance, $D$, travelled along the curve is given by

$$D(t) = \int_0^{\pi t} \sqrt{1 + 3\cos^2(x)}\, dx$$

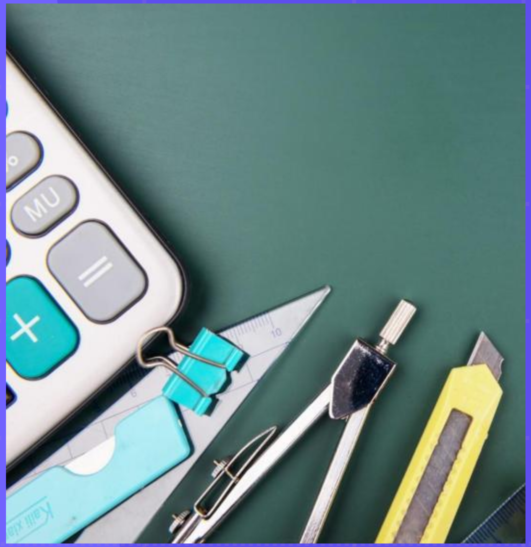
where $D$ is measured in metres and $t$ is measured in seconds.

(a) Determine the speed $s = \dfrac{dD}{dt}$ of the object when $t = 1$. (3 marks)



# WACE 2022 Question 15 (C)

(b) Use the increments formula to estimate the distance travelled by the object between $t = 1$ and $t = 1.02$. (2 marks)



# Unit 3 Topic 2

# Integration

# 3.2.1 – 3.2.22

# Syllabus points

**Anti-differentiation**

3.2.1 identify anti-differentiation as the reverse of differentiation

3.2.2 use the notation $\int f(x)dx$ for anti-derivatives or indefinite integrals

3.2.3 establish and use the formula $\int x^n dx = \frac{1}{n+1}x^{n+1} + c$ for $n \neq -1$

3.2.4 establish and use the formula $\int e^x dx = e^x + c$

3.2.5 establish and use the formulas $\int \sin x \, dx = -\cos x + c$ and $\int \cos x \, dx = \sin x + c$

3.2.6 identify and use linearity of anti-differentiation

3.2.7 determine indefinite integrals of the form $\int f(ax - b)dx$

3.2.8 identify families of curves with the same derivative function

3.2.9 determine $f(x)$, given $f'(x)$ and an initial condition $f(a) = b$

# Syllabus points

**Definite integrals**

3.2.10 examine the area problem and use sums of the form $\sum_i f(x_i)\,\delta x_i$ to estimate the area under the curve $y = f(x)$

3.2.11 identify the definite integral $\int_a^b f(x)dx$ as a limit of sums of the form $\sum_i f(x_i)\,\delta x_i$

3.2.12 interpret the definite integral $\int_a^b f(x)dx$ as area under the curve $y = f(x)$ if $f(x) > 0$

3.2.13 interpret $\int_a^b f(x)dx$ as a sum of signed areas

3.2.14 apply the additivity and linearity of definite integrals

# Syllabus points

**Fundamental theorem**

3.2.15 examine the concept of the signed area function $F(x) = \int_a^x f(t)dt$

3.2.16 apply the theorem: $F'(x) = \frac{d}{dx}\left(\int_a^x f(t)dt\right) = f(x)$, and illustrate its proof geometrically

3.2.17 develop the formula $\int_a^b f'(x)dx = f(b) - f(a)$ and use it to calculate definite integrals

# Syllabus points

**Applications of integration**

3.2.18 calculate total change by integrating instantaneous or marginal rate of change

3.2.19 calculate the area under a curve

3.2.20 calculate the area between curves determined by functions of the form $y = f(x)$

3.2.21 determine displacement given velocity in linear motion problems

3.2.22 determine positions given linear acceleration and initial values of position and velocity.

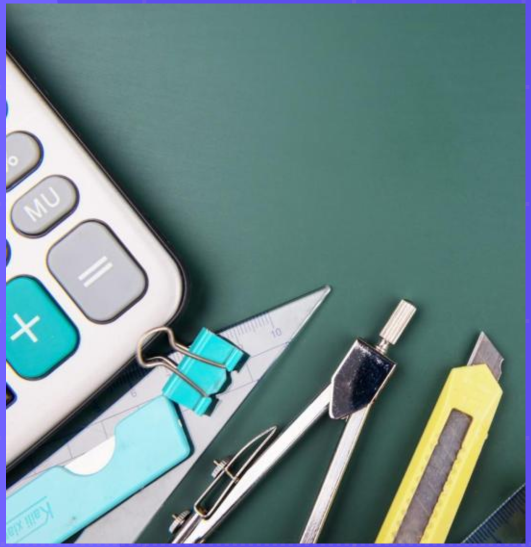
# Integration

Anti-differentiation is the opposite of differentiation. It is commonly called integration.

Integrating is the process of finding the primitive or antiderivative of a function.

Ww can find indefinite integrals (don't forget the +c), or definite integrals

Integration uses the symbol $\int$



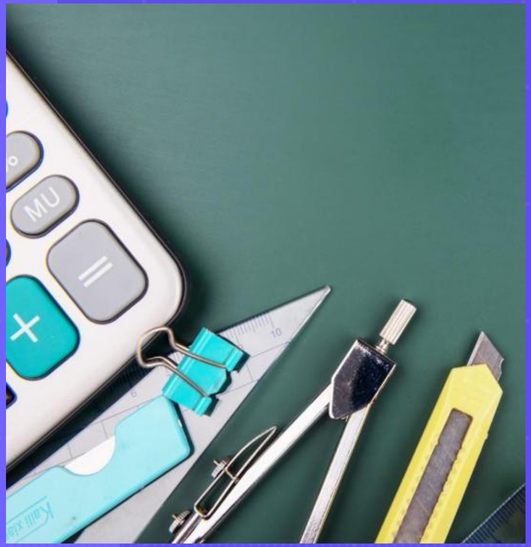
# Indefinite Integration

When we indefinitely integrate a function, we can only get a family of functions.

Eg $\frac{d}{dx}(x^2) = 2x$ , therefore $\int 2xdx = x^2 + c$

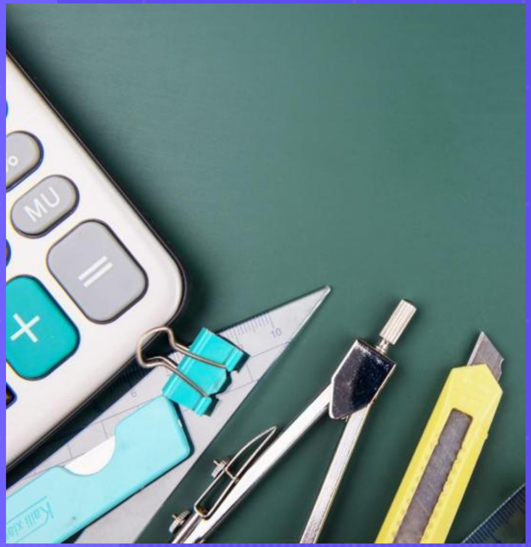
But also….. $\frac{d}{dx}(x^2 + 5) = 2x$ … and Eg $\frac{d}{dx}(x^2 \ - 4) = 2x$

So our indefinite integral will only give us the family of equations (unless we know a point on the original function and can solve for c)



# Integration

| |
|---|
| $\int x^n dx = \frac{x^{n+1}}{n+1} + c, \quad n \neq -1$ |
| $\int e^{ax} dx = \frac{1}{a} e^{ax} + c$ |
| $\int \frac{1}{x} dx = \ln x + c, \quad x > 0$ |
| $\int \frac{f'(x)}{f(x)} dx = \ln f(x) + c, \quad f(x) > 0$ |
| $\int \sin(ax - b)\, dx = -\frac{1}{a} \cos(ax - b) + c$ |
| $\int \cos(ax - b)\, dx = \frac{1}{a} \sin(ax - b) + c$ |



# Definite Integration

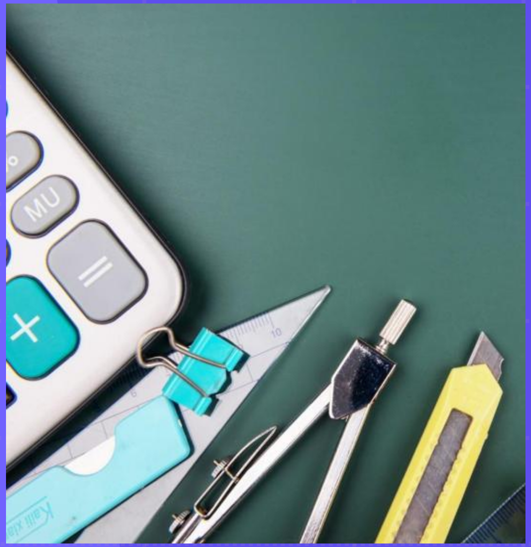
Definite integrals are shown by having limits on the integral sign

$\int_a^b f(x)\,dx$ denotes the area under the curve $f(x)$ from $x = a$ to $x = b$ where $f(x)$ is a non-negative function, where $a \leq x \leq b$

"Non-negative" because it is defined as the area 'under' the curve

$\int_a^b f(x)\,dx$ is where $f(x)$ is integrated with upper limit $b$ and lower limit $a$ with respect to $x$

A definite integrals has no units, but if the question requires you to work out an areas then appropriate units should be included.

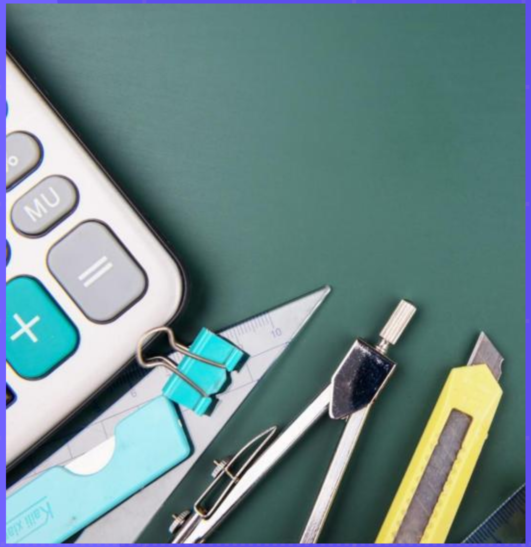


# Fundamental Theorem of Calculus

The fundamental theorem of calculus is a theorem that links the concept of differentiating a function (calculating its slopes, or rate of change at every point on its domain) with the concept of integrating a function (calculating the area under its graph).

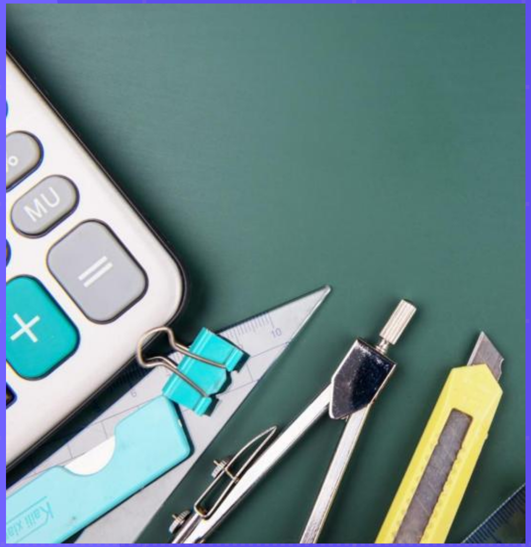
Roughly speaking, the two operations can be thought of as inverses of each other.

| | | | |
|---|---|---|---|
| Fundamental theorem | $\frac{d}{dx}\left(\int_a^x f(t)\,dt\right) = f(x)$ | and | $\int_a^b f'(x)\,dx = f(b) - f(a)$ |



# Linear Properties of Integration

| | |
|---|---|
| $\int_a^b f(x)dx$ | $-\int_b^a f(x)dx$ |
| $\int_a^b kf(x)dt$ | $k\int_a^b f(x)dt$ |
| $\int_a^b [f(x)+g(x)]dx$ | $\int_a^b f(x)dx + \int_a^b g(x)dx$ |
| $\int_a^b f(x)dx + \int_b^c f(x)dx$ | $\int_a^c f(x)dx$ |



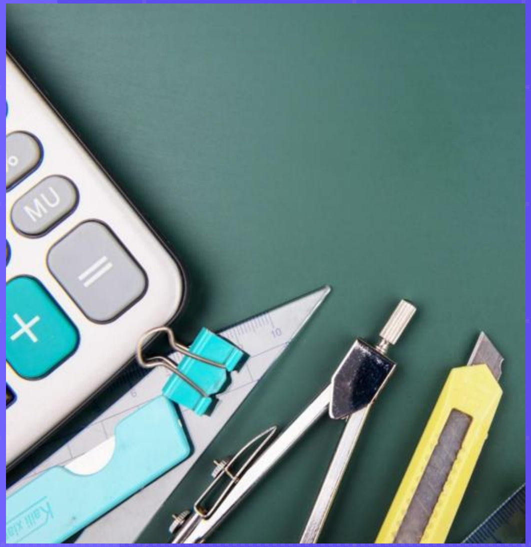
# Area under the curve

Practical problems will almost always require you to calculate an area, or more correctly interpret a definite integral for the situation being presented.

Area of the region trapped between $y = f(x)$, the $x$-axis and the lines $x = a$ and $x = b$ is:

$$A = \int_a^b |f(x)| dx$$

We must include the absolute value because our answer can only be positive (in practical cases).  We must also pay attention to instances where the curve crosses the x axis.



# Area between curves

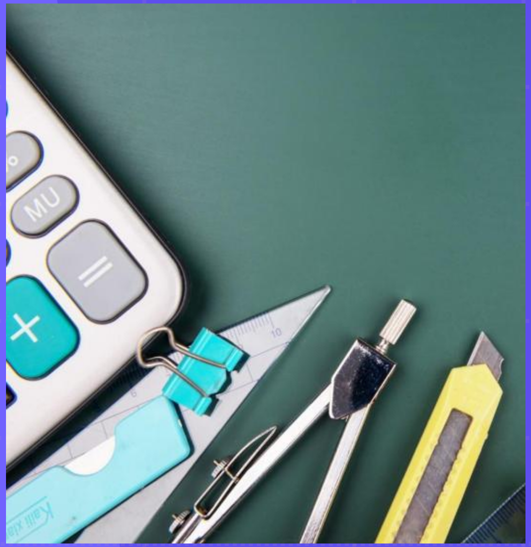
Another common question is to ask for the area between curves

Area of the region trapped between $y = f(x)$ and $y = g(x)$ is

$$A = \int_a^b |f(x) - g(x)| dx$$

Where f(x) > g(x), or f(x) sits higher on the Cartesian plane

We may need to do these questions in parts depending upon the complexity of the functions and their intersection points.



# Rectilinear motion

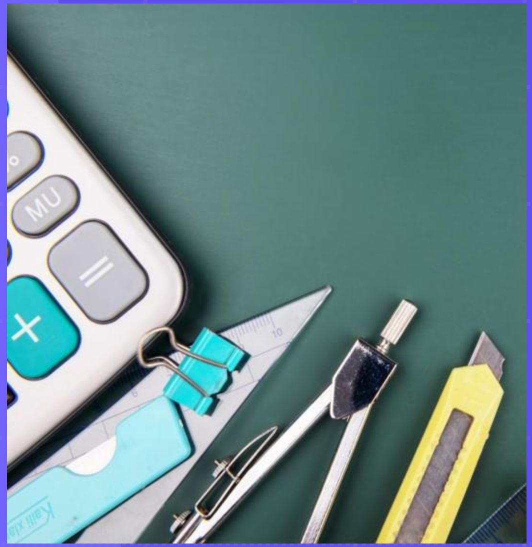
A fancy way of saying straight line motion.  For methods questions are usually limited to simple polynomial functions.  (Spec – you're not so lucky!)

When differentiating ….

Displacement → Velocity → Acceleration

When integrating (we need more information) ….

Acceleration → Velocity → Displacement

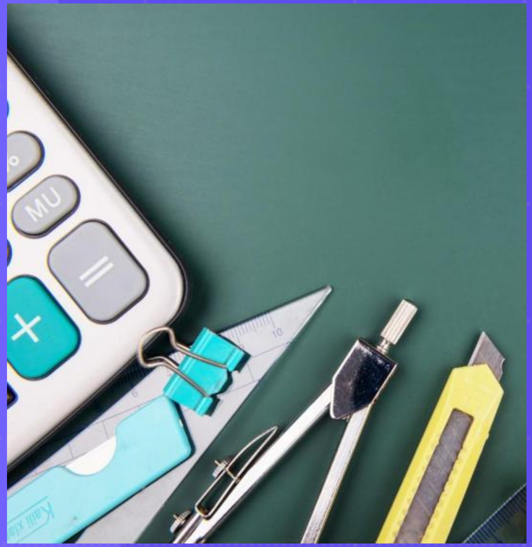


# Rectilinear motion

Scalar and Vector quantities:

Scalar – only has a magnitude.  Examples are distance, speed, mass, time, temperature, volume etc

Vector – has magnitude and direction.  Examples are displacement, velocity, acceleration, force, torque etc

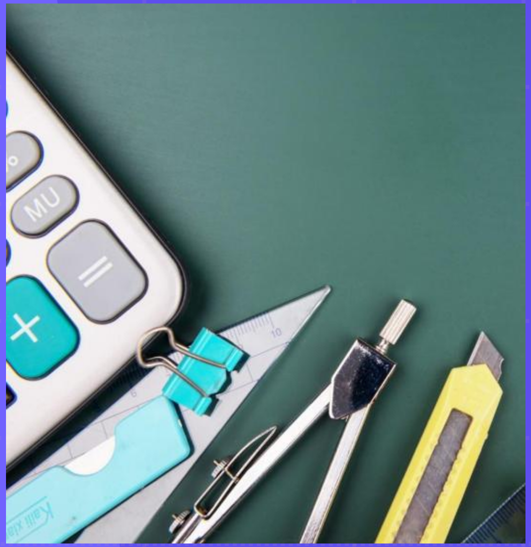


# Rectilinear motion

Displacement → $x = \int v\, dt$

Velocity → $v = \frac{dx}{dt} = \int a\, dt$

Acceleration → $a = \frac{dv}{dt} = \frac{d^2x}{dt^2}$



# Rectilinear motion

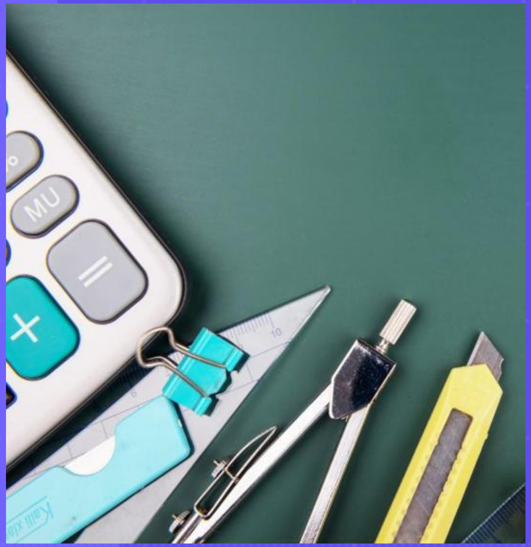
When x = 0, the body has returned to the origin.

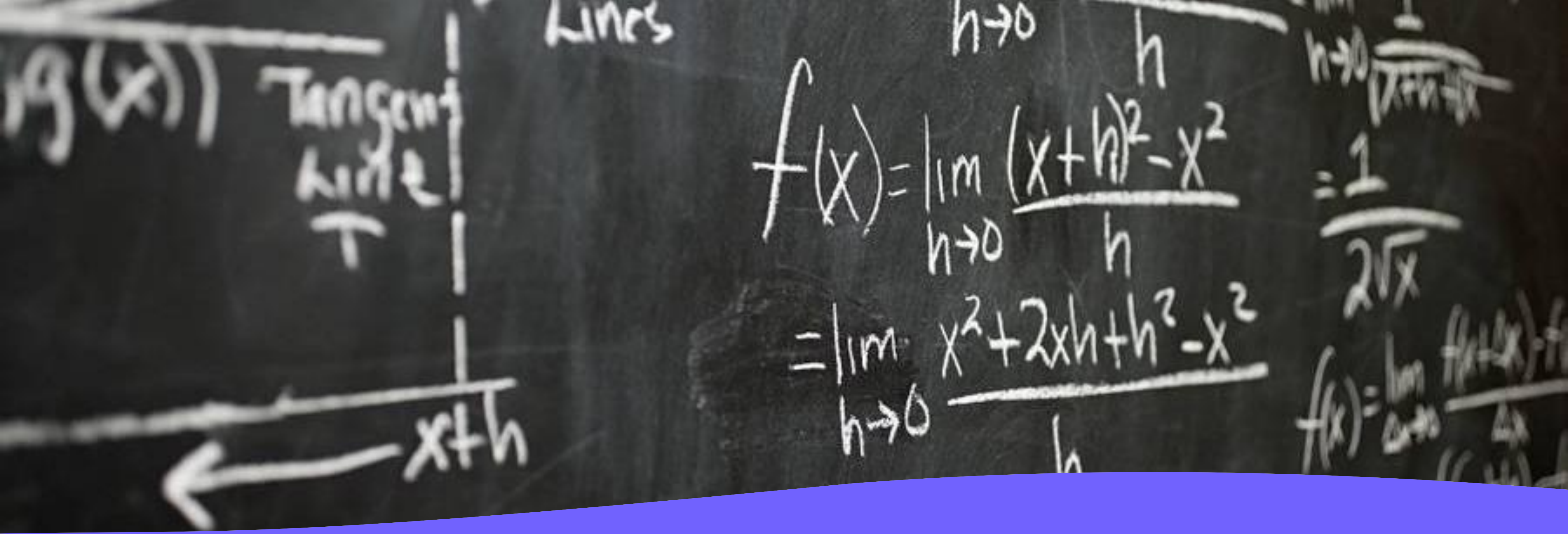
When $v = 0 \ or \ \frac{dx}{dt} = 0$, the body is not moving (and likely is changing direction)

When $a = 0 \ or \ \frac{dv}{dt} = 0 \ or \ \frac{d^2x}{dt^2} = 0$, the body is at a constant speed

Change in displacement (position) $a \leq t \leq b = \Delta x = \int_a^b v \, dt$

Distance travelled $a \leq t \leq b = \int_a^b |v| \, dt$






# Questions?

# WACE 2022 Question 10 (C)

**Question 10** **(9 marks)**

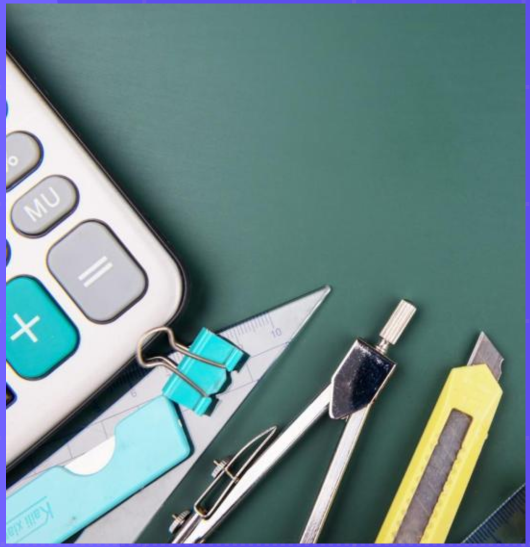
The displacement, $x$, of a mass on the end of a damped spring is given by

$$x(t) = 3e^{-t}\sin(t), \quad t \geq 0$$

where $x$ is in centimetres and $t$ is in seconds.

(a) Determine when the mass first returns to its starting position at $x = 0$. (2 marks)

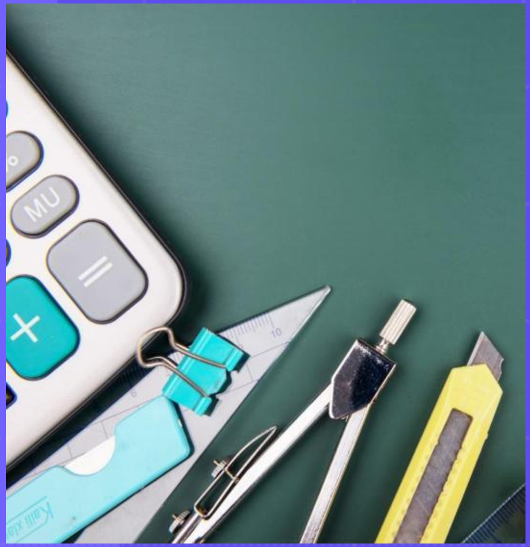
(b) Determine an expression for the velocity of the mass. (2 marks)



# WACE 2022 Question 10 (C)

(c) Determine the displacement of the mass when it first changes direction. (3 marks)

(d) The mass is considered to have stopped oscillating when the oscillation amplitude $A(t) = 3e^{-t}$ drops to 0.01 cm. How long does it take for the spring to stop oscillating? (2 marks)



# WACE 2023 Question 5 (NC)

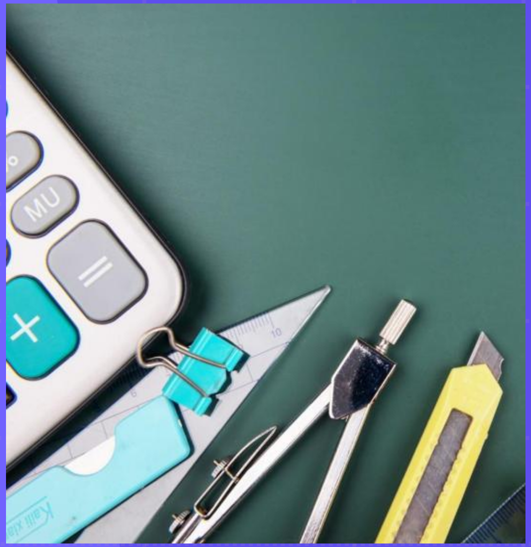
**Question 5** **(13 marks)**

The table below contains values of the polynomial function $f(x)$, its first and second derivatives, and the function $F(x) = \int_0^x f(t)\,dt$ for $x = 0, 1, 2, 3, 4, 5, 6$.

$f(x)$ has no stationary points at non-integer values of $x$, and the letters $a$, $b$, $c$, $d$ and $e$ represent unspecified constants.

| | $x=0$ | $x=1$ | $x=2$ | $x=3$ | $x=4$ | $x=5$ | $x=6$ |
|---|---|---|---|---|---|---|---|
| $f(x)$ | $a$ | $b$ | 4 | $c$ | 0 | $d$ | $e$ |
| $f'(x)$ | 16 | 0 | –4 | –2 | 0 | –4 | –20 |
| $f''(x)$ | –24 | –9 | 0 | 3 | 0 | –9 | –24 |
| $F(x)$ | 0 | 4.7 | 10.4 | 12.6 | 12.8 | 12.5 | 7.2 |

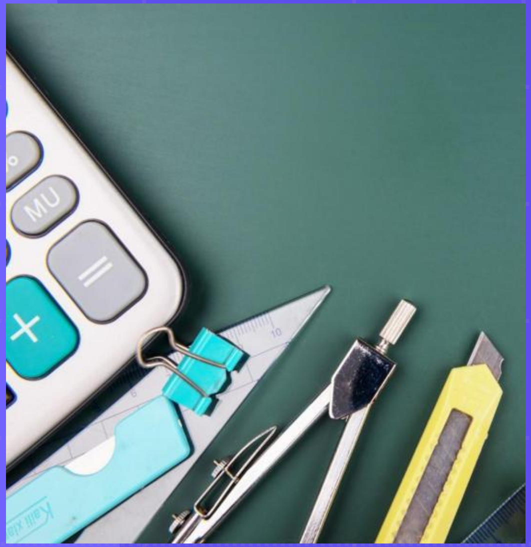
(a) Evaluate $\frac{d}{dx}\left(f(x)^2\right)$ when $x = 2$. (2 marks)



# WACE 2023 Question 5 (NC)

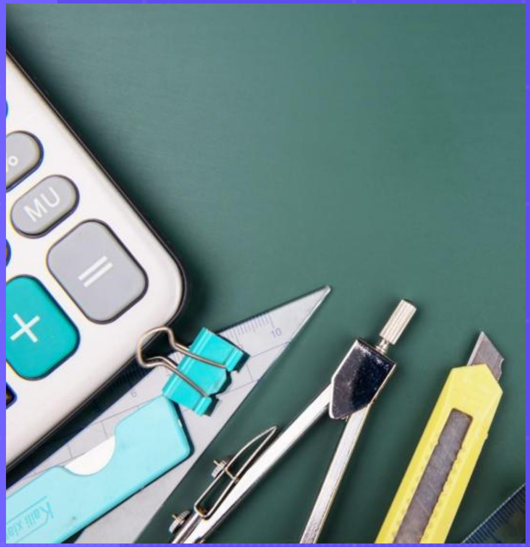
(b) Evaluate $\int_2^4 \left(f(x)+2\right)\,dx$ . (3 marks)

(c) Evaluate $\dfrac{d}{dx}\int_2^x f(t)\,dt$ when $x = 2$ . (2 marks)



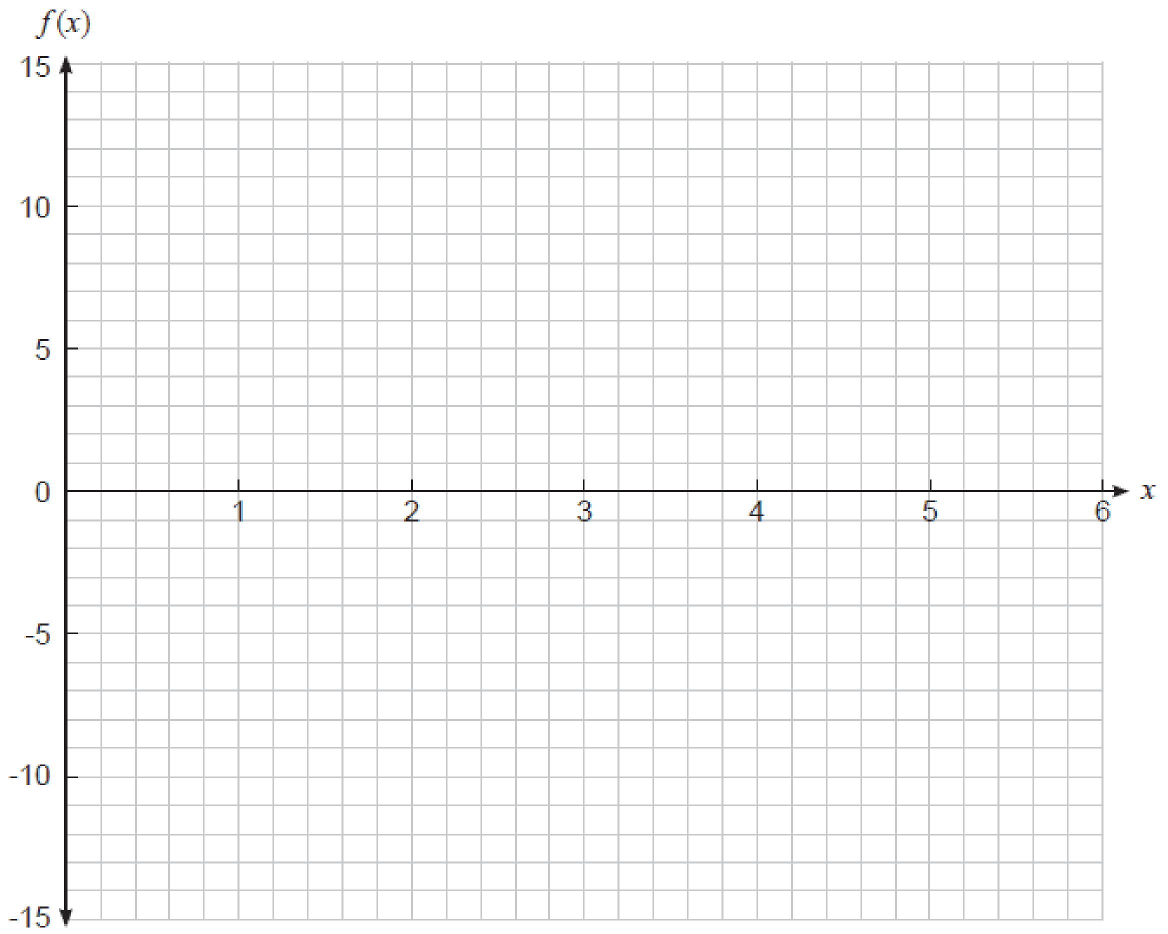
# WACE 2023 Question 5 (NC)

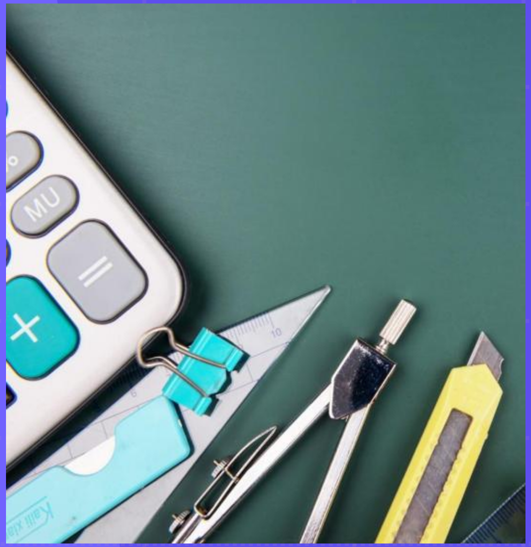
(d) Determine the $x$-coordinate of any stationary points and whether they are local maxima, local minima or inflection points. Justify your answer. (3 marks)

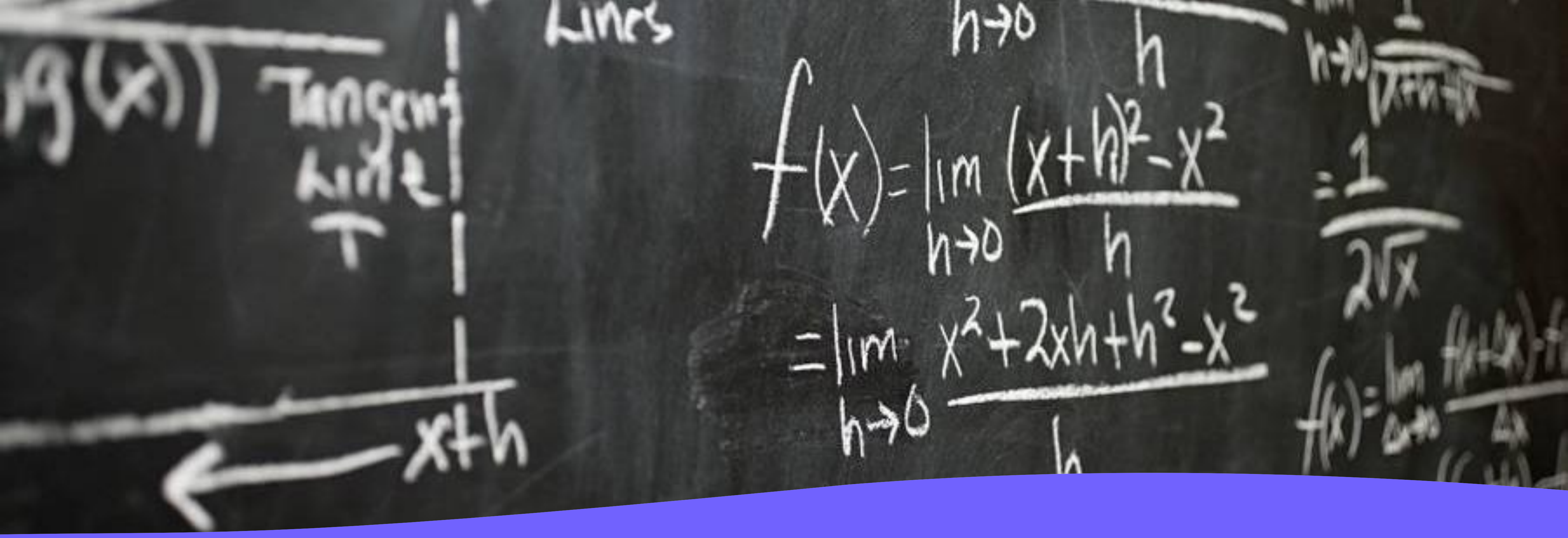


# WACE 2023 Question 5 (NC)

(e) Sketch a possible graph of $f(x)$ for $0 \leq x \leq 6$ on the axes below. (3 marks)






# Time to take a break!

10 minutes to stretch your legs, have a chat, use the restrooms!

Unit 3 Topic 3

DRV

3.3.1 – 3.2.16

# Syllabus points

**General discrete random variables**

3.3.1 develop the concepts of a discrete random variable and its associated probability function, and their use in modelling data

3.3.2 use relative frequencies obtained from data to obtain point estimates of probabilities associated with a discrete random variable

3.3.3 identify uniform discrete random variables and use them to model random phenomena with equally likely outcomes

3.3.4 examine simple examples of non-uniform discrete random variables

3.3.5 define the mean or expected value of a discrete random variable and interpret it as a measure of location, and evaluate it in simple cases

3.3.6 define the variance and standard deviation of a discrete random variable and interpret them as measures of spread, and evaluate them using technology

3.3.7 examine the effects of linear changes of scale and origin on the mean and the standard deviation

3.3.8 use discrete random variables and associated probabilities to solve practical problems

# Syllabus points

**Bernoulli distributions**

3.3.9 use a Bernoulli random variable as a model for two-outcome situations

3.3.10 identify contexts suitable for modelling by Bernoulli random variables

3.3.11 determine the mean $p$ and variance $p(1-p)$of the Bernoulli distribution with parameter $p$

3.3.12 use Bernoulli random variables and associated probabilities to model data and solve practical problems

# Syllabus points

**Binomial distributions**

3.3.13 examine the concept of Bernoulli trials and the concept of a binomial random variable as the number of 'successes' in $n$ independent Bernoulli trials, with the same probability of success $p$ in each trial

3.3.14 identify contexts suitable for modelling by binomial random variables

3.3.15 determine and use the probabilities $P(X = x) = \binom{n}{x} p^x (1 - p)^{n-x}$ associated with the binomial distribution with parameters $n$ and $p$; note the mean $np$ and variance $np(1 - p)$ of a binomial distribution

3.3.16 use binomial distributions and associated probabilities to solve practical problems

# Discrete Random Variables

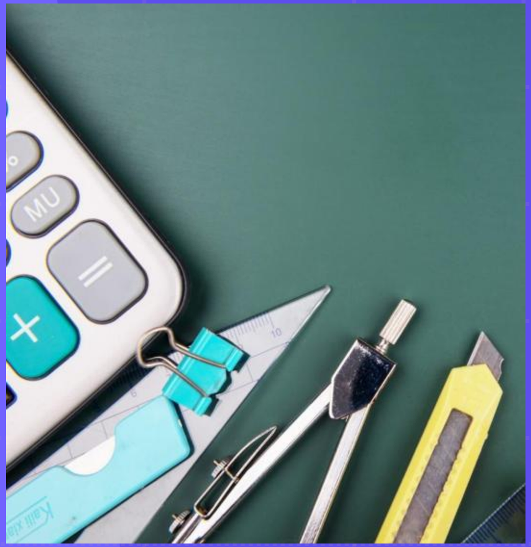
**Discrete** means a value can only take on countable numbers

**Random** means it can take on a range of values

**Variable** means the value can vary across a range of values

The random variable $X$ can only take on countable values. If we have a discrete random variable, $X$, then its probability distribution function P(x) must adhere to the following constraints:

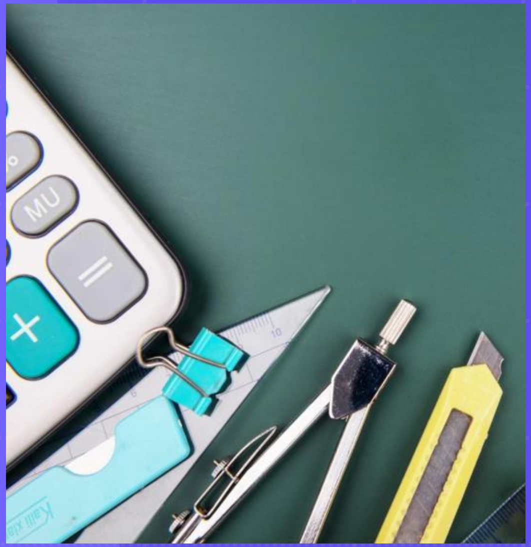
- Each value of $p(x)$ must be between 0 and 1, $0 \leq p(x) \leq 1$
- The sum of all the values of $p(x)$ must equal 1, $\sum p(x) = 1$



# Expected value

The expected value or mean is found by summing the products of each value of $X$ and the probability that $X$ takes on that value for $X$,

$$\mu = E(X) = \sum x \times p(x)$$



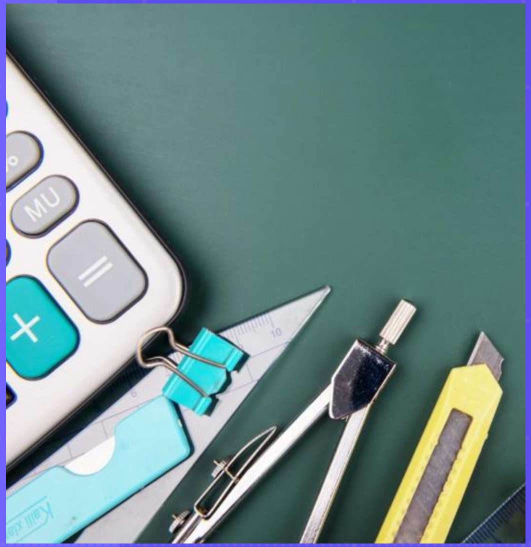
# Variance

The variance of a random variable $X$, is a measure of the spread of the probability about its mean

$$
\begin{aligned}
Var(X) = \sigma^2 &= \sum (x - \mu)^2 \times p(x) \\
&= E(X)^2 - [E(X)]^2 \\
&= \sum x^2 \times p(x) - \mu^2
\end{aligned}
$$

The standard deviation of a random variable $X$ is the square root of the variance

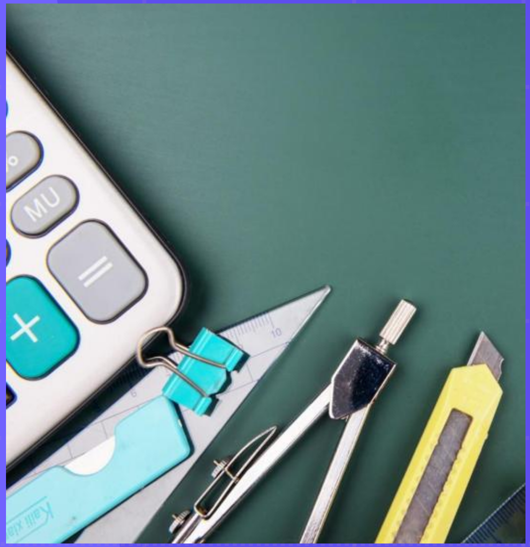
$SD(X) = \sigma = \sqrt{Var(X)}$



# Variance v Standard Deviation

Statisticians use variance to compare data sets, and there is further analysis that can be done with variance.

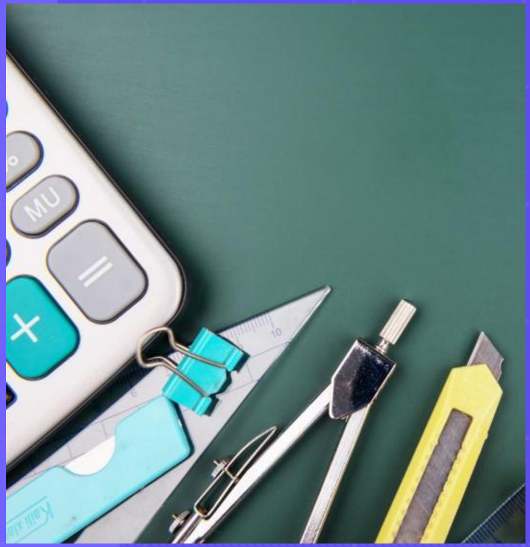
Standard Deviation is relevant to the data, hence in most situations, standard deviation is reported to non-statisticians.



# Effect of linear changes

**A change in scale** – stretches the data. Eg multiplying all data by a, stretches the range by a sf of a, the expected value by a sf of a and the variance by $a^2$, and the standard deviation by the absolute value of a.

**A change in origin** moves the data along the number line but makes no difference to the spacing of the data.  Eg adding b to all data increases the maximum and minimum by b, increases the expected value by b, but makes no difference to the variance or standard deviation

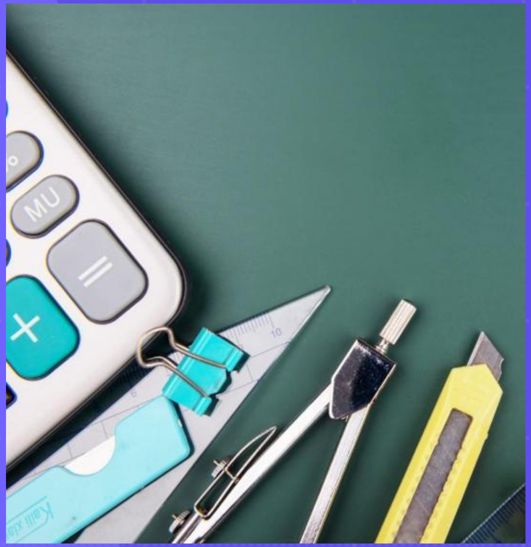


# Effect of linear changes

$$E(aX+b) = aE(X)+b$$

$$Var(aX+b) = a^2Var(X)$$

$$SD(aX+b) = |a|SD(X)$$



# Bernoulli distributions

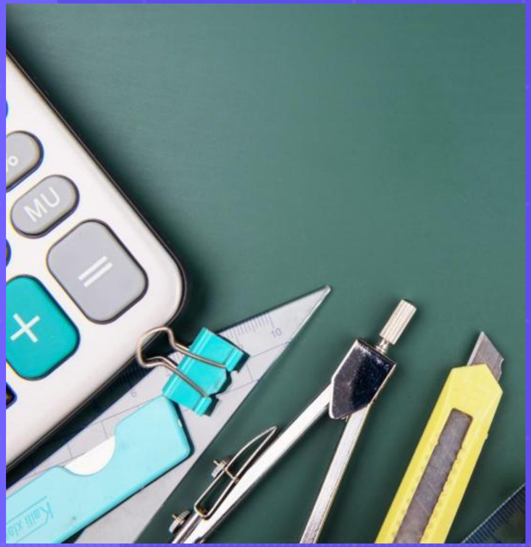
A Bernoulli distribution is the outcome of a single trial.

The outcome is either a success, usually ascribed a numerical value of 1, or a failure, usually ascribed a numerical value of 0.

If the probability of success on a single trial is given the variable, p, then the probability of failure on a single trial is (1-p).

Mean for $X, \mu = E(X) = p$

Variance for $X, \sigma^2 = E(X^2) - \mu^2 = p(1-p)$



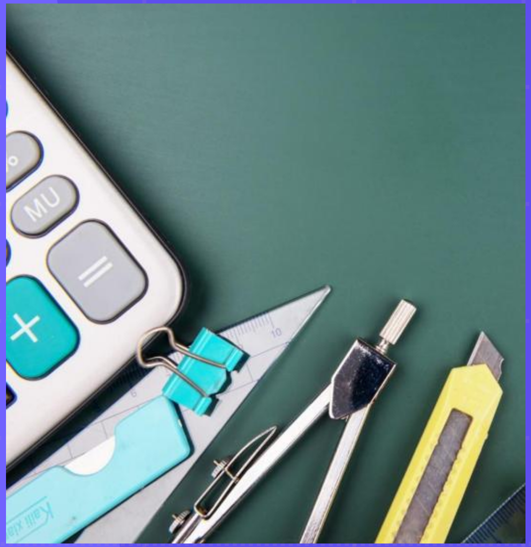
# Binomial distributions

Repeated Bernoulli trials with the same conditions is a binomial distribution.

As the syllabus says "…. binomial random variable as the number of 'successes' in $n$ independent Bernoulli trials, with the same probability of success $p$ in each trial"  dot point 3.1.13.

Each trial must have the same probability of success, and hence each trial must be independent.

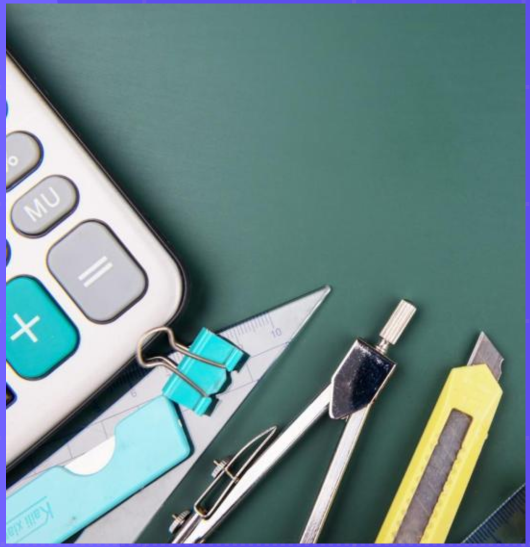
Eg Flipping  a coin is a binomial distribution as each flip has no memory or hang over from the past flip.  Where as drawing counters from a bag **without replacement** changes the probability of success for the next selection.



# Gambler’s Fallacy

“The **gambler's fallacy**, also known as the **Monte Carlo fallacy** or the **fallacy of the maturity of chances**, is the belief that an independent and equally probable outcome which happened less frequently than expected is more likely to happen in the future (or vice versa). The fallacy is commonly associated with gambling, where it may be mistakenly believed, for example, that the next dice roll is more likely to give '4' because there have recently been fewer '4's than expected, when in reality the probability of the next outcome being '4' is always 1/6, for each dice roll is an independent event.”

Wikipedia – accessed 29 August 2026



# Binomial Distributions

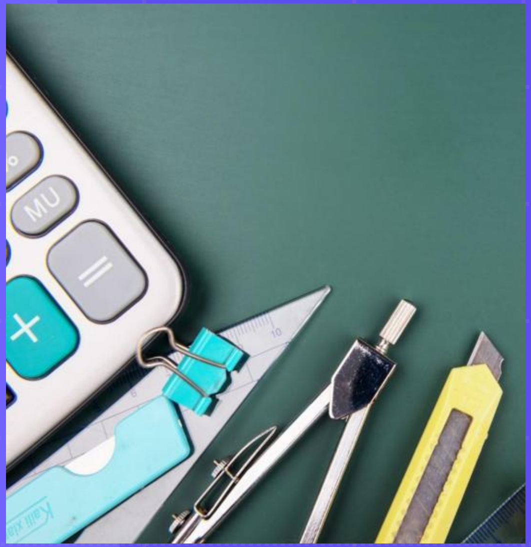
The probability distribution function for a binomial variable $X$ defined over $n$ trials with $p$ as the constant probability of producing a successful outcome for each trial is

$$p(x) = \binom{n}{x} p^x (1-p)^{n-x} \text{ where } x = 0, 1, 2, 3 \dots n.$$

Often Binomial Distribution is written as $X \sim B(n, p)$

We interpret and read this as "$X$ is binomially distributed with $n$ trials and a probability of success in each trial of $p$".

The mean of $X = np$ and the variance of $X = np(1-p)$



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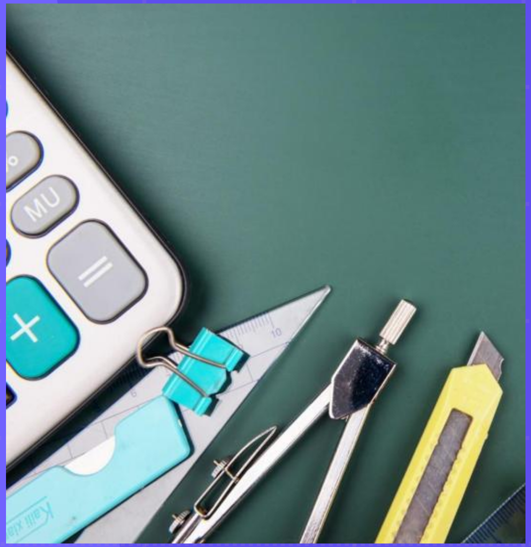
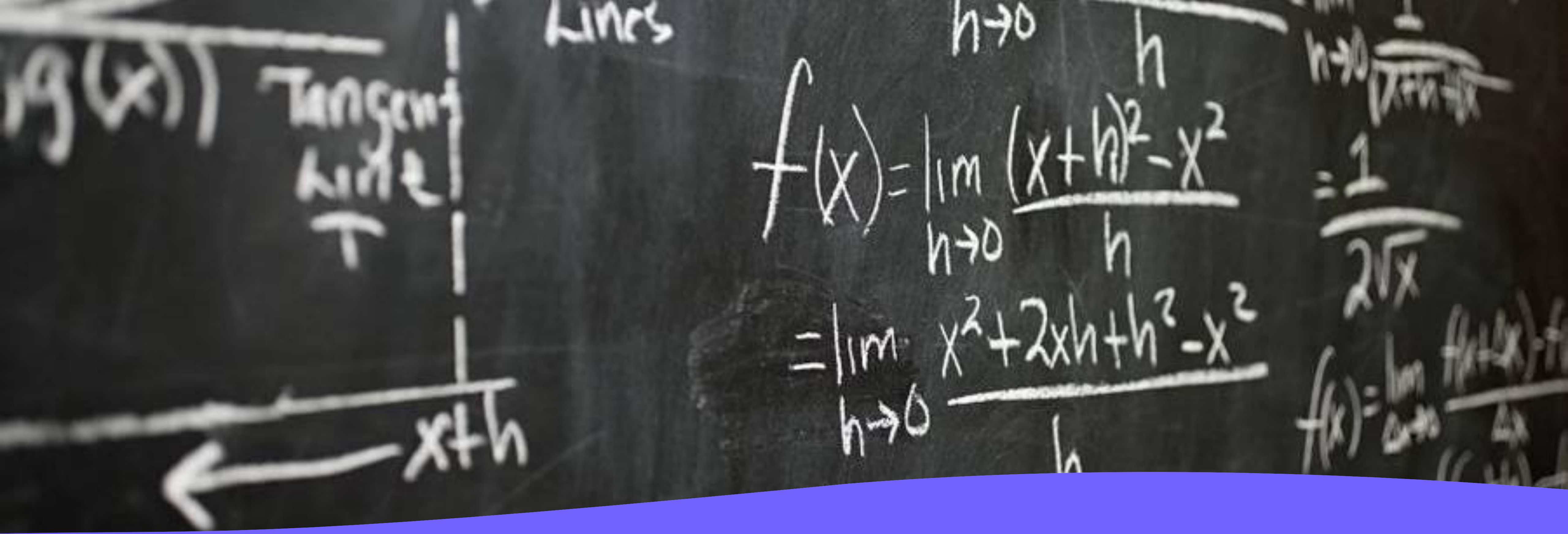





# Questions?

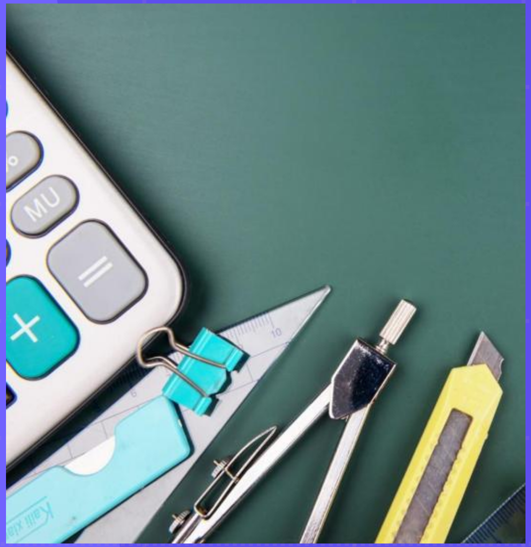
# WACE 2022 Question 9 (C)

**Question 9** **(14 marks)**

Andrew plans to run a game called Lucky Cup for a school fundraising event. All profits go toward the school's fundraising efforts. The game consists of three standard dice, each placed into a red cup. The red cup is shaken, the dice rolled, and the number of sixes recorded.

Let $X$ be a random variable denoting the number of sixes rolled in a game of Lucky Cup.

(a) State the distribution of $X$. (2 marks)



# WACE 2022 Question 9 (C)

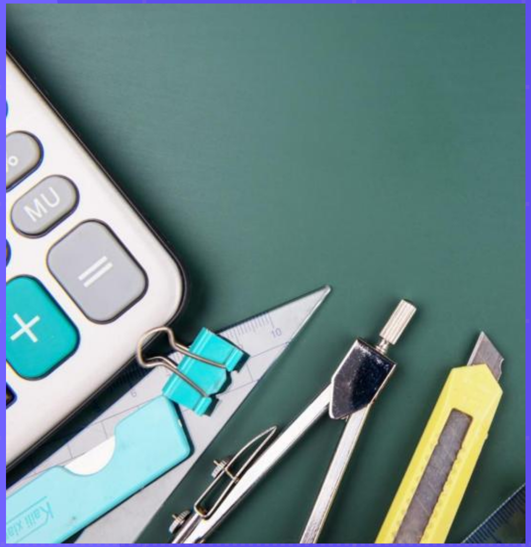
An incomplete probability distribution for $X$ is shown in the table below.

| $x$ | 0 | 1 | 2 | 3 |
|---|---|---|---|---|
| $P(X=x)$ | $\frac{125}{216}=0.5787$ | | | |

(b) Complete the table above, providing the missing probabilities. (2 marks)

Lucky Cup costs \$1 to play. If a player rolls one 6 they win \$1, if they roll two 6s they win \$2, and if they roll three 6s they win \$3.

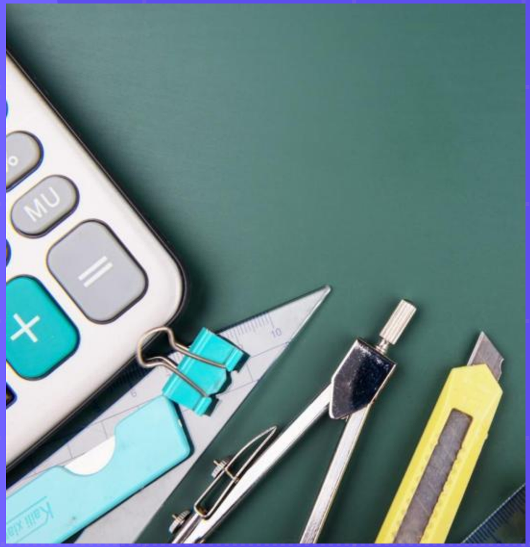
(c) Determine the school's expected profit/loss for each game of Lucky Cup. (3 marks)



# WACE 2022 Question 9 (C)

(d) Determine the probability that a player will make a profit in a game of Lucky Cup.

(2 marks)

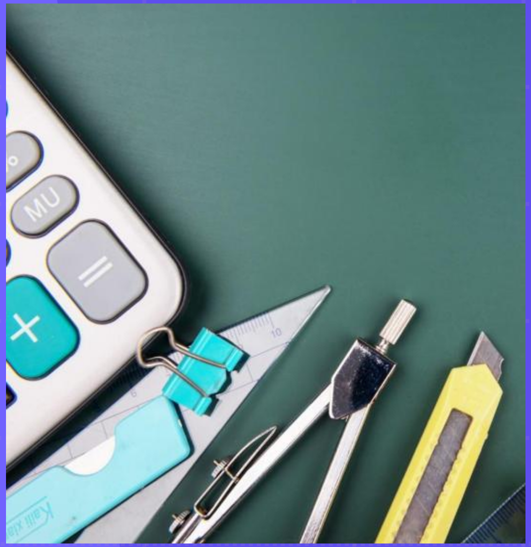


# WACE 2022 Question 9 (C)

Andrew wants to increase the attraction of the game by providing the opportunity for larger winnings. He modifies the rules of the game so that players only win money when two or more 6s are rolled, and the winnings for rolling three 6s is three times as much as the winnings for rolling two 6s. Each game still costs $1 to play. He estimates that he should be able to run 500 games and wants to make a profit of $200 for the school.

Andrew calls this game Lucky Cup II.

(e) Determine the winnings Andrew should set for rolling three 6s in Lucky Cup II. (3 marks)

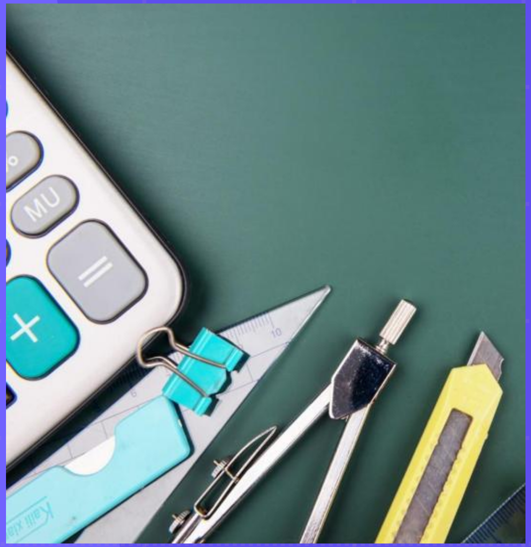


# WACE 2022 Question 9 (C)

Andrew decides to make the game even more dynamic and exciting. He adds a green cup with a die that he rolls at the beginning of each game. The value that Andrew rolls becomes the target value, and players must roll this target value to win. For example, if Andrew rolls a 2 from the green cup and a player rolls three 2s, the player wins the top prize.

Andrew calls this game Lucky Cup III.

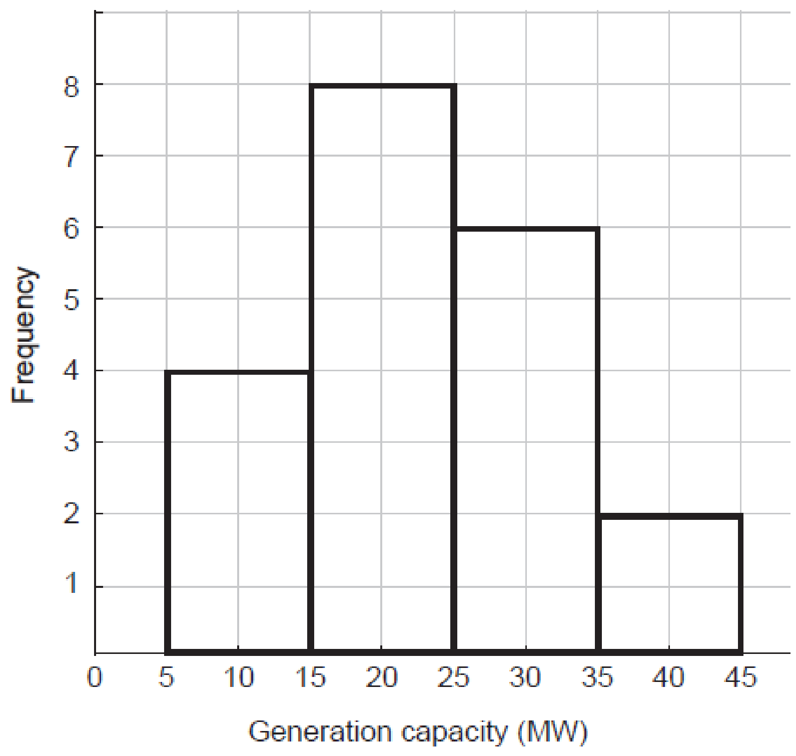
(f) Explain how this change affects a player's chance of winning compared with Lucky Cup II. (2 marks)

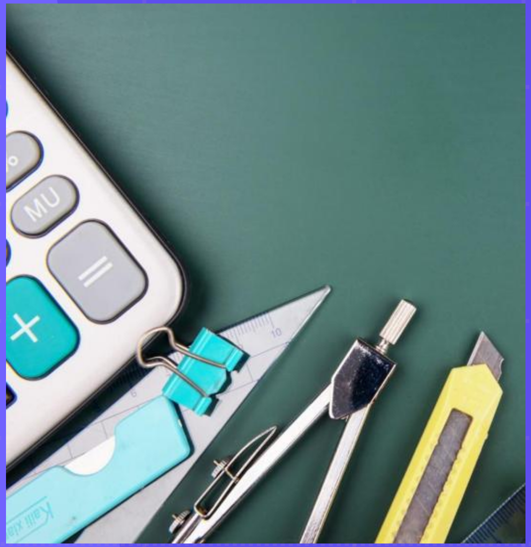


# WACE 2023 Question 3 (NC)

**Question 3** **(10 marks)**

Solcolwa is a green energy company that owns 20 solar farms across Western Australia. The generation capacities, in megawatts (MW), of the solar farms are displayed in the histogram below.



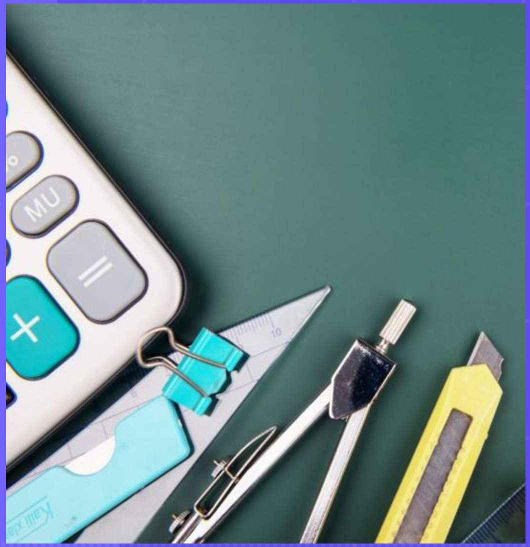
# WACE 2023 Question 3 (NC)

Suppose that one of the Solcolwa solar farms is selected at random. Let the random variable $W$ denote the generation capacity of the randomly-selected solar farm.

(a) Complete the following table of cumulative probabilities for $W$. (2 marks)

| $w$ | 5 | 15 | 25 | 35 | 45 |
|---|---|---|---|---|---|
| $P(W \le w)$ | | | | | |

(b) Determine $P(W \ge 35)$. (1 mark)

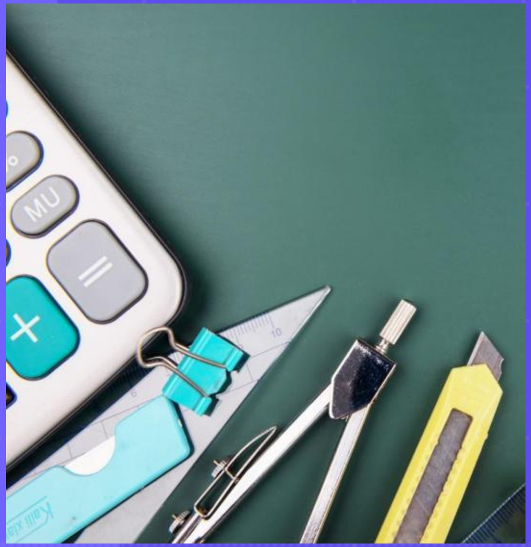


# WACE 2023 Question 3 (NC)

(c) Assuming the solar farms are uniformly distributed within each interval:

(i) estimate $P(W \geq 20)$. (2 marks)

(ii) estimate the expected value $E(W)$. (2 marks)



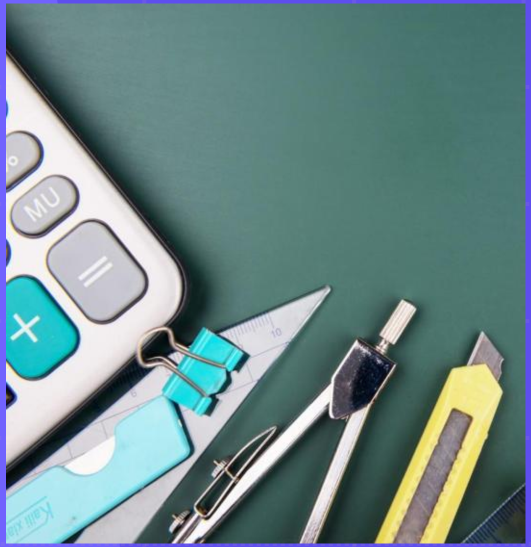
# WACE 2023 Question 3 (NC)

To increase the generation capacity of its solar farms, Solcolwa decides to upgrade all its solar panels with the latest technology. A new random variable $Y$ denotes the generation capacity of a randomly-selected upgraded solar farm. The random variables $W$ and $Y$ are related by

$$Y = aW$$

for some constant $a > 0$.

(d) Given that $W$ and $Y$ have variances $Var(W) = 81$ and $Var(Y) = 324$, determine the expected value $E(Y)$. (3 marks)



# Unit 4 Topic 1

# Logarithms

# 4.1.1 – 4.1.14

# Syllabus points

**Logarithmic functions**

4.1.1 define logarithms as indices: $a^x = b$ is equivalent to $x = \log_a b$ i.e. $a^{\log_a b} = b$

4.1.2 establish and use the algebraic properties of logarithms

4.1.3 examine the inverse relationship between logarithms and exponentials: $y = a^x$ is equivalent to $x = \log_a y$

4.1.4 interpret and use logarithmic scales

4.1.5 solve equations involving indices using logarithms

4.1.6 identify the qualitative features of the graph of $y = \log_a x$ $(a > 1)$, including asymptotes, and of its translations $y = \log_a x + b$ and $y = \log_a(x - c)$

4.1.7 solve simple equations involving logarithmic functions algebraically and graphically

4.1.8 identify contexts suitable for modelling by logarithmic functions and use them to solve practical problems

# Syllabus points

**Calculus of the natural logarithmic function**

4.1.9 define the natural logarithm $\ln \mathrm{x} = \log_{\mathrm{e}} \mathrm{x}$

4.1.10 examine and use the inverse relationship of the functions $y = e^x$ and $y = \ln x$

4.1.11 establish and use the formula $\frac{d}{dx}(\ln x) = \frac{1}{x}$

4.1.12 establish and use the formula $\int \frac{1}{x} dx = \ln\ x\ + c$, for $x > 0$

4.1.13 determine derivatives of the form $\frac{d}{dx}(\ln f(x))$ and integrals of the form $\int \frac{f'(x)}{f(x)} dx$, for $f\ (x) > 0$

4.1.14 use logarithmic functions and their derivatives to solve practical problems

# Index rules

base $\rightarrow$ $a^{m}$ $\leftarrow$ index, exponent, power

$$a^m \times a^n = a^{m+n}$$

$$a^m \div a^n = a^{m-n}$$

$$(a^m)^n = a^{mn}$$

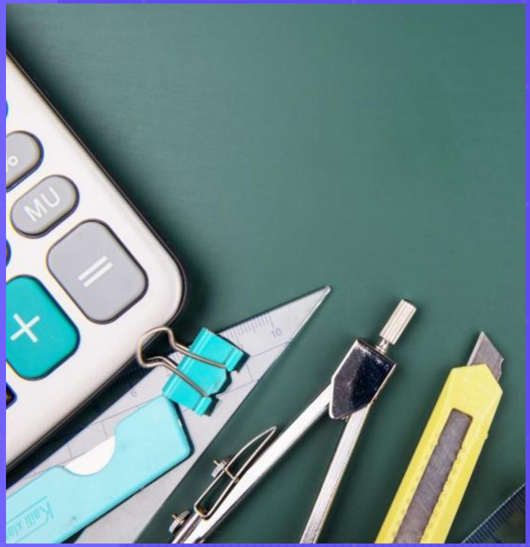
$$(ab)^m = a^m b^m$$

$$\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$$

$$a^0 = 1$$

$$a^{-m} = \frac{1}{a^m}$$

$$a^{m/n} = \sqrt[n]{a^m}$$



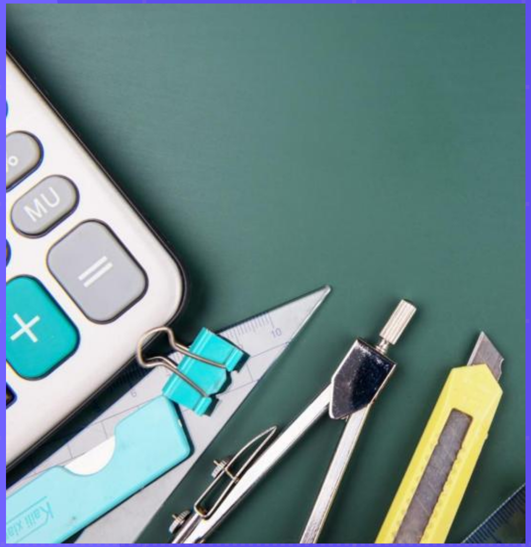
# Index rules ←→Logarithms

If $a^x = b$ then $\log_a b = x$

In $a^x = b$ → the base is a, the exponent is x and the result is b

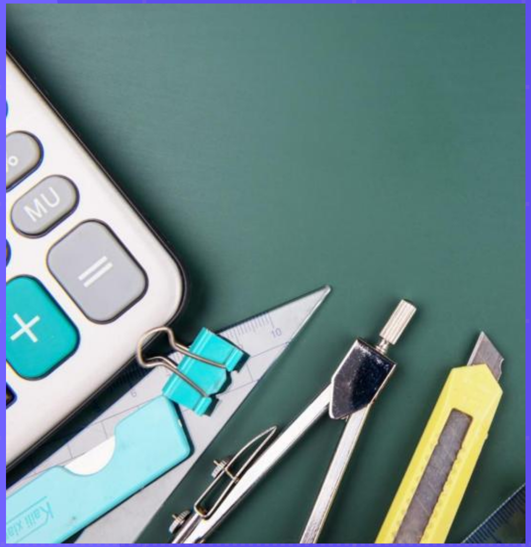
In $\log_a b = x$ → the log of b in base a is x.

$$\log_a b = x$$



# Logarithm rules

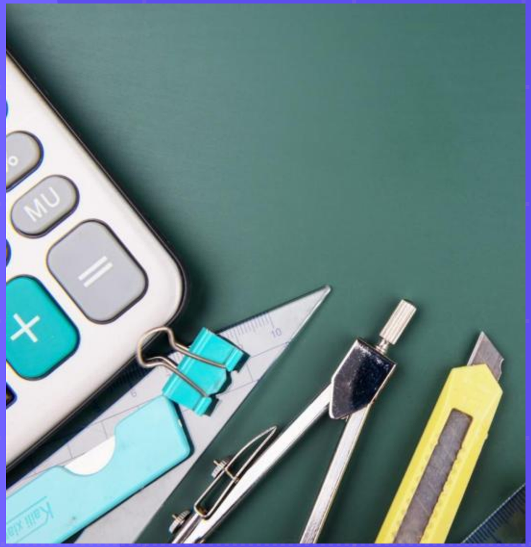
| | |
|---|---|
| $x = \log_a b \Leftrightarrow a^x = b$ | $a^{\log_a b} = b$ and $\log_a(a^b) = b$ |
| $\log_a mn = \log_a m + \log_a n$ | $\log_a \frac{m}{n} = \log_a m - \log_a n$ |
| $\log_a(m^k) = k \log_a m$ | $\log_e x = \ln x$ |



# Change of base

These can be handy to manipulate equations into a format which is more usable.

$$\log_a x = \frac{\log_b x}{\log_b a} \qquad \text{and} \qquad a^x = b^{(\log_b a)x}$$

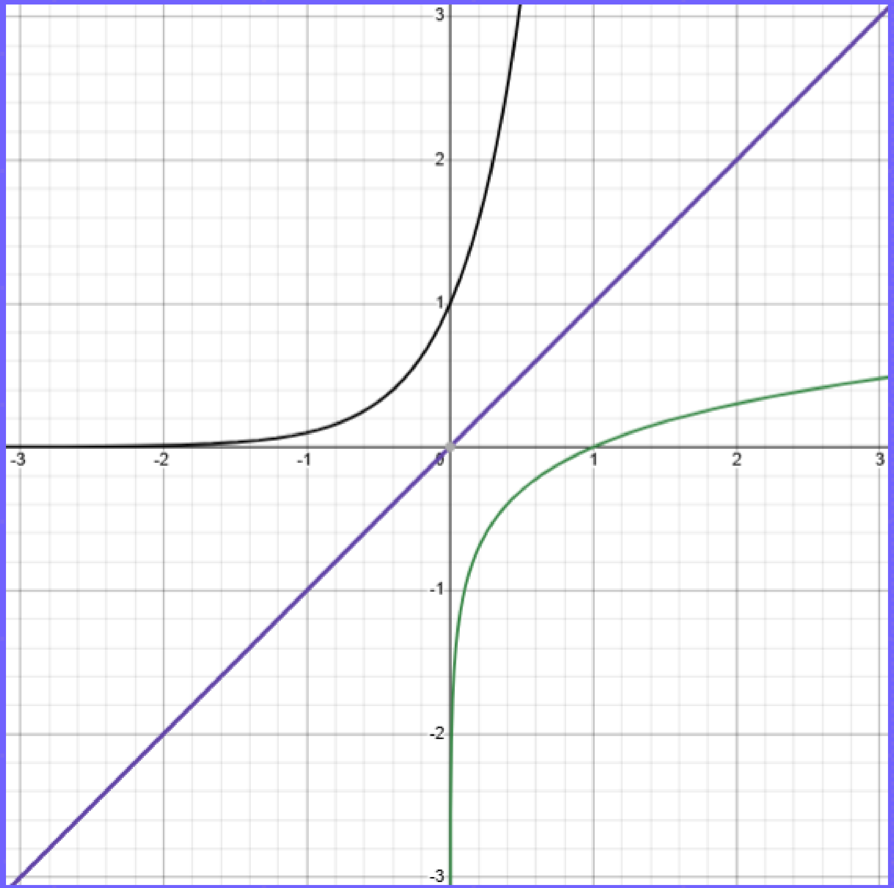


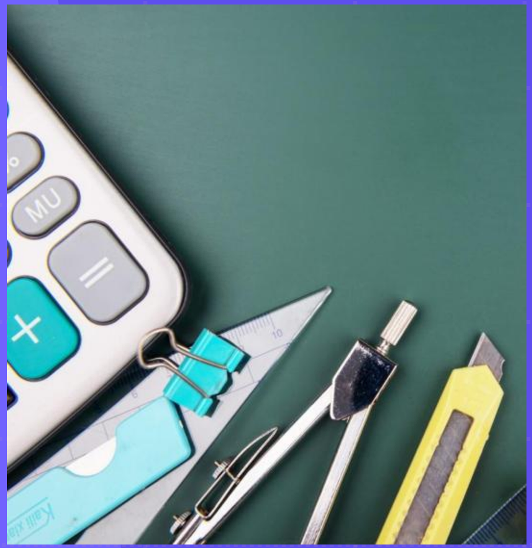
# Inverse relationship

The two relationships below are inverses of each other (that means they reflect over the line y = x, or if you swap the x and y value and rearrange).

$f(x) = a^x$

$\mathrm{f(x)} = \log_a x$

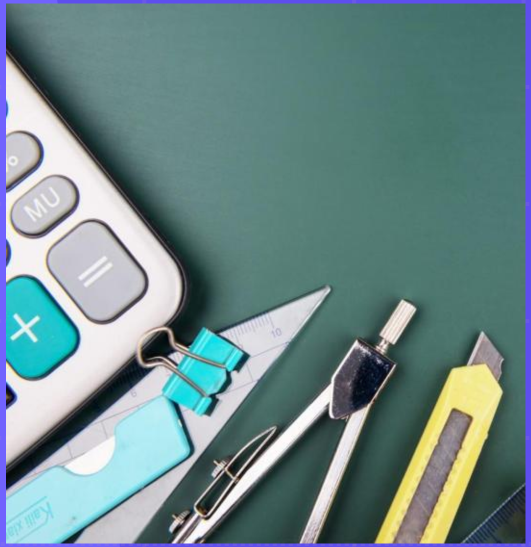


# Key points of $f(x) = \log_a x$

For the function $f(x) = \log_a x$

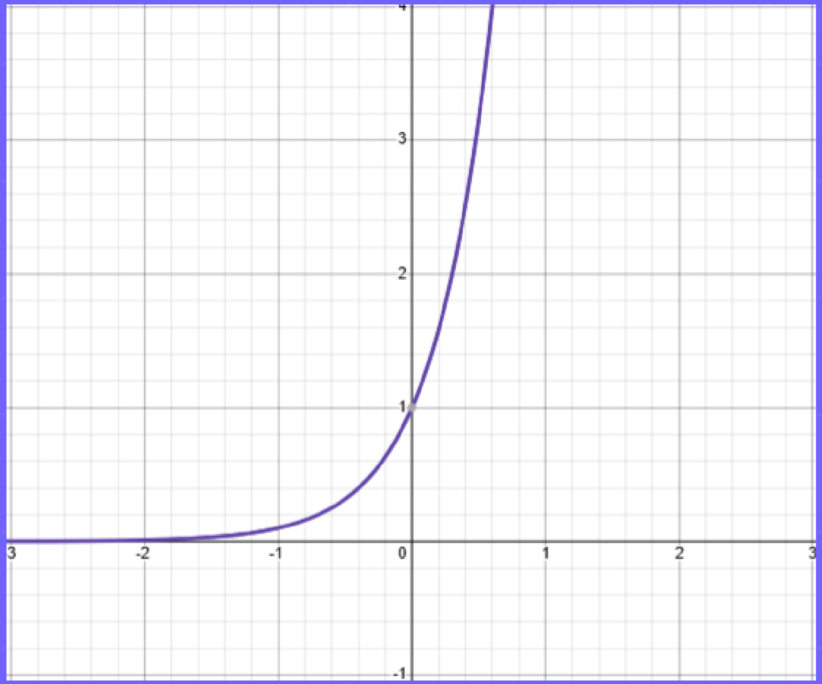
- The x-intercept is at (1,0)
- A second point is at (a,1)
- The y-axis is an asymptote (the graph approaches, but never touches)
- $f(x) = \log_a x + b$ is a vertical translation up b units
- $f(x) = \log_a(x - c)$ is a horizontal translation c units to the right

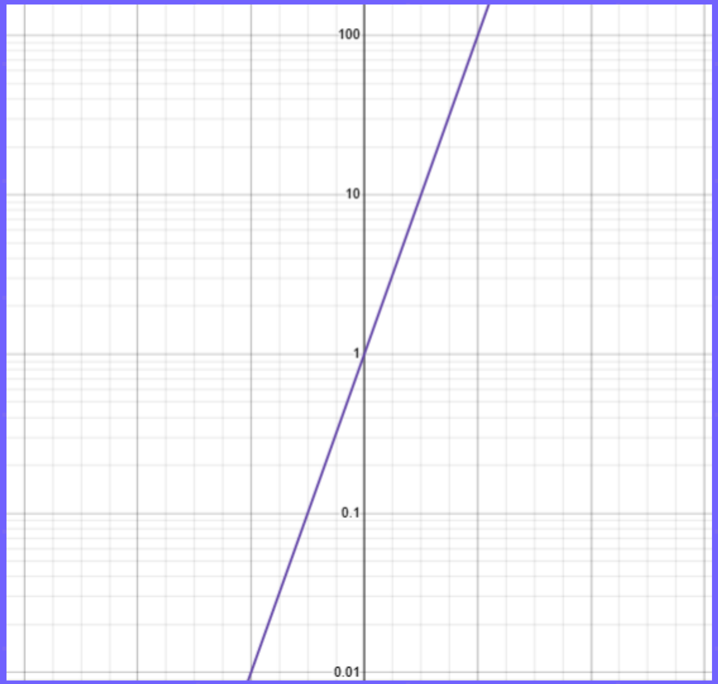


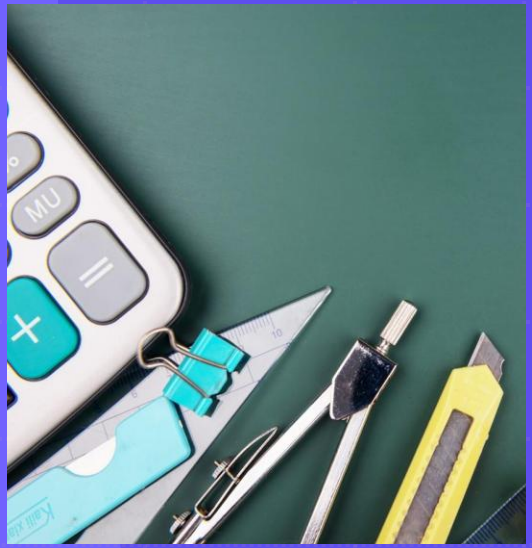
# Log – linear graphs

When graphing a logarithmic relationship such that the x axis is a linear scale and the y axis is a logarithmic scale, the graph will appear as a straight line.

Most used with sound (dB), earthquakes (Richter scale),



# Differentiating logarithmic functions

With the base ‘e’ to differentiate a logarithmic function, we must rearrange to make x the subject of the equation first. If the base is not ‘e’ then we can use change of base formula to convert. (Remember ln is just natural log or the log of base e)

Let $\quad \mathrm{y} = \ln x$

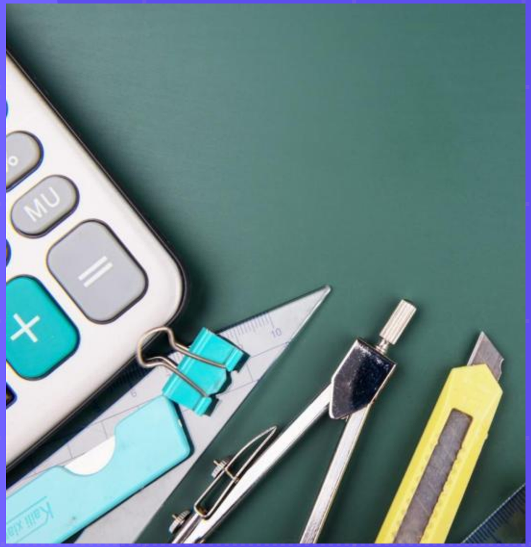
Then $\quad \mathrm{x} = e^{y}$

$$\frac{\mathrm{d}x}{\mathrm{dy}} = e^{y}$$

$$\frac{\mathrm{d}y}{\mathrm{dx}} = \frac{1}{e^{y}}$$

$$\frac{\mathrm{dy}}{\mathrm{dx}} = \frac{1}{x}$$

| |
|---|
| $\frac{d}{dx} \ln x = \frac{1}{x}$ |
| $\frac{d}{dx} \ln f(x) = \frac{f'(x)}{f(x)}$ |



# Integrating logarithmic functions

If $\quad y = \ln x$

Then $\quad \frac{\mathrm{dy}}{\mathrm{dx}} = \frac{1}{x}$

Or $\quad y' = \frac{1}{\mathrm{x}}$

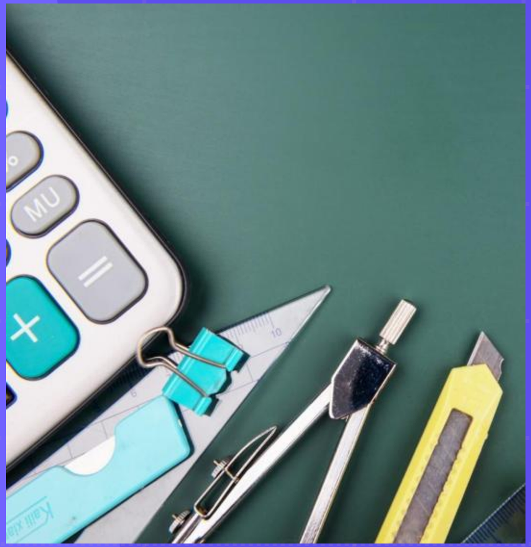
$$\int \frac{1}{x}\,dx = \ln x + c, \quad x > 0$$

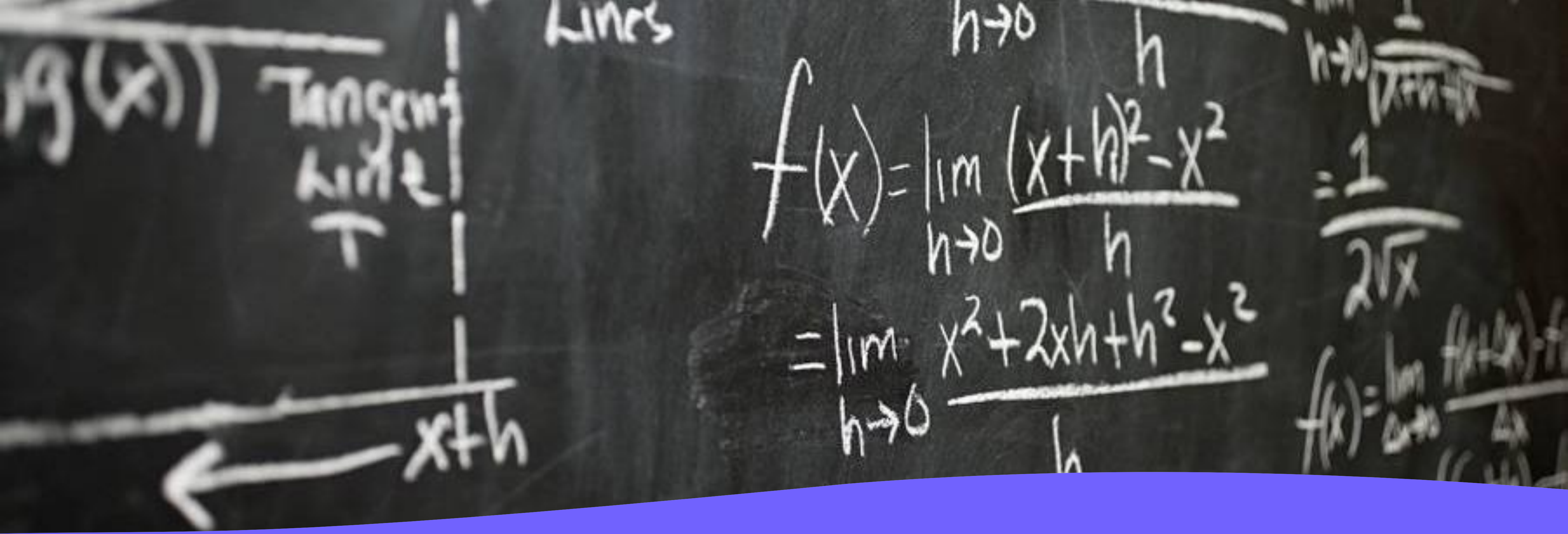
$$\int \frac{f'(x)}{f(x)}\,dx = \ln f(x) + c, \quad f(x) > 0$$

Therefore, if $\quad y' = \frac{1}{\mathrm{x}}$

Then $\quad y = \int \frac{1}{x}\,dx$

And hence $\quad y = \ln x + c$






# Questions?

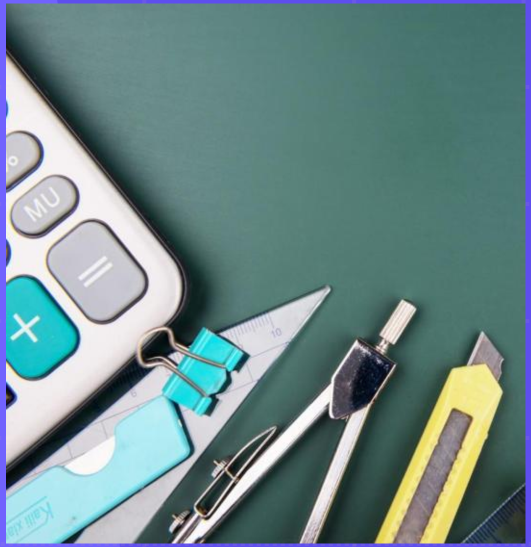
# WACE 2022 Question 1 (NC)

**Question 1** **(9 marks)**

Consider the derivative function $f'(x) = \dfrac{4x}{x^2 + 3}$.

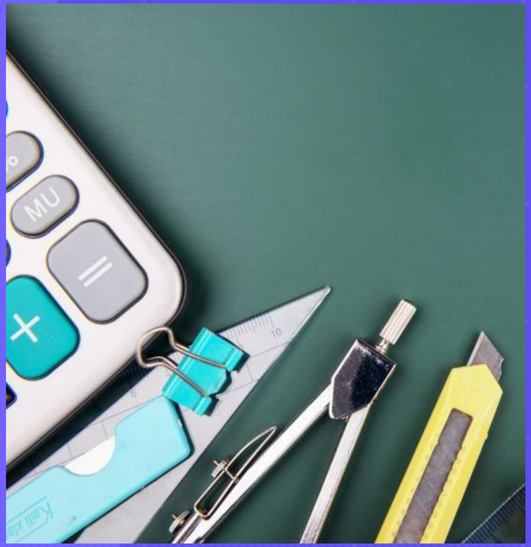
(a) Determine the rate of change of $f'(x)$ when $x = 1$. (3 marks)

(b) Determine $f(x)$ given that $f(1) = \ln(32)$. (4 marks)



# WACE 2022 Question 1 (NC)

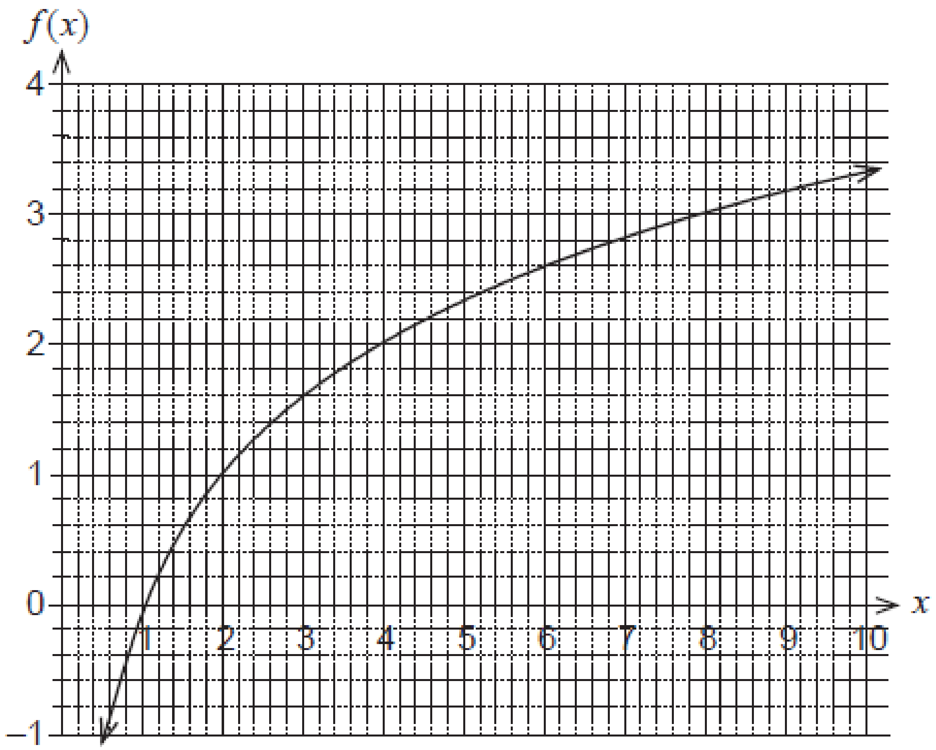
(c) Determine $\dfrac{d}{dt}\displaystyle\int_t^3 f(x)\,dx$. (2 marks)

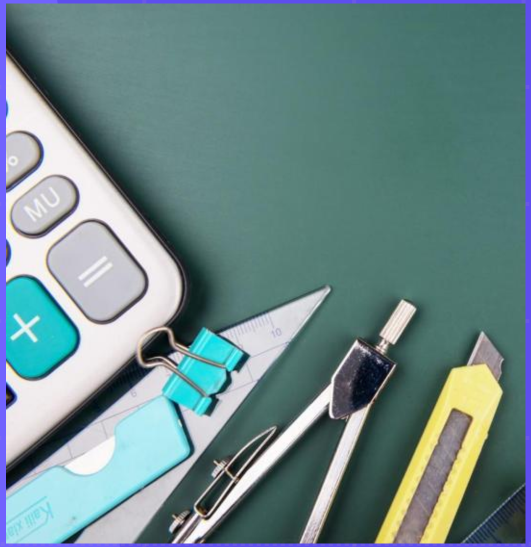


# WACE 2022 Question 4 (NC)

**Question 4** **(12 marks)**

The graph of the function $f(x) = \log_2(x)$ is shown below.



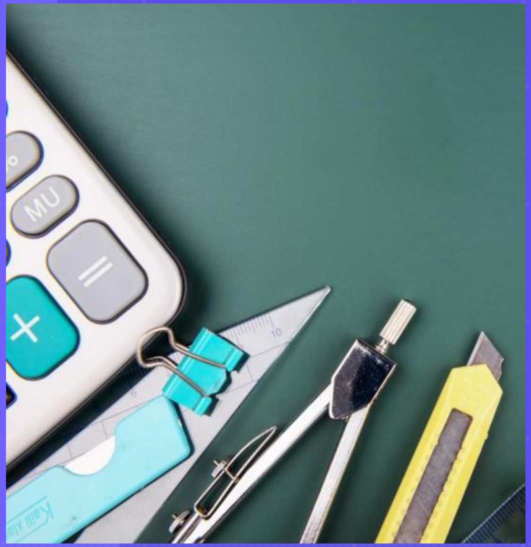



# WACE 2022 Question 4 (NC)

(a) Using the graph:

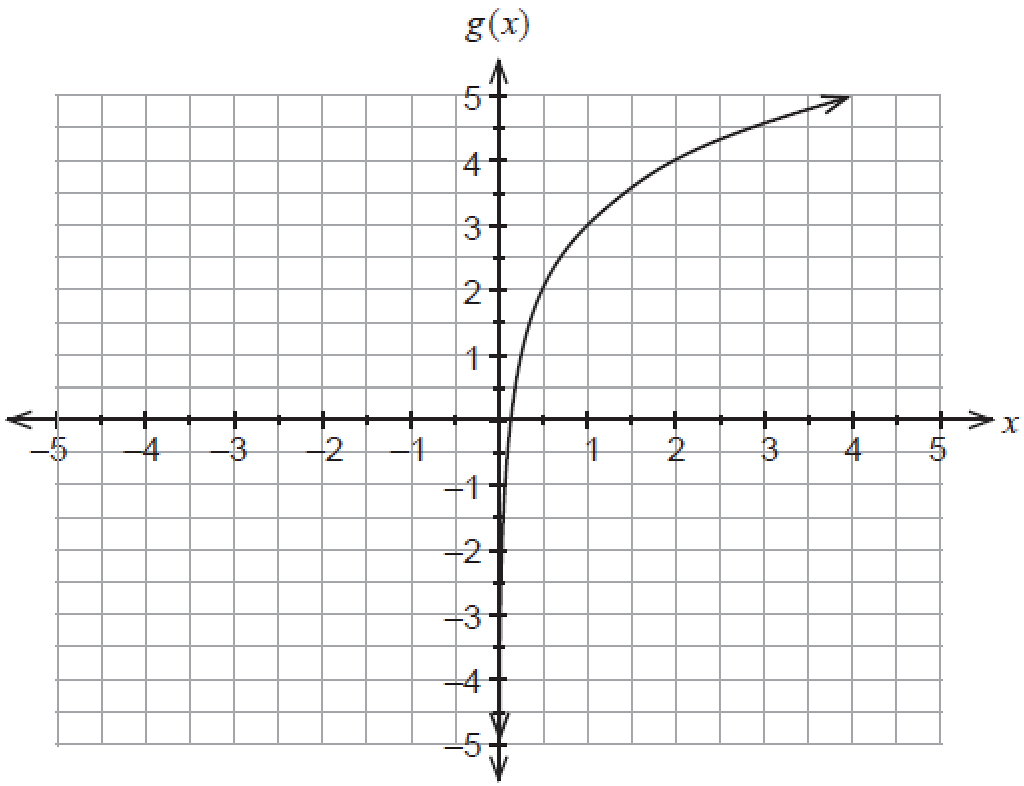
(i) solve $\log_2(x-5)=3$. (2 marks)

(ii) determine $\sqrt{7}$, correct to one decimal place. (Hint: let $x=\sqrt{7}$.) (3 marks)

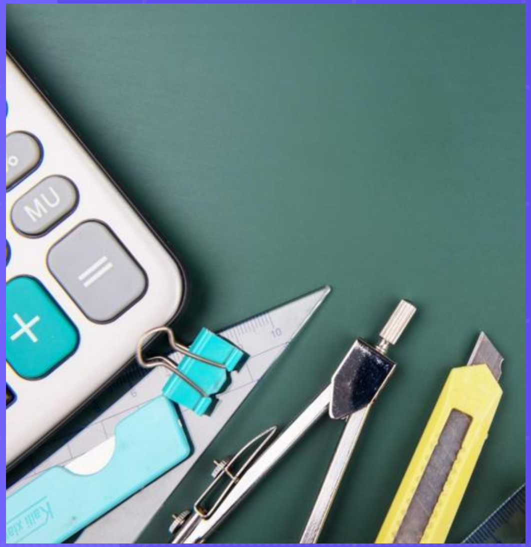


# WACE 2022 Question 4 (NC)

(b) The function $f(x) = \log_2(x)$ is translated to give the new function $g(x)$, which is shown in the graph below.



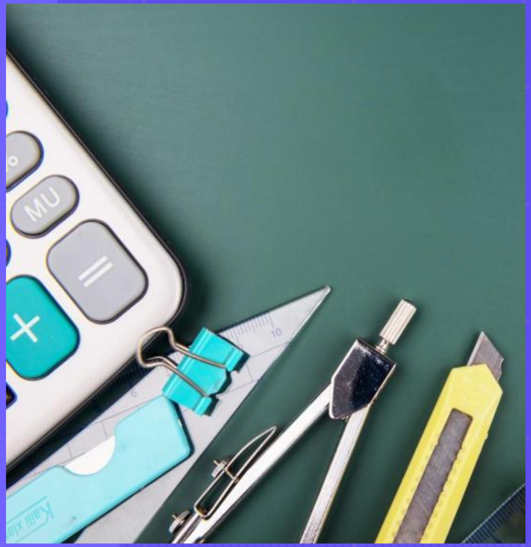

Determine the equation for $g(x)$. (2 marks)



# WACE 2022 Question 4 (NC)

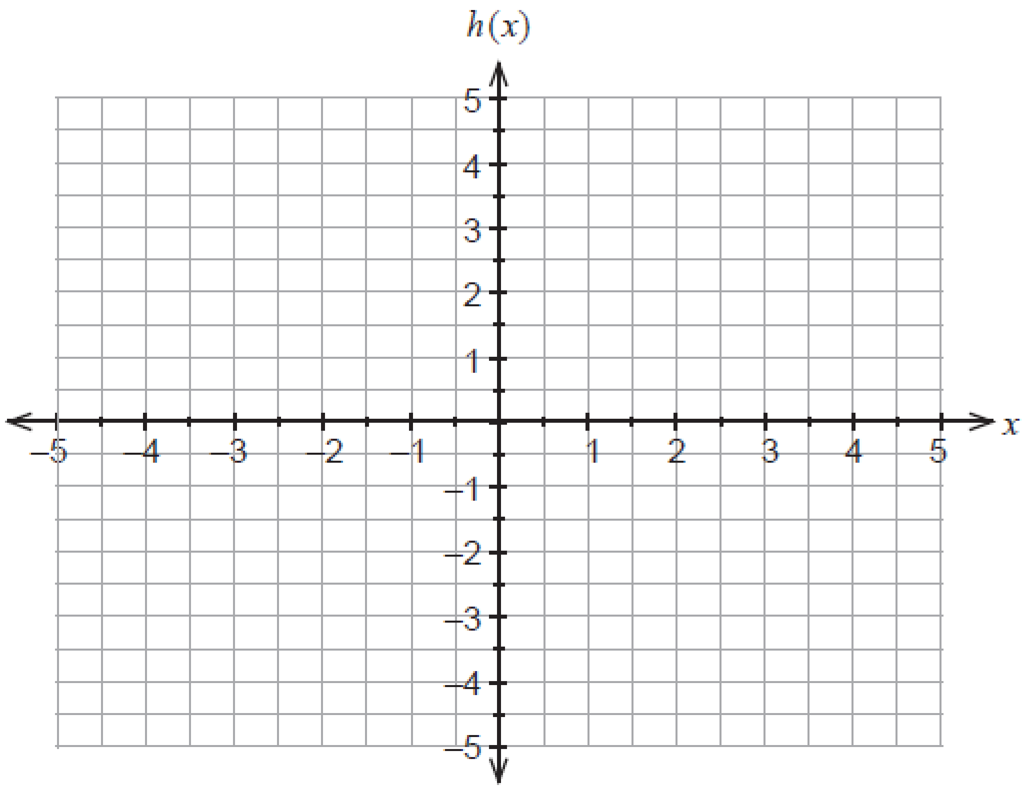
**Question 4** (continued)

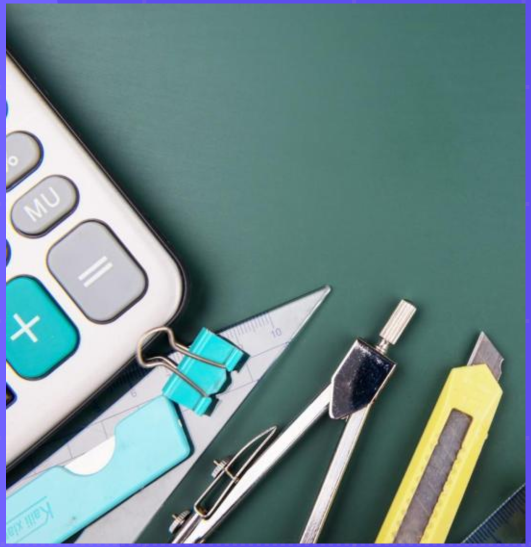
(c) (i) Show that $\log_2\left(\dfrac{1}{x-1}\right) = -\log_2(x-1)$. (2 marks)

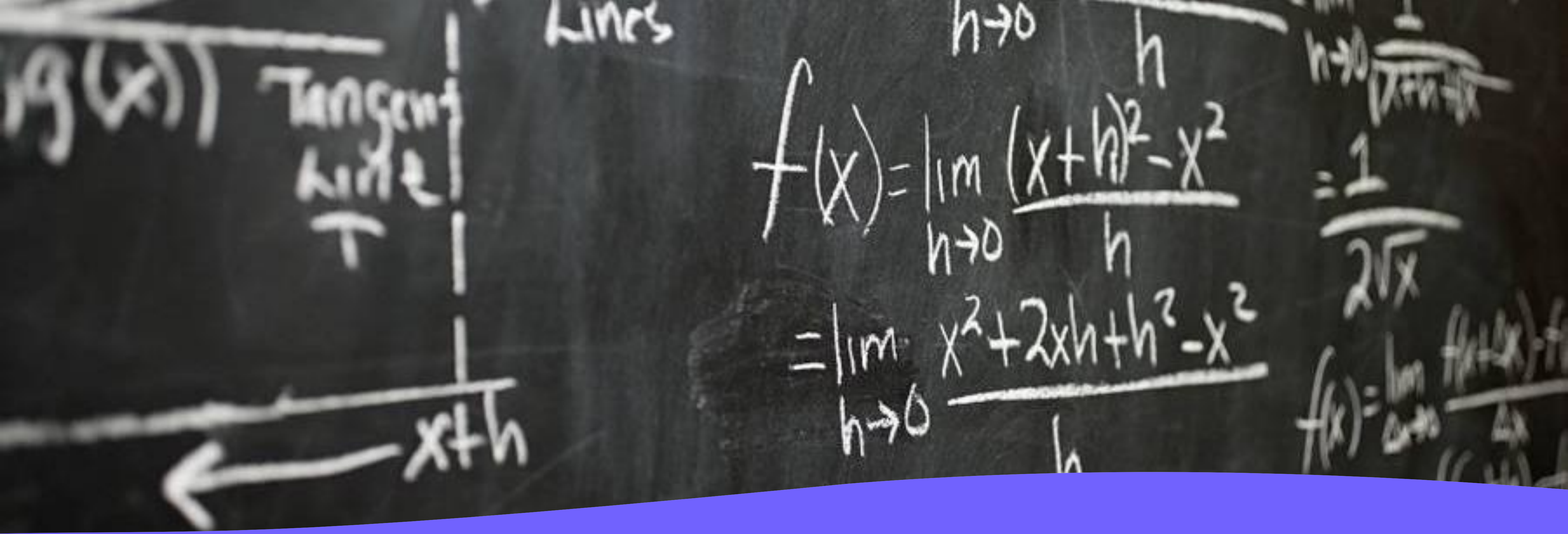


# WACE 2022 Question 4 (NC)

(ii) Hence sketch the graph of $h(x) = \log_2\left(\frac{1}{x-1}\right)$ on the axes below. (3 marks)






# Time to take a break!

10 minutes to stretch your legs, have a chat, use the restrooms!

# Unit 4 Topic 2

## CRV and ND

## 4.2.1 – 4.2.7

# Syllabus points

**General continuous random variables**

4.2.1 use relative frequencies and histograms obtained from data to estimate probabilities associated with a continuous random variable

4.2.2 examine the concepts of a probability density function, cumulative distribution function, and probabilities associated with a continuous random variable given by integrals; examine simple types of continuous random variables and use them in appropriate contexts

4.2.3 define the expected value, variance and standard deviation of a continuous random variable and evaluate them using technology where required

4.2.4 examine the effects of linear changes of scale and origin on the mean and the standard deviation

# Syllabus points

**Normal distributions**

4.2.5 identify contexts, such as naturally occurring variation, that are suitable for modelling by normal random variables

4.2.6 identify features of the graph of the probability density function of the normal distribution with mean $\mu$ and standard deviation $\sigma$ and the use of the standard normal distribution

4.2.7 calculate probabilities and quantiles associated with a given normal distribution using technology, and use these to solve practical problems

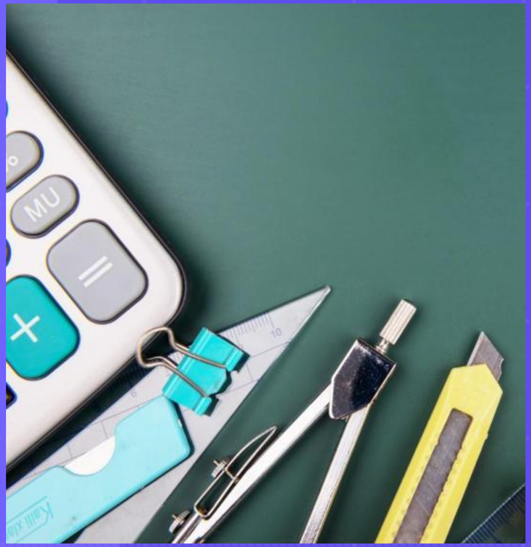
# Continuous Random Variables

A **continuous** random variable are a set of values that may take any value across a domain. (Contrast to a DRV where only selected values are allowed)

If $X$ is a continuous random variable, then for its probability density function $f(x)$ the following conditions must hold true:

- Within the defined domain, $f(x) \geq 0$
- $\int_a^b f(x)\,dx = 1$

| | |
|---|---|
| Continuous random variable: $P(a \le X \le b) = \int_a^b p(x)dx$ | |
| Expected value: $\mu = E(X) = \int_{-\infty}^{\infty} x\,p(x)dx$ | Variance: $\sigma^2 = \int_{-\infty}^{\infty} (x-\mu)^2 p(x)\,dx$ |



# Continuous Random Variables

The probability for a CRV is found by determining the area under the graph between the limits required.

$$P(a \le x \le b) = \int_a^b f(x)\,dx$$

The mean or the expected value is $\mu = E(X) = \Sigma(x \times f(x)) = \int_a^b x \times f(x)\,dx$

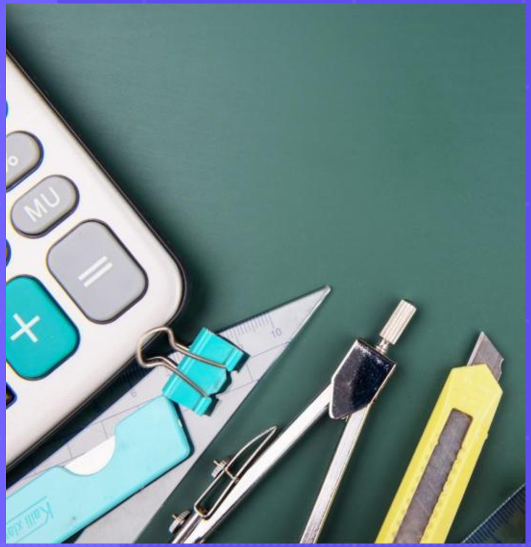
The variance is $\sigma^2 = Var(X) = \int_a^b (x-\mu)^2 \times f(x)\,dx$

$$= E(X^2) - [E(X)]^2$$

$$= \int_a^b x^2 \times f(x)\,dx - \mu^2$$

The standard deviation of a random variable $X$ is the square root of the variance

$SD(X) = \sigma = \sqrt{Var(X)}$



# Median of a CRV

Given that we saw

$$P(a \leq x \leq b) = \int_a^b f(x)\,dx$$

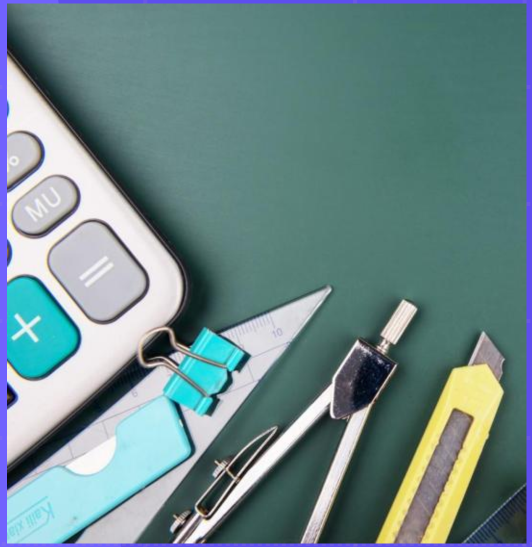
The median of a CRV is where the probability is equal to 0.5.

Hence $P(a \leq x \leq b) = 0.5$

$$P(-\infty \leq x \leq c) = \int_{-\infty}^{c} f(x)\,dx = 0.5$$

or

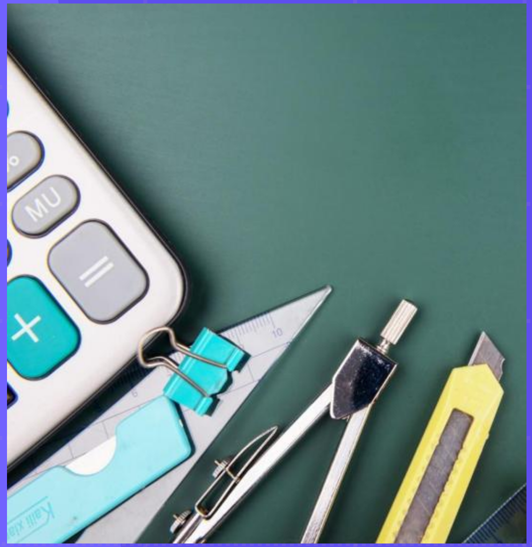
$$P(c \leq x \leq \infty) = \int_{c}^{\infty} f(x)\,dx = 0.5$$



# Effect of linear changes

**A change in scale** – stretches the data. Eg multiplying all data by a, stretches the range by a sf of a, the expected value by a sf of a and the variance by $a^2$, and the standard deviation by the absolute value of a.

**A change in origin** moves the data along the number line but makes no difference to the spacing of the data.  Eg adding b to all data increases the maximum and minimum by b, increases the expected value by b, but makes no difference to the variance or standard deviation

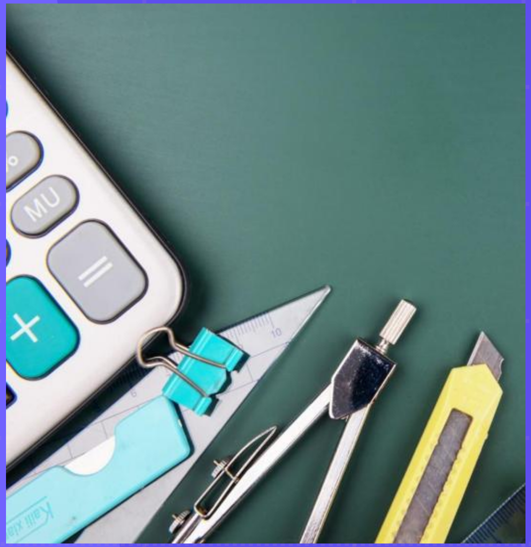


# Effect of linear changes

$$E(aX + b) = aE(X) + b$$

$$Var(aX + b) = a^2Var(X)$$

$$SD(aX + b) = |a|SD(X)$$



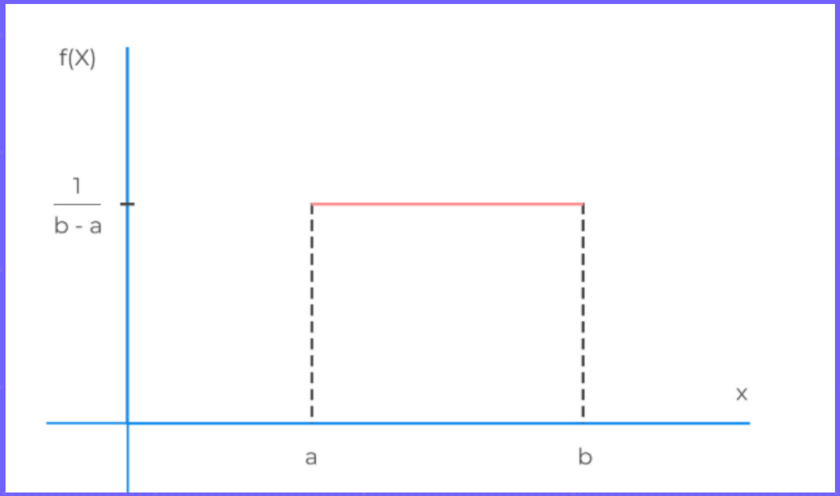
# Uniform distributions *

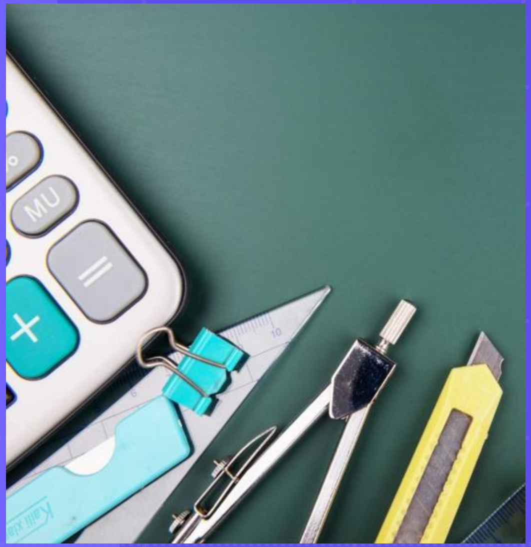
A uniformly distributed continuous random variable is when the random variable is equally likely to take any value in the interval $[a, b]$.

The distribution is rectangular in shape with the height of the rectangle equal to the probability and the length of the rectangle equal to b – a.

The pdf is defined as $f(x) = \begin{cases} \frac{1}{b-a}, & a \leq x \leq b \\ 0, & \text{elsewhere} \end{cases}$

The expected value is $E(X) = \frac{a+b}{2}$



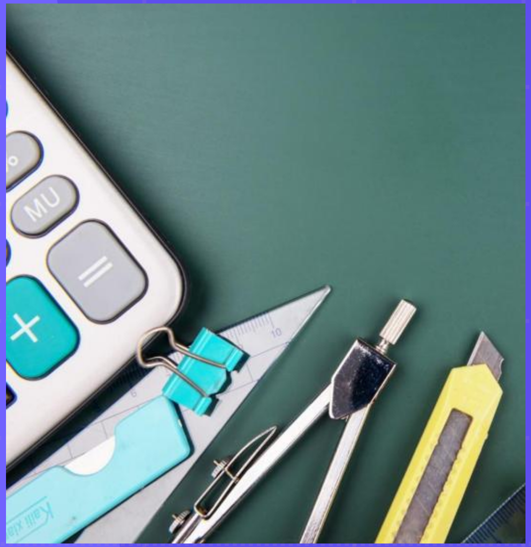



# Normal Distributions

A normal distribution or Gaussian distribution is a continuous probability distribution that is symmetric about its mean. In this distribution, values close to the mean occur more frequently than values farther away from the mean.

Key facts of a Normal Distribution is

- The area under the curve of a normal distribution is always equal to 1
- Values further away from the mean are less likely
- Values can potentially extend to both $\infty$ and $-\infty$.



# Normal Distributions

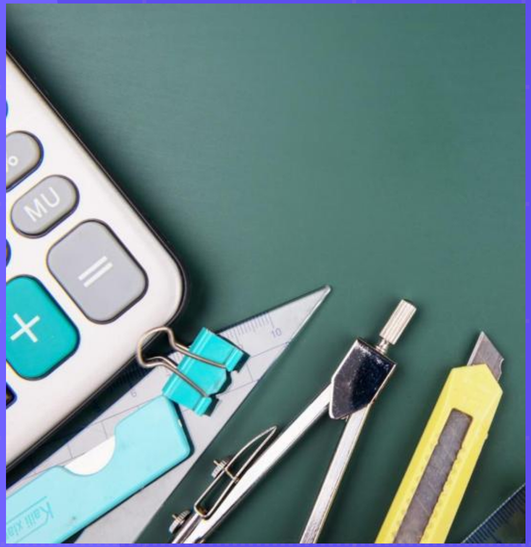
Symmetry: The normal distribution is symmetric around its mean. That is the left side of the distribution mirrors the right side.

Mean, Median, and Mode: In a normal distribution, the mean, median, and mode are all equal and located at the center of the distribution.

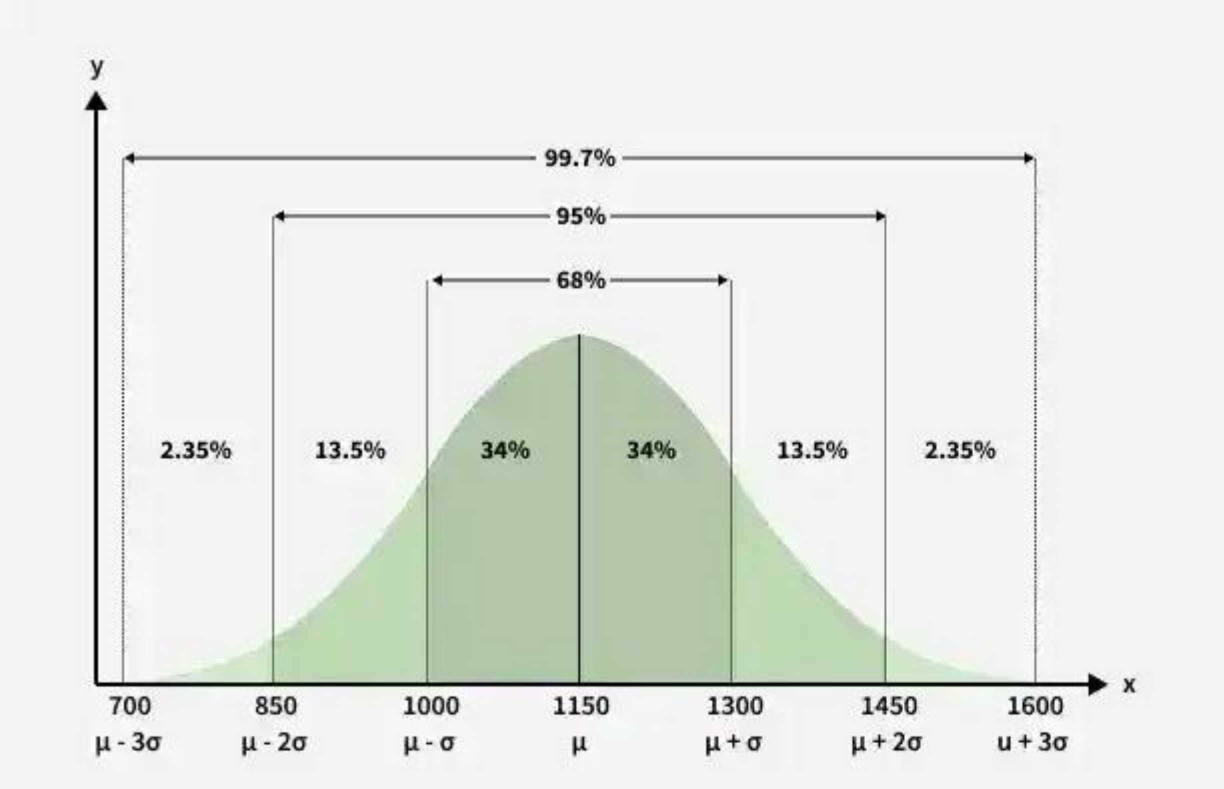
Bell-shaped Curve: The curve is bell-shaped, indicating that most of the observations cluster around the central peak and the probabilities for values further away from the mean taper off equally in both directions.

Standard Deviation: The spread of the distribution is determined by the standard deviation. About 68% of the data falls within one standard deviation of the mean, 95% within two standard deviations, and 99.7% within three standard deviations.

Asymptotic tails: The curve never touches the x-axis but extends infinitely.



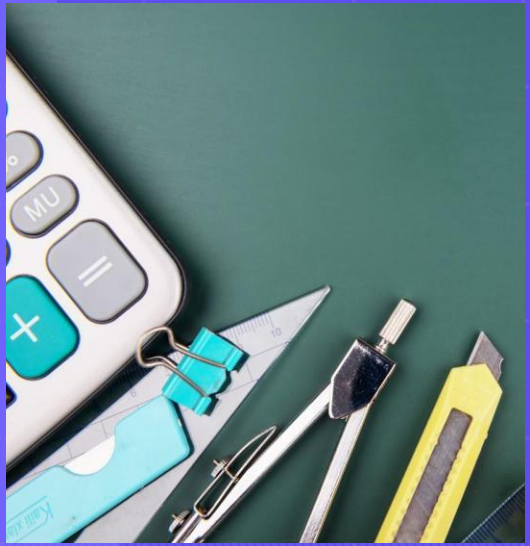
# Normal Distributions




$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

*where:*

- $x$ = *value of the random variable*
- $\mu$ = *mean of the distribution*
- $\sigma$ = *standard deviation*
- $\pi \approx 3.14159$
- $e \approx 2.71828$



# Parameters of Normal Distributions

Mean (μ) - the mean represents the center of the distribution and determines the location of the peak.

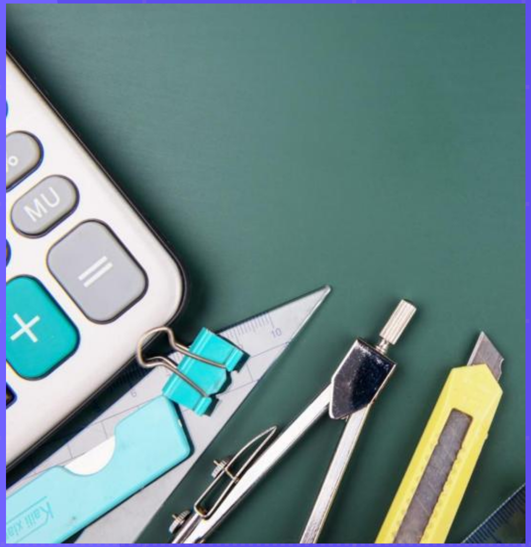
Increasing μ shifts the curve to the right.

Decreasing μ shifts the curve to the left.

Standard Deviation (σ) - the standard deviation determines how spread out the data is around the mean.

Smaller σ produces a narrower and taller curve.

Larger σ produces a wider and flatter curve.



# Empirical rule of Standard Deviations

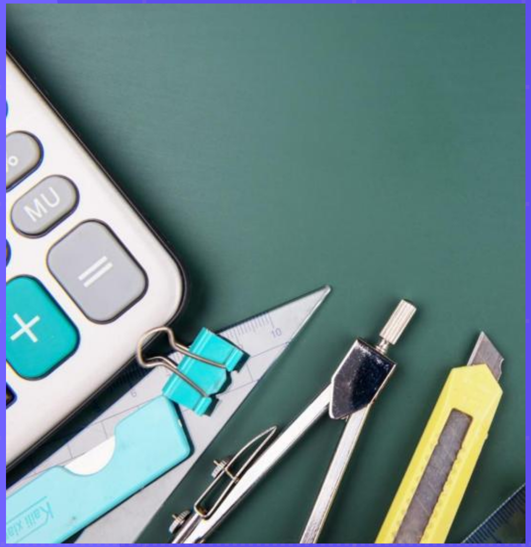
Generally, the normal distribution has a **positive standard deviation**, and the standard deviation divides the area of the normal curve into smaller parts, and each part defines the percentage of data that falls into a specific region. This is called the Empirical Rule of Standard Deviation in Normal Distribution.

Approximately 68% of data lies within 1 standard deviation of the mean.

Approximately 95% of data lies within 2 standard deviations of the mean.

Approximately 99.7% of data lies within 3 standard deviations of the mean.

This rule is known as the Empirical Rule or 68–95–99.7 Rule.

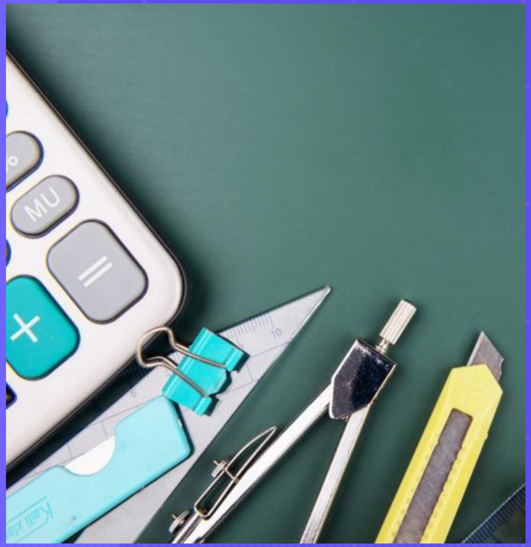


# Normal distributions

A normally distributed continuous random variable is usually express as

$$X \sim N(\mu, \sigma^2)$$

which is read as $X$ is normally distributed with a mean of $\mu$ and a standard deviation of $\sigma$ (or variance of $\sigma^2$).



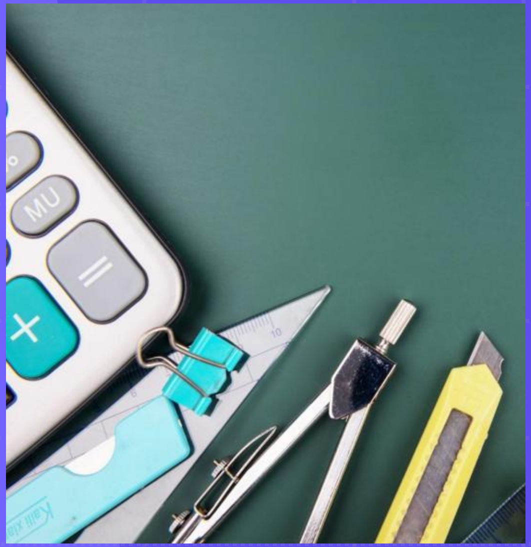
# The Standard Normal Distributions

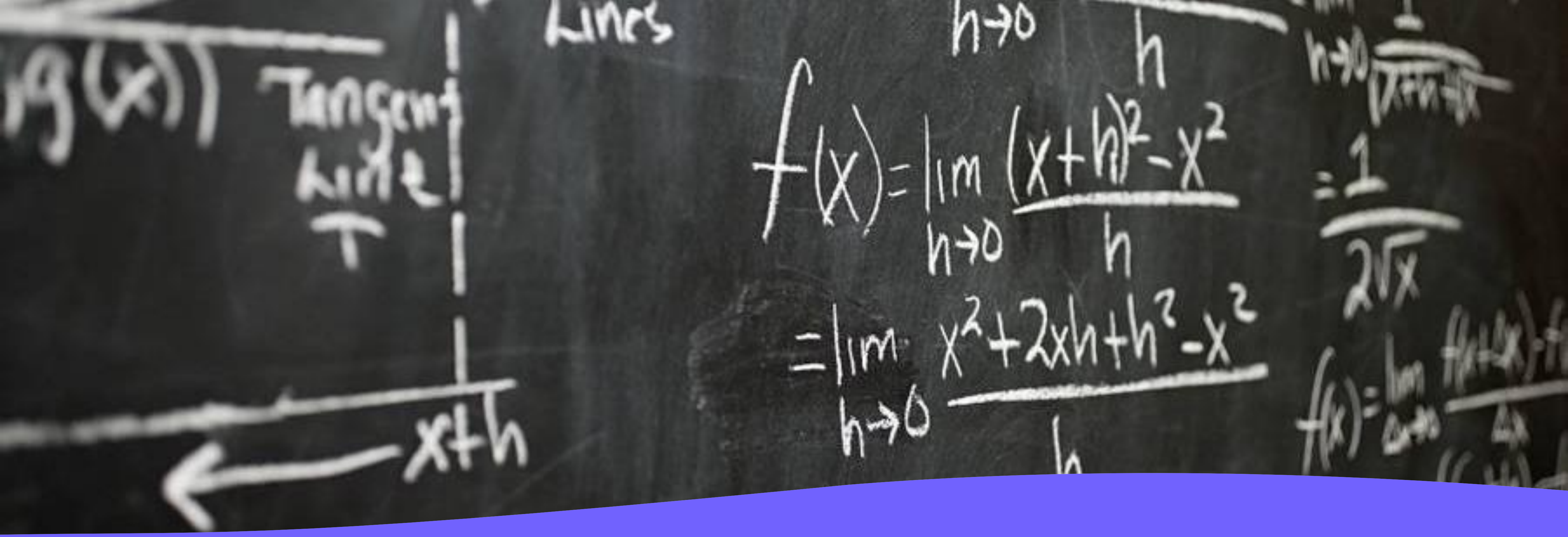
The standard normal variable $Z$ has a mean of 0 and a standard deviation of 1 and is calculated with $z = \frac{x-\mu}{\sigma}$

The value of $z$ is the standardised score which indicates the number of standard deviations ($\sigma$) the value $x$ is away from the mean ($\mu$).

If the standardised score is known, the position of the score in the Normal Distribution can be understood, without knowing the original data.

Eg. If you achieved a z score of 1.5 on your last test you outscored about 70% of students, if you achieved a  z score of 0, 50% of students beat you, and 50% of students did not.






# Questions?

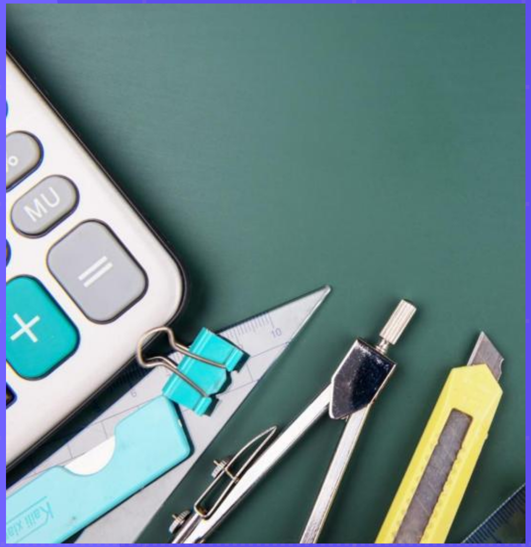
# WACE 2022 Question 6 (NC)

**Question 6** **(11 marks)**

The table of values below may be used to assist you in answering part (b) of this question.

| | | | | |
|---|---|---|---|---|
| $\sin(0) = 0$ | $\sin\left(\frac{\pi}{6}\right) = \frac{1}{2}$ | $\sin\left(\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2}$ | $\sin\left(\frac{\pi}{3}\right) = \frac{\sqrt{3}}{2}$ | $\sin\left(\frac{\pi}{2}\right) = 1$ |
| $\cos(0) = 1$ | $\cos\left(\frac{\pi}{6}\right) = \frac{\sqrt{3}}{2}$ | $\cos\left(\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2}$ | $\cos\left(\frac{\pi}{3}\right) = \frac{1}{2}$ | $\cos\left(\frac{\pi}{2}\right) = 0$ |

(a) (i) Determine $\frac{d}{dx}\left(x \sin\left(\frac{\pi x}{4}\right)\right)$. (2 marks)

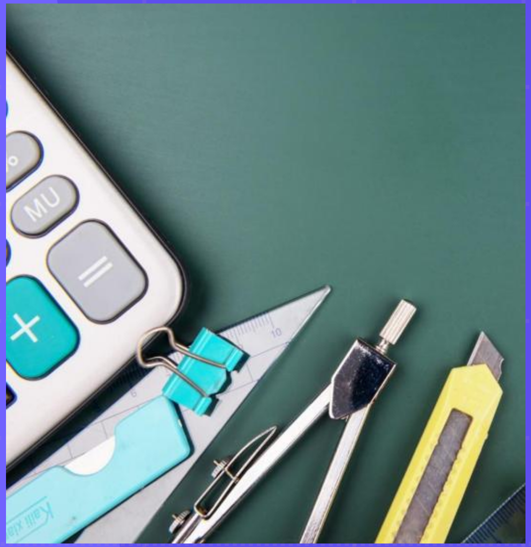


# WACE 2022 Question 6 (NC)

(ii) Hence show that

$$\int \frac{\pi x}{4} \cos\left(\frac{\pi x}{4}\right) dx = x \sin\left(\frac{\pi x}{4}\right) + \frac{4}{\pi} \cos\left(\frac{\pi x}{4}\right) + c$$

where c is a constant. (3 marks)

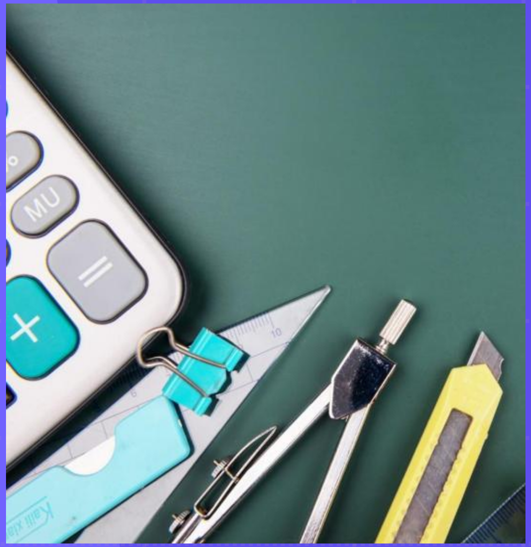


# WACE 2022 Question 6 (NC)

(b) The time in minutes, $T$, between incoming phone calls at a call centre is a random variable with probability density function

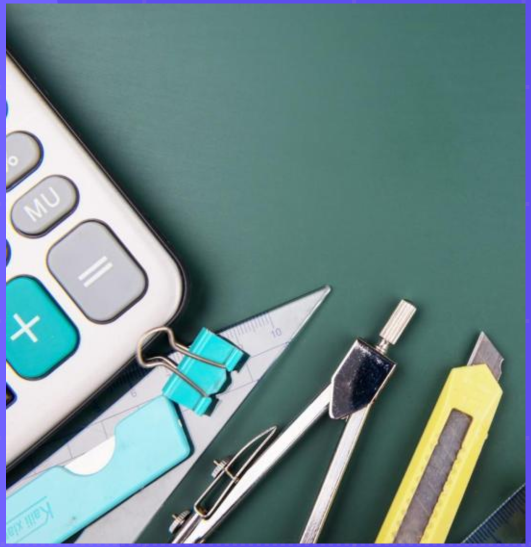
$$p(t) = \begin{cases} \dfrac{\pi}{4}\cos\left(\dfrac{\pi t}{4}\right), & 0 \leq t \leq 2 \\ 0, & \text{otherwise} \end{cases}$$

(i) Determine the probability that the time between two consecutive phone calls is less than 40 seconds. State your answer exactly. (3 marks)



# WACE 2022 Question 6 (NC)

(ii) Use the result from part (a)(ii) to determine the expected time between consecutive phone calls. (3 marks)



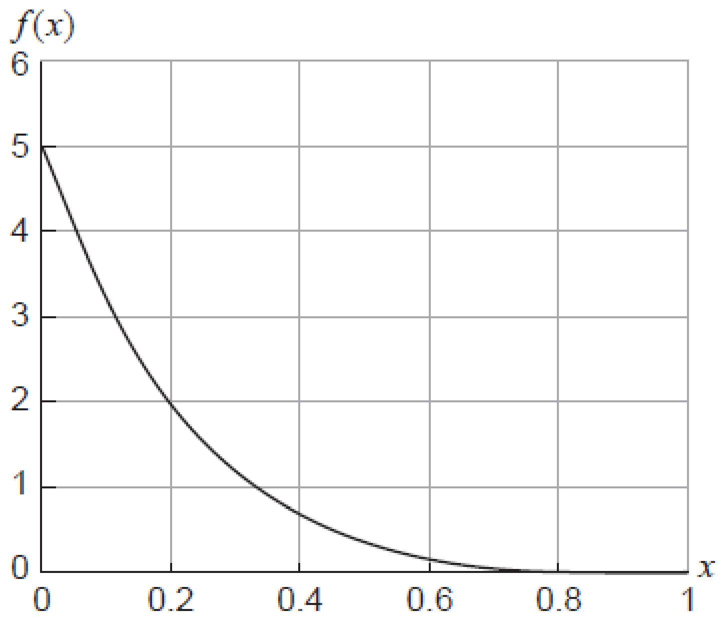
# WACE 2022 Question 8 (C)

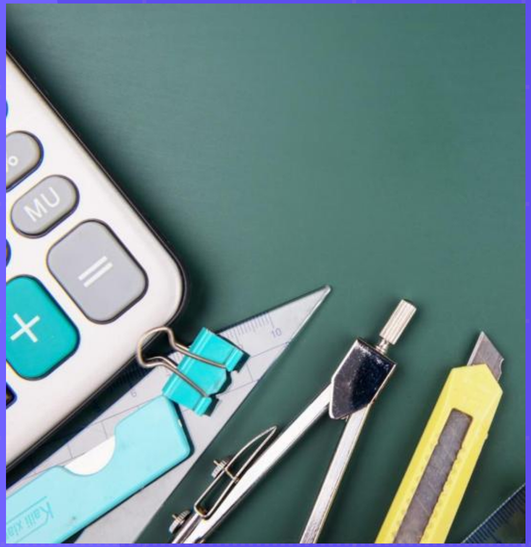
**Question 8** **(10 marks)**

A small outback petrol station receives a weekly delivery of petrol. The volume of petrol sold in a week, $X$, (in units of 10 000 litres) is a random variable with probability density function

$$f(x) = 5(1-x)^4, \quad 0 \leq x \leq 1$$

as shown in the graph below.

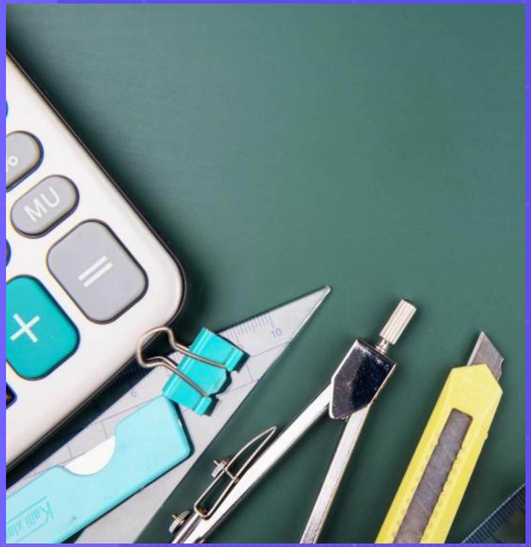


# WACE 2022 Question 8 (C)

(a) Determine, using appropriate units, the expected value and variance of the amount of fuel sold in a week. (4 marks)

(b) What storage tank capacity will ensure that there is only a 1% chance of running out of petrol in a given week? State your answer to the nearest litre. (3 marks)

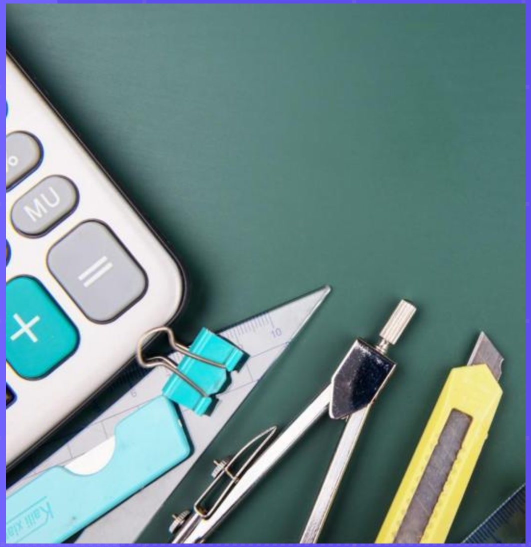


# WACE 2022 Question 8 (C)

(c) When the petrol is delivered, it is pumped into the storage tank. The rate of change of the petrol level in the tank, $h(t)$, (measured in metres) at time $t$ (measured in minutes) is given by

$$h'(t) = \frac{5}{2t+3}$$

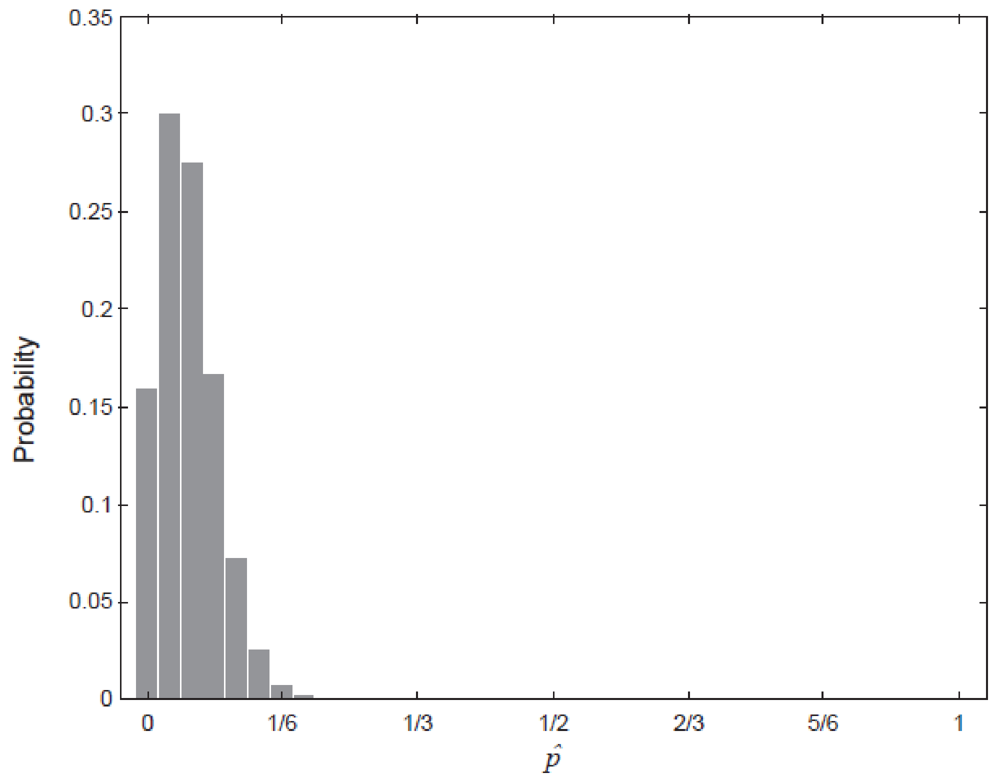
Determine the height of the storage tank if it takes 20 minutes to fill. (3 marks)

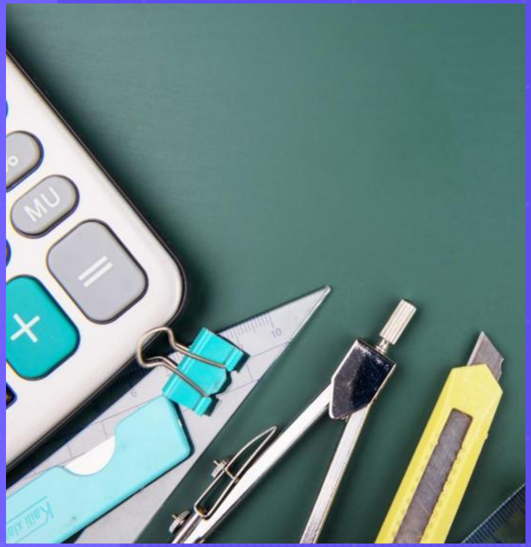


# WACE 2023 Question 10 (C)

**Question 10** **(7 marks)**

Fingerprints can be classified broadly as loop-shaped, whirl-shaped or arch-shaped. Only 5% of the world's population have arch-shaped fingerprints. Consider a random sample of 36 people and let $\hat{p}$ denote the sample proportion of people with arch-shaped fingerprints. The probability distribution for $\hat{p}$ is shown below.



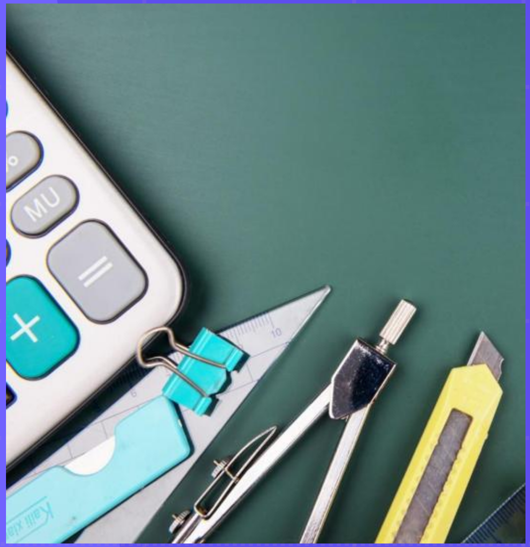
# WACE 2023 Question 10 (C)

(a) On the basis of the diagram above, is it appropriate to use the normal distribution to approximate the distribution of $\hat{p}$? Justify your answer. (2 marks)

A larger sample of 500 people is selected at random.

(b) Determine the probability that more than 30 people in the sample have arch-shaped fingerprints. (3 marks)

(c) Use the approximate normality of the distribution to determine the probability that the sample proportion of people with arch-shaped fingerprints is greater than 0.06. (2 marks)



# Unit 4 Topic 3

# Interval estimates for proportions

## 4.3.1 – 4.3.10

# Syllabus points

**Random sampling**

4.3.1 examine the concept of a random sample

4.3.2 discuss sources of bias in samples, and procedures to ensure randomness

4.3.3 use graphical displays of simulated data to investigate the variability of random samples from various types of distributions, including uniform, normal and Bernoulli

# Syllabus points

**Sample proportions**

4.3.4 examine the concept of the sample proportion $\hat{p}$ as a random variable whose value varies between samples, and the formulas for the mean $p$ and standard deviation $\sqrt{\frac{p(1-p)}{n}}$ of the sample proportion $\hat{p}$

4.3.5 examine the approximate normality of the distribution of $\hat{p}$ for large samples

4.3.6 simulate repeated random sampling, for a variety of values of $p$ and a range of sample sizes, to illustrate the distribution of $\hat{p}$ and the approximate standard normality of $\frac{\hat{p}-p}{\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}}$ where the closeness of the approximation depends on both $n$ and $p$

# Syllabus points

**Confidence intervals for proportions**

4.3.7 examine the concept of an interval estimate for a parameter associated with a random variable

4.3.8 use the approximate confidence interval $\left(\hat{p} - z\sqrt{\left(\frac{\hat{p}(1-\hat{p})}{n}\right)}, \hat{p} + z\sqrt{\left(\frac{\hat{p}(1-\hat{p})}{n}\right)}\right)$ as an interval estimate for $p$, where $z$ is the appropriate quantile for the standard normal distribution

4.3.9 define the approximate margin of error $E = z\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$ and understand the trade-off between margin of error and level of confidence

4.3.10 use simulation to illustrate variations in confidence intervals between samples and to show that most, but not all, confidence intervals contain $p$

# Sampling

Population – the entire group upon which data can be collected.  Usually, we'd be talking about the entire school, club, state, country etc. The population can be defined to suit the sampling needed.

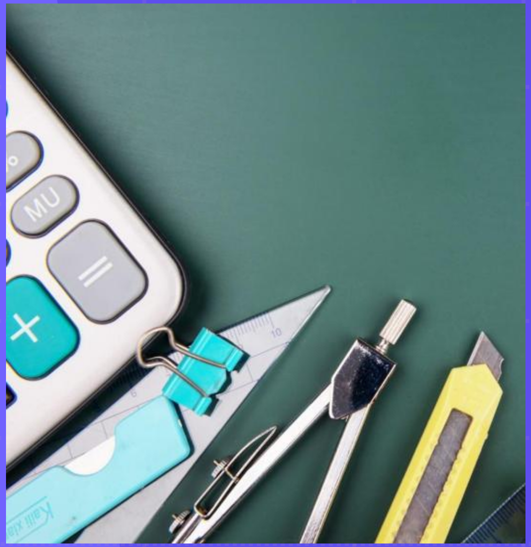
Census – To collect data from the entire population

Sample – To collect data from a subset of the population

Fair sampling – this is the million-dollar question!

A fair sample must represent the entire population and ensure that every data point in the population has an equal chance of being included.

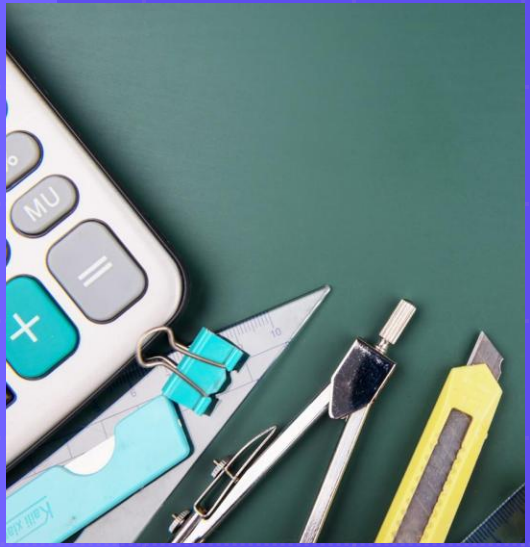
We often conduct random sampling to attempt to achieve equal chance



# Sampling

Types of sampling can include:

- Random sampling where each item has an equal chance of being selected using a random number generator.
- Stratified sampling where a random choice of representatives are chosen from a strata in proportion to the size of each group.
- Cluster sampling, where the population is separated into clusters and then a cluster or clusters are randomly chosen.
- Systematic sampling where a choice of representatives from a group are chosen by selecting every $k^{\text{th}}$ item.

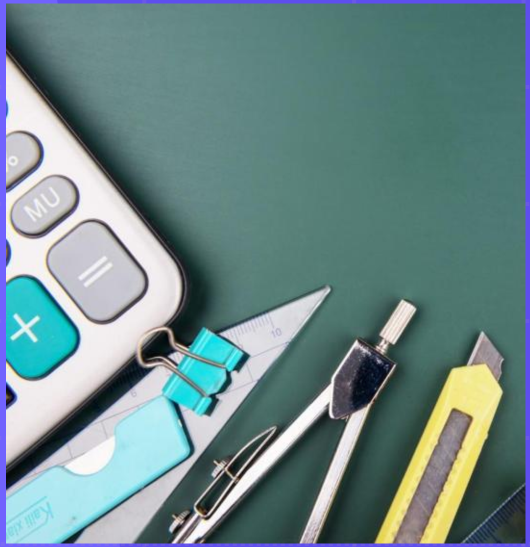


# Bias Sampling

If strong controls are not employed during sampling, bias can creep in.

Examples include

- Selection in a way that increases the chances of some of the population being selected more than others.
- Sample design bias where the structure of the sampling (questions, etc) offer bias towards or away from one group of the population.
- Offering the sample by way of self-selection.
- Accepting nonresponse where selected population do not offer a response.



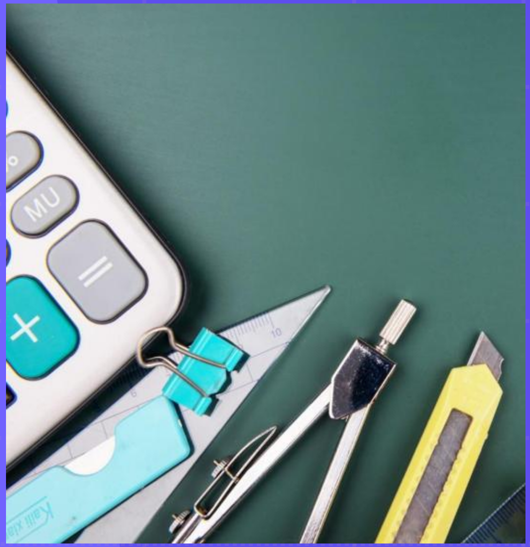
# Sample proportions

A sample proportion ($\hat{p}$) is the proportion of the sample that contains a certain characteristic.

$$\hat{p} = \frac{X}{n}$$

If I sample 10 people and 4 of them are male, the proportion of males is 0.4. I could do another sample immediately after the first and sample a different (or perhaps some the same) 12 people and 4 of those are male, giving a sample proportion of 0.33.

Both are completely fair, correct and reasonable estimates of the proportion in the wider population.

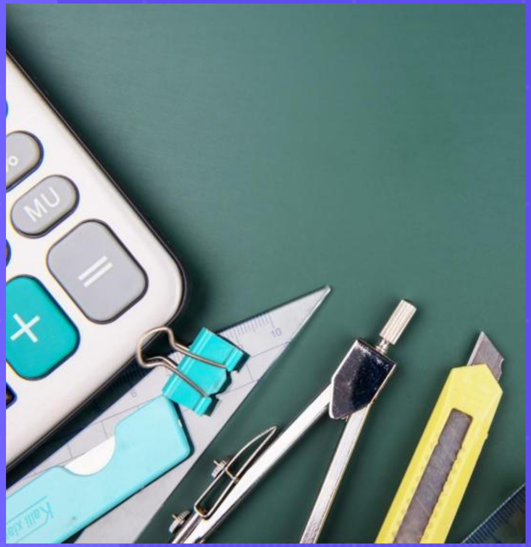


# Central Limit Theorem (CLT)

Central limit theorem states that as n (sample size) becomes large, the distribution of the sample proportions will approximate a normal distribution with a mean of $p$ and a standard deviation of $\sqrt{\frac{p(1-p)}{n}}$

| Mean: | Standard deviation: |
|---|---|
| $E(\hat{p}) = p$ | $\sigma = \sqrt{\frac{p(1-p)}{n}}$ |

That means the closer we get to sampling the entire population (as n→ population), the closer our sample proportion will get to the population proportion.

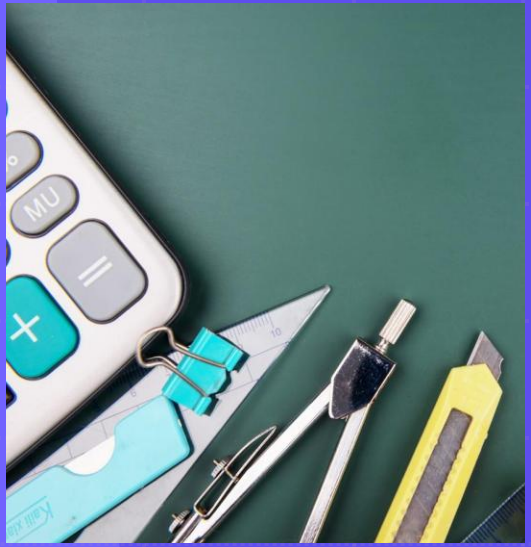


# Using Central Limit Theorem (CLT)

To approximate the population proportion using sample proportions we must adhere to the following constraints

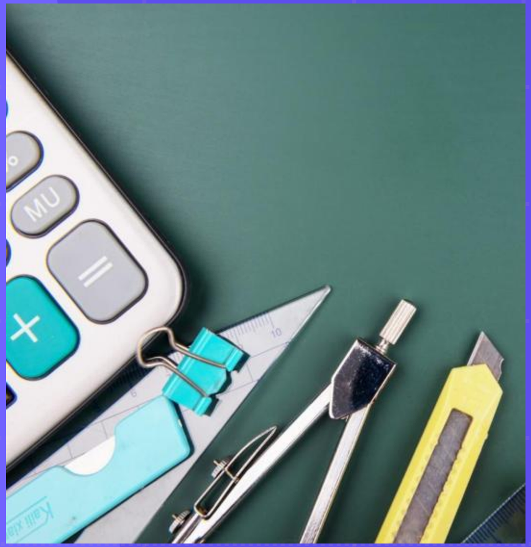
$np > 5$ and $n(1-p) > 5$ and $n > 30$

If we have a large or small proportion (below 0.2, above 0.8) we must be careful and take a greater sample (increase n) to ensure we meet the criteria above.



# Using Central Limit Theorem (CLT)

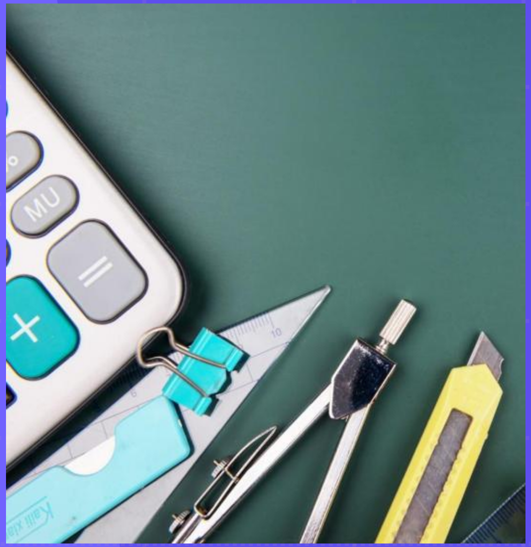
In the ideal case as $n \to \infty$ and $p \to 0.5$ the distribution of $\frac{\hat{p}-p}{\sqrt{\frac{p(1-p)}{n}}}$ approaches a standard normal distribution.



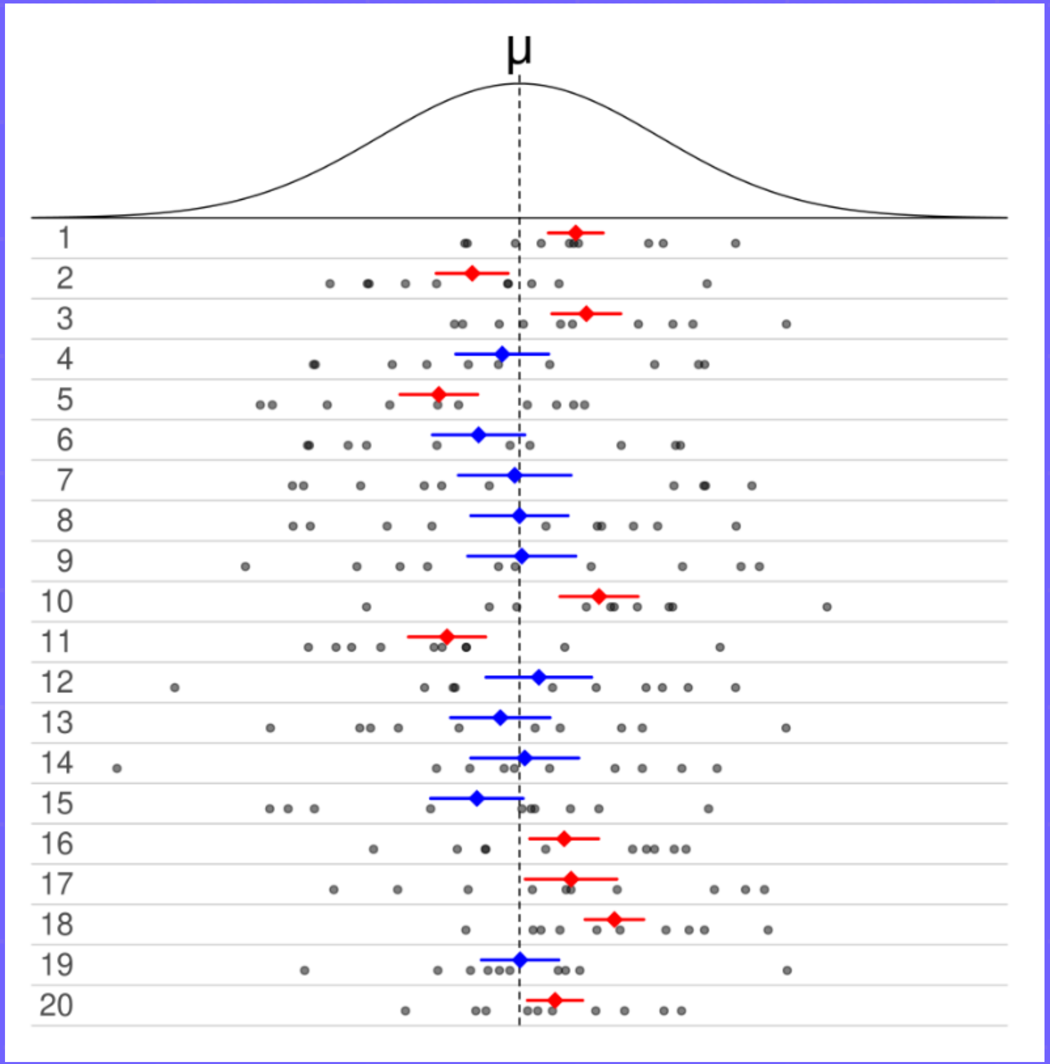
# Confidence intervals

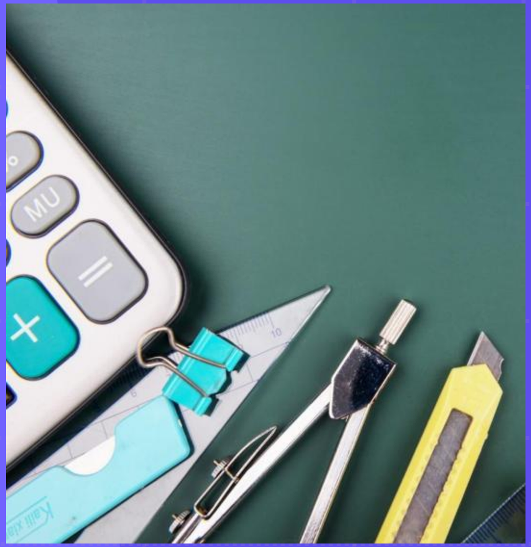
A **confidence interval** is a range of values which is likely to contain (in repeated sampling) the true value of an unknown statistical parameter, such as a population mean.

The key here is we have an unknown but fixed parameter, usually the mean or proportion.  It is unknown as the population is extremely large and sampling the entire population is very difficult.  Hence, we estimate it, but within a range.



# Confidence intervals



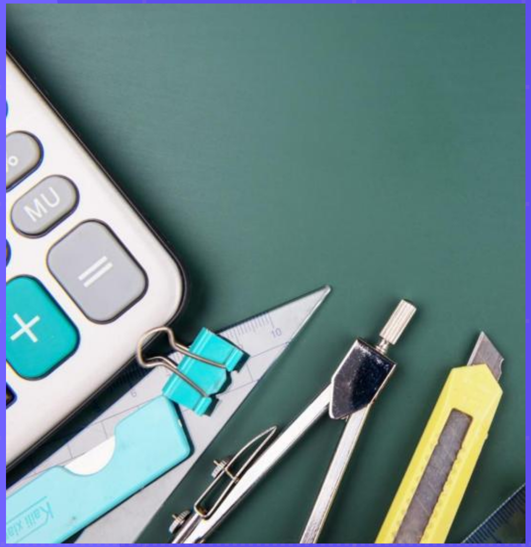



# Interpreting Confidence intervals

A 95% confidence level does not imply a 95% probability that the true parameter lies within a particular calculated interval.

The confidence level instead reflects the long-run reliability of the method used to generate the interval. In other words, if the same sampling procedure were repeated 100 times from the same population, approximately 95 of the resulting intervals would be expected to contain the true population mean.

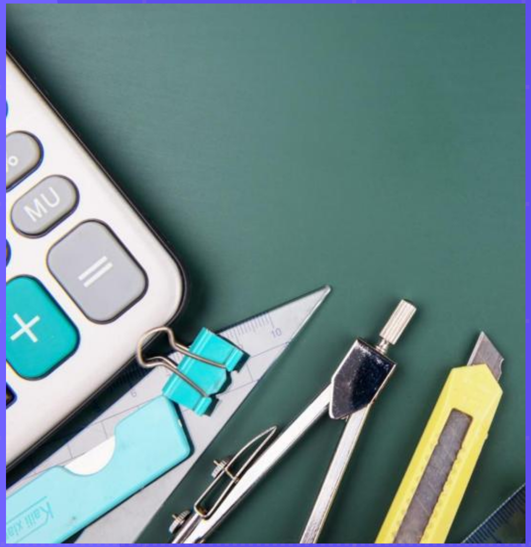
We see the true population mean as a fixed unknown constant, while the confidence interval is calculated using data from a random sample. Because the sample is random, the interval endpoints are random variables.



# Interpreting Confidence intervals

Contrary to common misconceptions, a 95% confidence level **does *not* mean** that:

- for a given confidence interval there is a 95% probability that the population parameter lies within the interval.
- 95% of the sample data lie within the confidence interval.
- there is a 95% probability of the mean from a repeat of the sample falling within the confidence interval computed from a given sample.

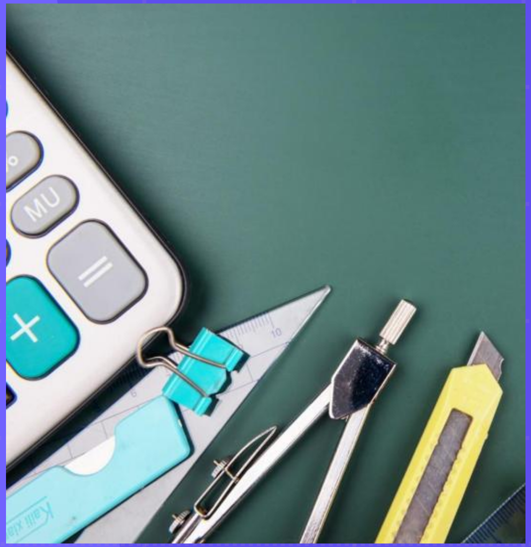


# Interpreting Confidence intervals

Actually……..

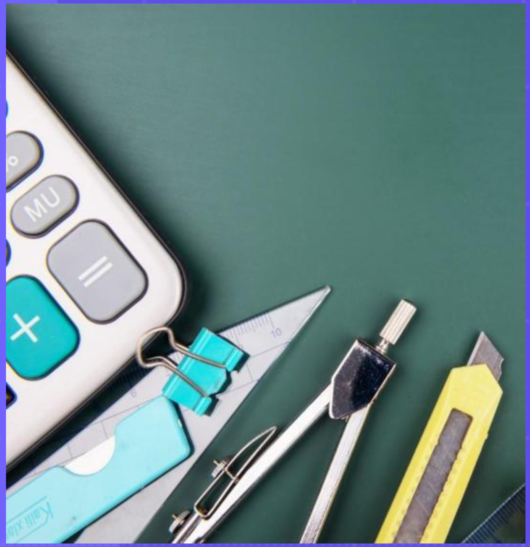
“A 95% confidence interval means that if we were to rerun the sampling 100 times, we would expect 95 of these to contain the population proportion”

We must remember the sample proportion is a continuous random variable and so we can not be sure that 95 out of 100 will contain the population proportion, but only that we expect 95 of 100 to contain the population proportion.



# Example of Confidence intervals

For example, suppose a factory produces metal rods, and a random sample of 25 rods gives a 95% confidence interval of 36.8 to 39.0 mm for the population mean length.

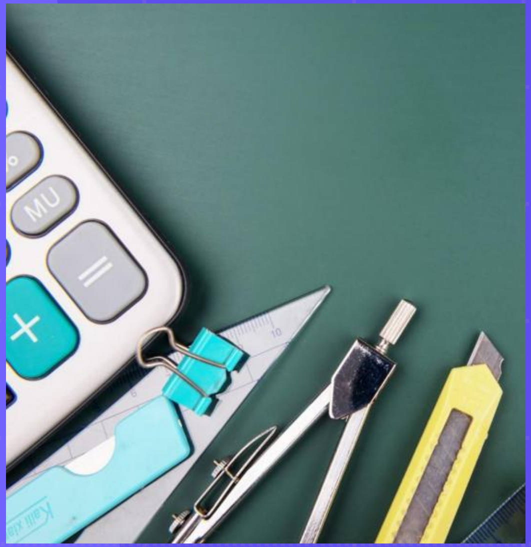


# Example of Confidence intervals

- It is incorrect to say that there is a 95% probability that the true population mean lies within this interval: the true mean is fixed, not random. The true mean could be 37 mm, which is within the confidence interval, or 40 mm, which is not; in any case, whether it falls between 36.8 and 39.0 mm is a matter of fact, not probability.

- It is not necessarily true that the lengths of 95% of the sampled rods lie within this interval. In this case, it cannot be true: 95% of 25 is not an integer, so simply on numerical analysis this can not be the case!

- It is not generally true that there is a 95% probability that the sample mean length (an estimate of the population mean length) in a second sample would fall within this interval. In fact, if the true mean length is far from this specific confidence interval, it could be very unlikely that the next sample mean falls within the interval.

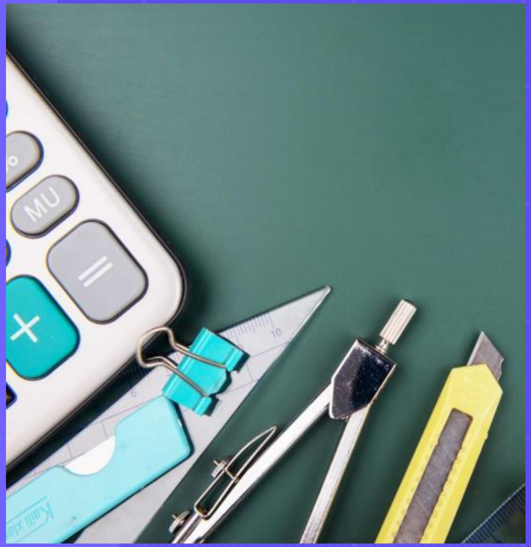


# Example of Confidence intervals

For example, suppose a factory produces metal rods, and a random sample of 25 rods gives a 95% confidence interval of 36.8 to 39.0 mm for the population mean length.

Okay so what can we say?

Instead, the 95% confidence level means that if we took 100 such samples, we would expect the true population mean to lie within approximately 95 of the calculated intervals.



# Calculation of Confidence intervals

A confidence interval is calculated by using

- The proportion calculated from the sample
- A confidence level (often, 90%, 95% or 99%)
- Creating an interval known as the margin of error

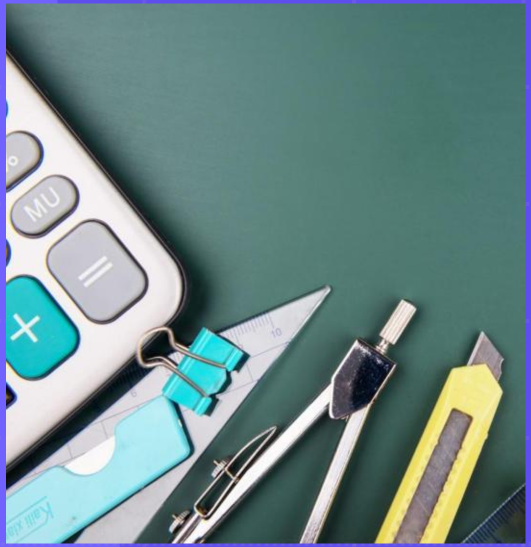
The confidence level is calculated by using a standard normal distribution

$(\mu = 0, \sigma = 1)$

| Confidence interval | 90% | 95% | 99% |
|---|---|---|---|
| $z$ score | 1.645 | 1.960 | 2.576 |

The margin of error is calculated by

$$E = z\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$



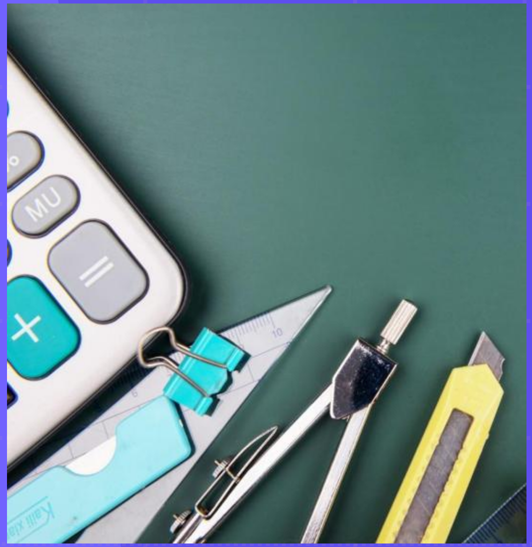
# Calculation of Confidence intervals

So, the confidence interval is calculated by

Upper $\hat{p} + z\sqrt{\left(\frac{\hat{p}(1-\hat{p})}{n}\right)}$

Lower $\hat{p} - z\sqrt{\left(\frac{\hat{p}(1-\hat{p})}{n}\right)}$

Or $\hat{p} - E \leq P \leq \hat{p} + E$



# Calculation of sample size

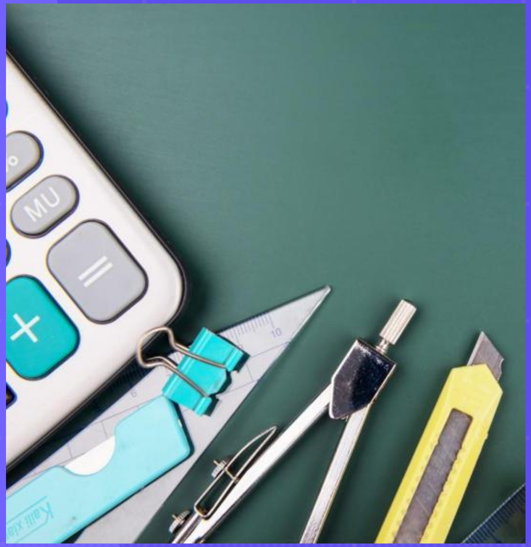
Not on the syllabus but frequently appearing is the calculation of the size of the sample needed to achieve a certain confidence.

$$n = \left(\frac{z}{E}\right)^2 \hat{p}(1 - \hat{p})$$

Always round $n$ up to the nearest whole number to ensure you have at least your chosen confidence interval.

As z increases, n will increase.

As p, approaches the extremes (closer to 0 or 1), n will increase



# Calculation of sample size

Looking into this formula

$$n = \left(\frac{z}{E}\right)^2 \hat{p}(1-\hat{p})$$

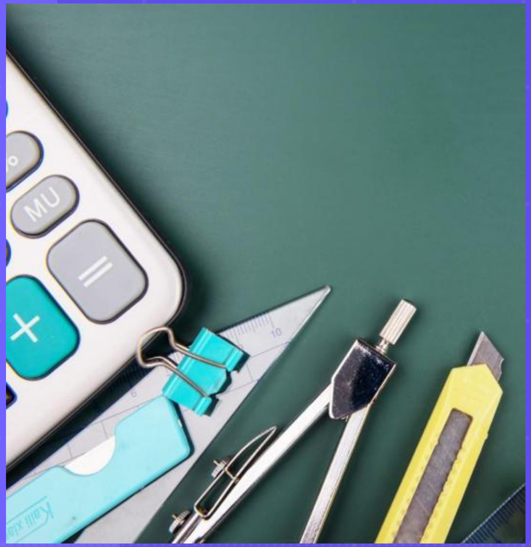
$$\text{RHS} = \left(\frac{z}{z\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}}\right)^2 \hat{p}(1-\hat{p})$$

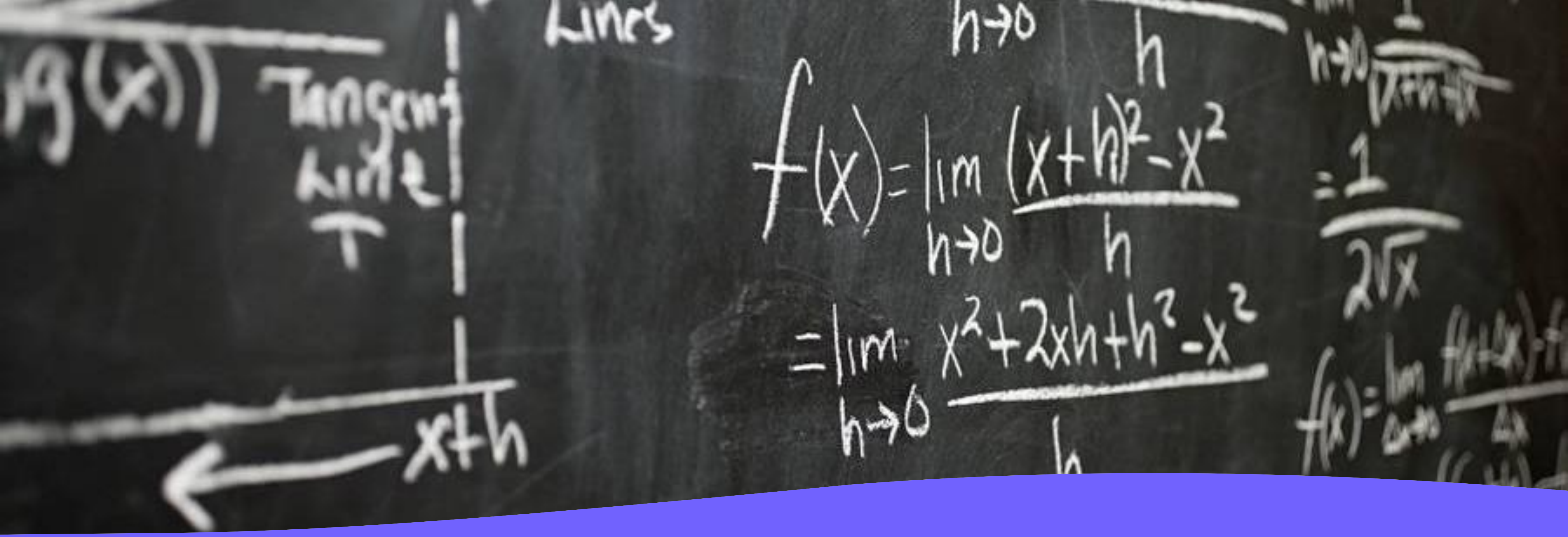
$$= \left(\frac{1}{\frac{\hat{p}(1-\hat{p})}{n}}\right)\hat{p}(1-\hat{p})$$

$$= \left(\frac{n}{\hat{p}(1-\hat{p})}\right)\hat{p}(1-\hat{p})$$

$$= n$$

$$= LHS$$




Lines
Tangent line
T
x+h
$f(x) = \lim_{h \to 0} \frac{(x+h)^2 - x^2}{h}$
$= \lim_{h \to 0} \frac{x^2+2xh+h^2-x^2}{h}$
$= \frac{1}{2\sqrt{x}}$

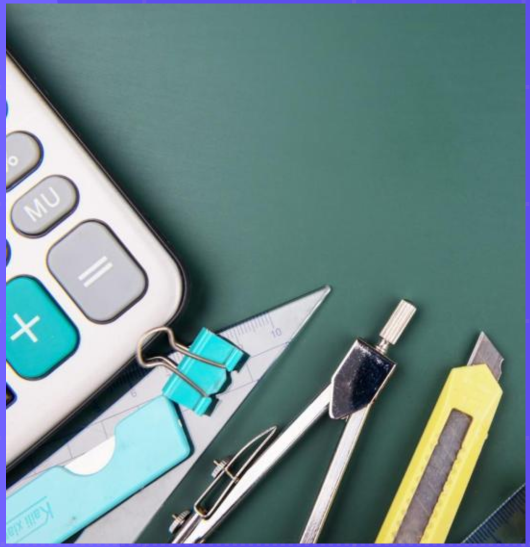
# Questions?

# WACE 2022 Question 12 (C)

The Larje Machine Co manufactures metal rods for large industrial equipment. Their standard manufacturing process produces rods whose lengths are normally distributed with a mean of 400 cm, and a standard deviation of 5 cm. A rod is considered 'useable' if its length is between 395 cm and 405 cm.

Let $X$ be a random variable denoting the length of a rod manufactured by the Larje Machine Co.

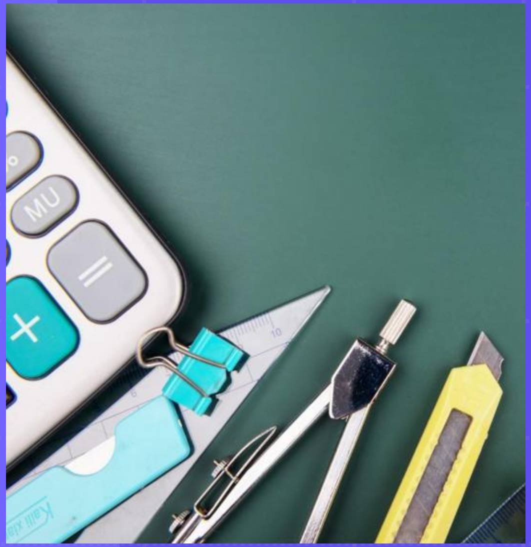
(a) Determine the probability that a rod manufactured by the Larje Machine Co is useable. Round your answer to three decimal places. (3 marks)



# WACE 2022 Question 12 (C)

Recently the Larje Machine Co introduced a new manufacturing process that industry experts claim will improve the percentage of useable rods produced to 80%. The quality control department decides to investigate whether this standard is being achieved and plan to collect a random sample of rods manufactured using the new process.

(b) What condition must the sample satisfy in order to use a normal distribution to model the sample proportion of useable rods? (1 mark)



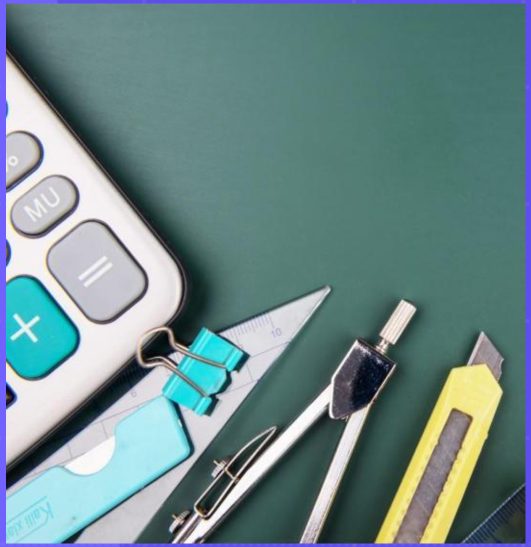
# WACE 2022 Question 12 (C)

The quality control department collects a sample of 100 rods.

(c) What is the approximate distribution of the sample proportion of useable rods? (2 marks)

Upon measuring the sample of 100 rods, it is found that 75 are useable.

(d) Calculate a 95% confidence interval for the population proportion of useable rods.
(3 marks)

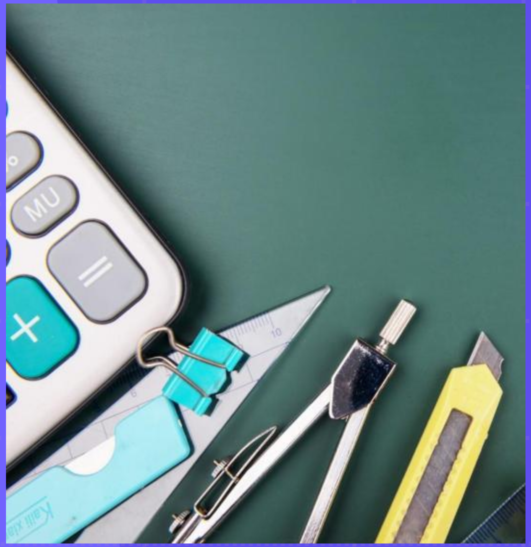


# WACE 2022 Question 12 (C)

(e) The quality control department would like to obtain a confidence interval with a smaller margin of error. State **two** methods that it could use to achieve this. (2 marks)

(f) The quality control department decides to select a new sample for which the maximum possible margin of error for a 95% confidence interval is 0.05. What sample size will achieve this requirement? (3 marks)

(g) The new sample yields the 95% confidence interval $(0.717, 0.803)$. On the basis of this sample, is the proportion of useable rods different from what was claimed by the industry experts? Justify your answer. (2 marks)



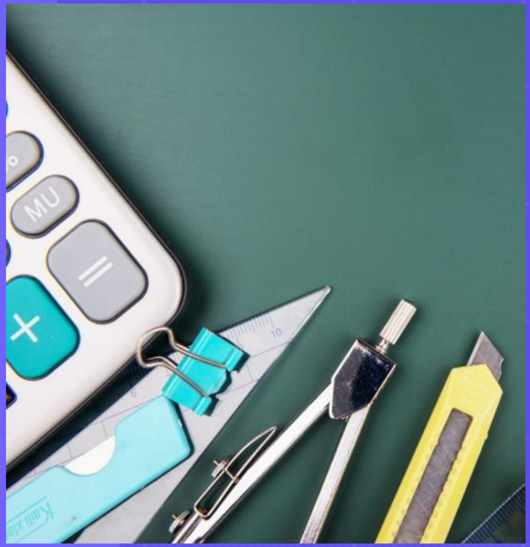
# WACE 2023 Question 7 (C)

**Question 7** **(9 marks)**

The Carnaby's Black Cockatoo is a bird native to southwest Australia. A birdwatcher is interested in estimating the proportion, $p$, of birds living in a Western Australian national park that are Carnaby's Black Cockatoos. The birdwatcher visited the national park one morning and, standing at their favourite bird-watching location, observed 200 birds flying past. Seventy-six of those birds were Carnaby's Black Cockatoos.

(a) On the basis of the sample, determine a point estimate for $p$. (1 mark)

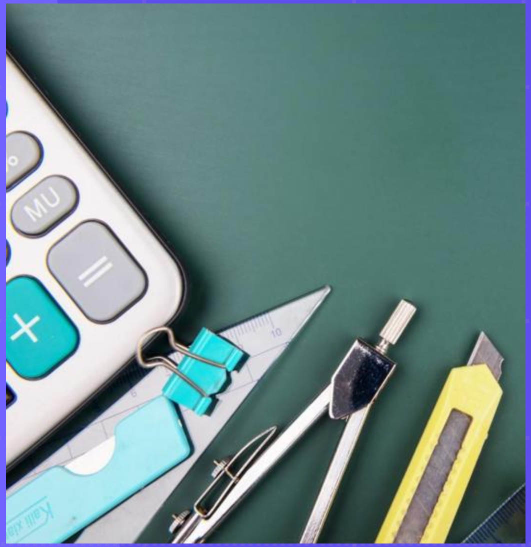
(b) On the basis of the sample, determine a 95% confidence interval for $p$. (2 marks)



# WACE 2023 Question 7 (C)

(c) What is the minimum number of birds that would need to be sampled to ensure that the margin of error of the 95% confidence interval for $p$ is at most 0.02? (2 marks)

(d) Identify and explain **two** sources of bias in the birdwatcher's sampling method. (4 marks)



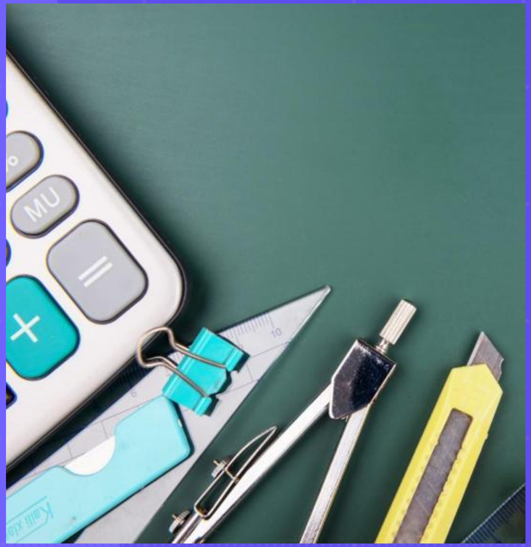
# Personal Notes and Formula Sheet

Through out the presentation I have included snips of the Formula Sheet you will have access to in the exam (will be provided).

On your personal notes, do not waste space and time rewriting the Formula Sheet!

Should further equations, information, etc be needed it will be provided in the question

*Note:* *Any additional formulas identified by the examination panel as necessary will be included in the body of the particular question.*



# Formula Sheet

The following parts of your formula sheet have not appeared in this presentation.

Mensuration would be helpful in optimisation question.

Probability will pop up occasions, especially conditional probability.

The trigonometric identities perhaps if you need to rearrange to integrate????

**Mensuration**

| | | |
|---|---|---|
| Parallelogram | $A = bh$ | |
| Triangle | $A = \frac{1}{2}bh$ or $A = \frac{1}{2}ab\sin C$ | |
| Trapezium | $A = \frac{1}{2}(a+b)h$ | |
| Circle | $A = \pi r^2$ and $C = 2\pi r = \pi d$ | |

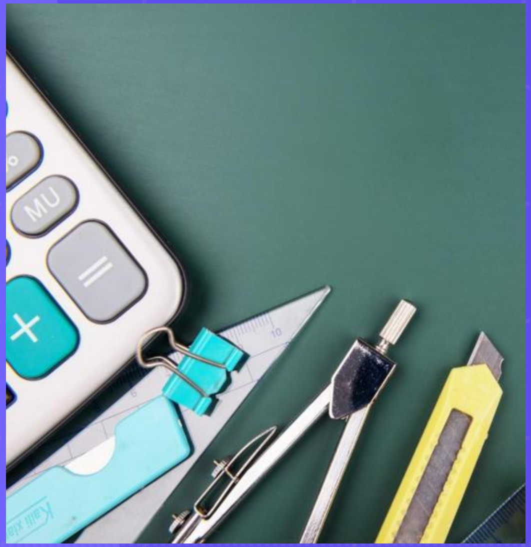
| | | |
|---|---|---|
| Prism | $V = Ah$, where $A$ is the area of the cross section | |
| Pyramid | $V = \frac{1}{3}Ah$, where $A$ is the area of the base | |
| Cylinder | $V = \pi r^2 h$ | $TSA = 2\pi rh + 2\pi r^2$ |
| Cone | $V = \frac{1}{3}\pi r^2 h$ | $TSA = \pi rs + \pi r^2$, where $s$ is the slant height |
| Sphere | $V = \frac{4}{3}\pi r^3$ | $TSA = 4\pi r^2$ |

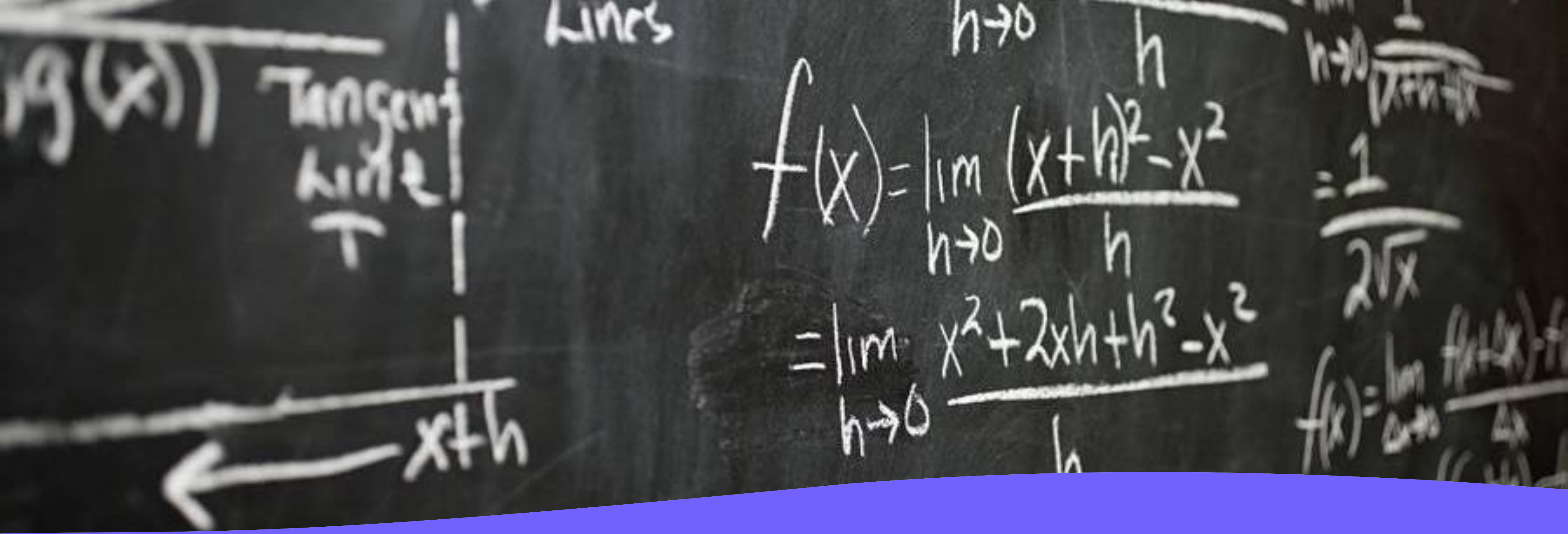
**Probability**

| | |
|---|---|
| For any event $A$ and its complement $A'$ | $P(A') = 1 - P(A)$ |
| $P(A \cup B) = P(A) + P(B) - P(A \cap B)$ | $P(A \mid B) = \frac{P(A \cap B)}{P(B)}$ |

**Trigonometry**

| | |
|---|---|
| $\sin^2 x + \cos^2 x = 1$ | $\tan x = \frac{\sin x}{\cos x}$ |






# Time to take a break!

10 minutes to stretch your legs, have a chat, use the restrooms!

# WACE Exam – the details

Game time is…..

Monday 2nd November – 1 month away

| | | | | |
|---|---|---|---|---|
| | Monday | 2 November | AST: Agricultural Science and Technology<br>*MAM: Mathematics Methods | AIT: A<br>*CSL: |

Start time is 9:20am, be there for 8:30am

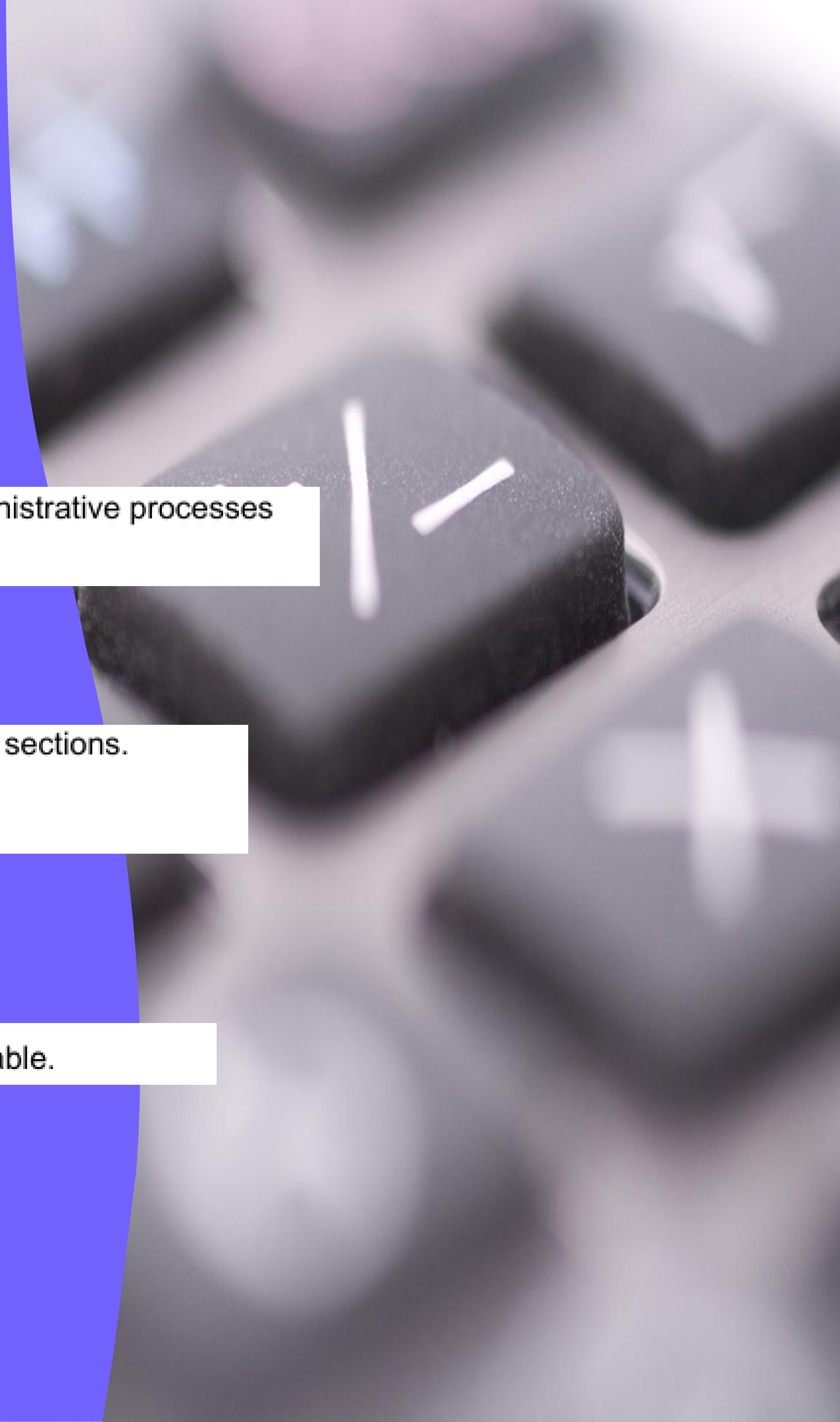
If you have an exam at a venue which isn't your school, make sure you know where it is!

# WACE Exam – the details

- Candidates must arrive 30 minutes prior to the examination start time. This will allow time for seating, administrative processes and the reading of instructions.

- * Mathematics Applications, Mathematics Methods and Mathematics Specialist examinations consist of two sections.
  - Section one: Calculator-free has 5 minutes reading time and 50 minutes working time.
  - Section two: Calculator-assumed has 10 minutes reading time and 100 minutes working time.

- No allowance can be made for candidates who miss the examination session through misreading the timetable.



# WACE Exam – Handwriting!

- Write neatly and legibly!
- Make sure your fours (4) look like a 4, sevens (7) are also pretty bad!
- This isn't English or Lit, there is no reward for writing more….

# WACE Exam – Hints and tips

Recognise the go no where question!

You read the question,

You try something,

You try again,

You stare at it……..

5 minutes disappears!!!! Or maybe 10 minutes gone!!!! And maybe 1 mark gained…

# WACE Exam – Hints and tips

**Instead**

## 🛑 STOP

➡️ Mark the question that you need to come back to it
➡️ Move on and get your marks elsewhere
➡️ Complete questions you can answer and build confidence
➡️ Return with fresh thinking if you have time

Why? Because 5 minutes spent stuck = marks lost elsewhere.

# WACE Exam – Hints and tips

- 1 mark ≈ 1 minute
- Stuck? Move on
- Spending some time on a question? Getting somewhere, stick with it; not getting anywhere, move on
- 1 mark is for one key skill shown, it isn't for how many lines of working you do
- Some answers can be two or three lines for 5 marks! But sometimes you'll need multiple lines to get 1 or 2 marks.

# WACE Exam – Hints and tips

Reading time

- Decide the 'easy', 'medium', 'hard' questions
- Spot your timewasters – "I have no idea!"
- Plan your approach
- Use your formula sheet/notes to plan some questions
- The wordier questions, have a read/reread to ensure you're understanding them correctly.

# WACE Exam – Hints and tips

Working time

- Write neatly!!!!
- Don't start writing half way across the page!
- One mark is one skill, 10 lines isn't 10 marks
- Show your logical thought through your working
- Make your final answers clear
- If you mess a question up pretty bad, ask for another booklet!

# WACE Exam – Hints and tips

Classpad skills

- Don't write "I did it on my Classpad"!!
- Show the substitution into the equation → then write the answer from Classpad ☺
- If your Classpad freezes/jams up/gives up on a question, you did something wrong in the question!
- If you haven't changed your batteries…. Do it now!

# WACE Exam – big questions?

What can I do between now and the exam to make my marks more predictable?

What can I do between now and the exam to push my marks higher?

What can I do between now and the exam to build my understanding?

# WACE Exam – Think about?

What part of the exam worries you most?

What do you already feel confident about?

What has helped from today's session?

Maybe another question you have?

# WACE Exam – Know yourself

What are your strengths?

Examples:

- Content knowledge
- Visualisation of question
- Mental maths calculations
- Classpad use
- Time management
- Reading and deciphering questions
- Remembering and applying terminology

# WACE Exam – Know yourself

What are your weaknesses?

Examples:

- Panic (why? What are your triggers? What can you do before the exam to minimise?)
- Running out of time – spending too long on some questions?
- Misreading questions –understanding? Maybe rushing? Lack of content knowledge?

# WACE Exam – Know yourself

What are your weaknesses?

Examples:

- Not showing working!!!! This is often a big one for some people
- Writing too much – yes! for some
- Writing too little –yes! For some
- Getting stuck (and not moving on?)

# WACE Exam – Know yourself

Before the exam, I will focus on improving ….

In exams my biggest mistake to avoid is….

One strategy I need to use more in the exam is…

Make your responses specific, measurable and implementable.
"Study more" isn't a good response.

# WACE Exam – the markers?

## Put yourself in the place of the marker

- Is marking papers easy… well mostly yes, but not always
- Following the thought pattern of students is the hardest part
- Big jumps in thinking, do they really know what they're doing? Or is this rote learnt?
- When a marker is lacking confidence in a student, marks are unlikely
- We can only mark what we see, not what you're thinking
- Neatness and logical thought process are always the winner!

# WACE Exam – Study/revision

Don't rely only on:

📖 Re-reading textbook chapters

📖 Looking over notes/examples

🖍️ Highlighting/underlining etc

👀 Watching videos

# WACE Exam – Study/revision

What does effective revision look like then for Mathematics?

To revise Mathematics, we need to triangulate three key aspects

1. Building content knowledge – syllabus, book, knowledge
2. Understand what the questions are asking and testing
3. Be able to complete the mathematics to show our content knowledge

# WACE Exam – Study/revision

Okay, great how do we do that then?

# WACE Exam – Study/revision

1. Building content knowledge – syllabus, book, knowledge

For each topic (6 in our course), ask:

**WHAT?** What do I need to revise? Check syllabus!

**HOW?** What method will I use? More questions, more understanding, more what?

**WHEN?** When will I do it?

**HOW LONG?** How much time will I realistically spend?

# WACE Exam – Study/revision

## 2. Understand what the questions are asking and testing

To do well here, we need:

- To know our syllabus
- To recognise the types of questions that come up
- Be able to show the required skills that get rewarded with marks

# WACE Exam – Study/revision

3. Be able to complete the mathematics to show our content knowledge

If the first two are being done well, then the third step should naturally follow but we must pay attention to

- Computation errors
- Paying attention to units, scales, sizes, orders of magnitude
- Showing what you are doing and not jumping steps

# WACE Exam – Study/revision

Some other points you need to consider in your revision are

- ☕ Brain breaks
- 🏃 Movement
- 📉 Downtime
- 💧 Water
- 😴 Sleep
- 📵 Focused study time
- 🏢 Work/sport/family etc

# WACE Exam – Study/revision

**REVIEW IT**

Refresh the knowledge (syllabus, text book, notes, videos, past questions etc)

**RETRIEVE IT**

Close the notes and recall it (think through the content, the skills, the questions)

**APPLY IT**

Complete a question (find a past WACE question from the topic you’re revising)

**MARK IT**

Identify what earned/lost you marks.

# WACE Exam – Study/revision

**FIX IT**

**Learn from the mistakes**

**REVISE IT**

Relook at the aspect(s) that weren't strong  for you

**RETRY IT**

Flag the question to attempt it again later

# WACE Examiner’s Reports

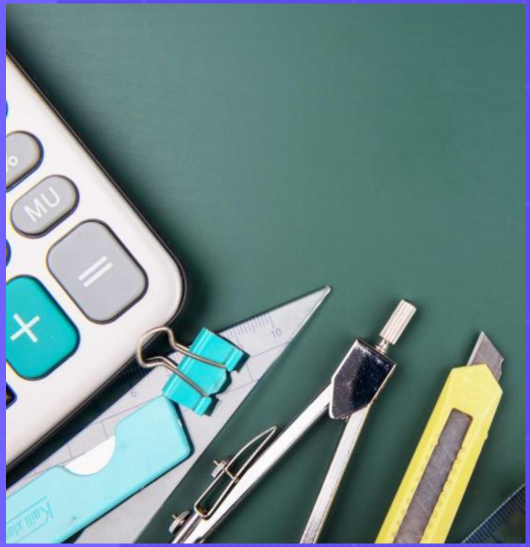
These are a treasure trove of information!  You must consult them and learn from other’s mistakes!

There are two versions – public and ‘for teachers’

2025 Examination report (319.9 KB) Last updated: 19 Mar 2026 2:28pm

2025 Summary examination report for candidates (254.6 KB) Last updated: 19 Mar 2026 2:28pm

You must read the full version, the summary is somewhat useless!

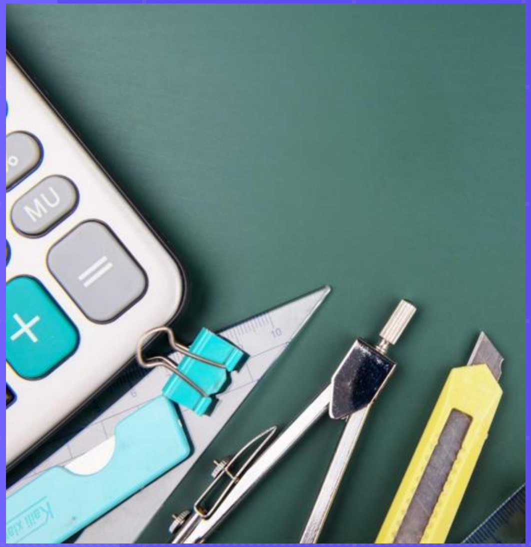


# WACE Examiner's Reports

***General comments***

Candidates performed well on most examination questions. Specific areas of content where candidates experienced difficulty were questions involving logarithms (Questions 7 and 17), rectilinear motion (Question 6) and trigonometric functions (Question 14). Although candidates answered routine questions in calculating confidence intervals well, interpretating these intervals against claims was performed less successfully.

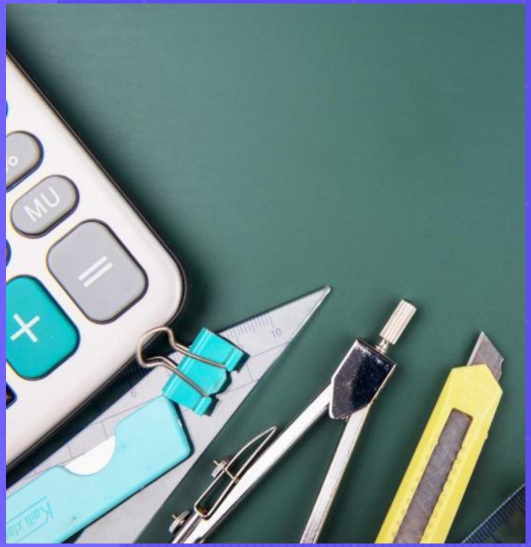
Candidates underperformed in questions requiring written explanations, with answers often lacking clarity, appropriate terminology and/or precision. Many candidates presented working in a disorganised manner, which in some cases contributed to errors in solutions. In questions requiring candidates to draw or use graphs, answers did not always follow instructions or accurately place key features. In questions requiring candidates to show a result, answers often did not include sufficient intermediate steps to fully justify the final answer.



# WACE Examiner’s Reports

*Advice for candidates*

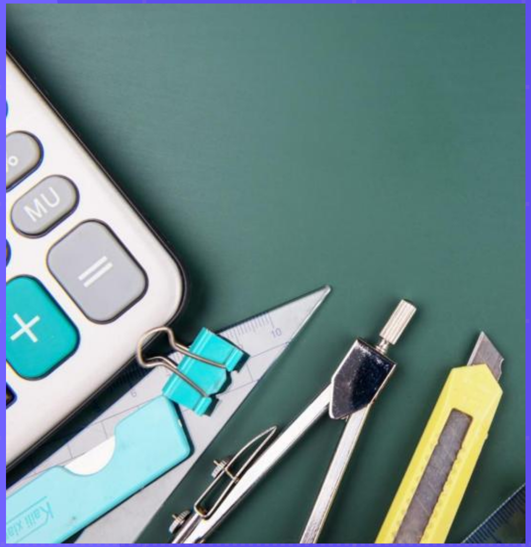
- Write clearly and legibly, and use pens that do not smudge, to ensure answers are easy to read.
- Communicate answers clearly by providing concise and mathematically accurate responses. Use the number of marks allocated as a guide to the depth required.
- Use appropriate mathematical terms, notation and definitions from the syllabus.
- Develop a clear understanding of when to use a CAS calculator and when to show detailed working (for example, a routine question versus a ‘show’ question), and balance efficient calculator use with sufficient steps to support your solutions.
- Read each question carefully, ensuring your answer directly addresses what is being asked and checking its reasonableness, such as probabilities lying between 0 and 1 or providing an integer value when required.
- Practise and strengthen basic fraction and numerical skills, particularly for the Calculator-free section.
- Include appropriate units when answering questions set in a contextual or applied setting.
- Follow instructions carefully when asked to use a graph and provide evidence of this process rather than relying on calculator-based methods.



# WACE Examiner's Reports

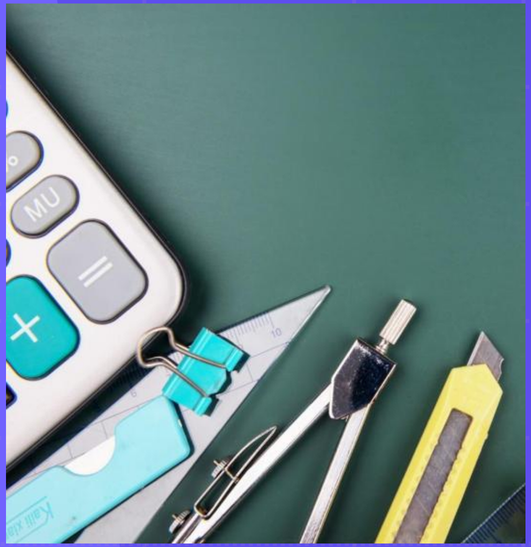
*Advice for teachers*

- Develop students' understanding of syllabus points related to logarithms.
- Develop students' understanding of interpreting confidence intervals in the context of statistical significance, as distinct from proving a result.
- Strengthen students' understanding of the relationship between rectilinear motion quantities and their interpretation.
- Remind students to remain in radian mode when performing differentiation or integration of trigonometric functions.
- Provide opportunities for students to develop clarity and precision in written answers through practice and feedback.
- Develop students' ability to use their CAS calculator efficiently while documenting key steps of their solution process.
- Encourage students to write correct mathematical notation rather than calculator language/syntax.



# WACE Examiner's Reports

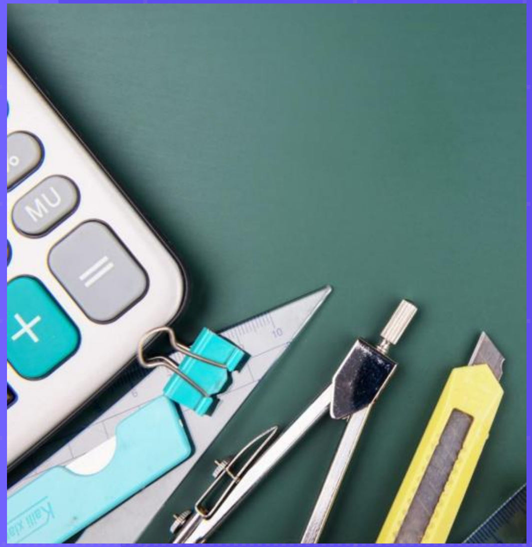
2025-ATAR-course-examination-report-MAM.PDF



# 2026 WACE exam front covers

2026-MAM-Examination-Calculator-Free-Front-Cover.PDF

2026-MAM-Examination-Calculator-Assumed-Front-Cover.PDF



# WACE Questions

Now let’s use what we’re revised and learnt to hit some WACE questions

Ahead are 6 questions, one from each topic

# WACE 2024 Question 2 (U3T1)

**Question 2** **(10 marks)**

An advertising graphic moves across the bottom of a television screen during a sporting game, changing direction to attract viewer attention. The position of the graphic is modelled by

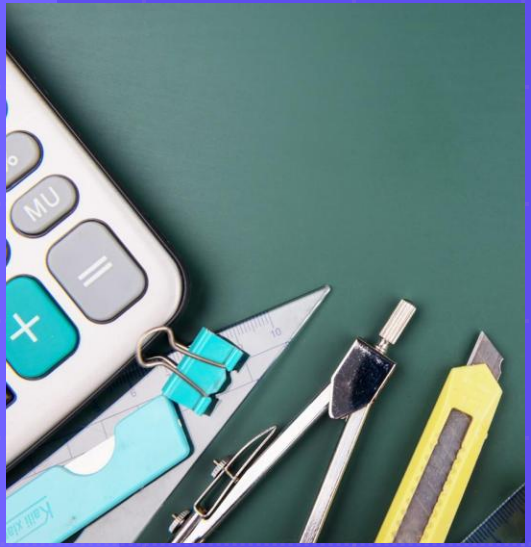
$$x(t) = \frac{1}{3}t^3 - 7t^2 + 40t$$

where $x$, in centimetres, is the position of the graphic relative to the left side of the screen, and $t$, in seconds, is the time from when the graphic first appears on the screen.

The position of the graphic at integer time increments is given in the table below.

| $t$ | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|---|
| $x(t)$ | 0 | $33\frac{1}{3}$ | $54\frac{2}{3}$ | 66 | $69\frac{1}{3}$ | $66\frac{2}{3}$ | 60 | $51\frac{1}{3}$ |

| $t$ | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 |
|---|---|---|---|---|---|---|---|---|
| $x(t)$ | $42\frac{2}{3}$ | 36 | $33\frac{1}{3}$ | $36\frac{2}{3}$ | 48 | $69\frac{1}{3}$ | $102\frac{2}{3}$ | 150 |



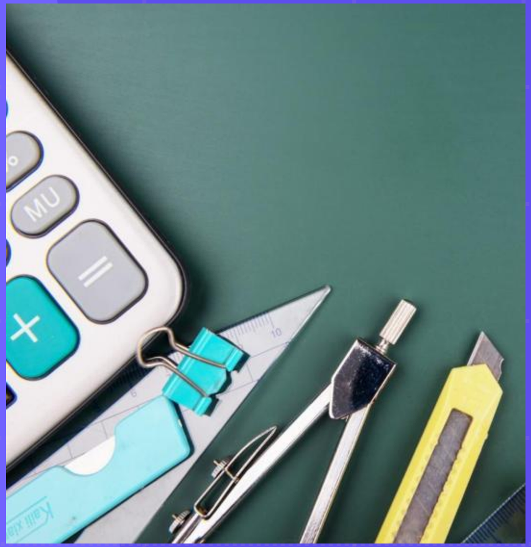
# WACE 2024 Question 2 (U3T1)

(a) Determine the velocity of the graphic when it first appears on the screen. (2 marks)

(b) Is the graphic initially speeding up or slowing down? Justify your answer. (2 marks)

(c) Evaluate $\int_3^9 v(t)\,dt$ and explain what this integral represents. (3 marks)

(d) Calculate the total distance travelled by the graphic from the time it enters the screen to the time it leaves the screen 15 seconds later. (3 marks)



# WACE 2022 Question 11 (U3T2)

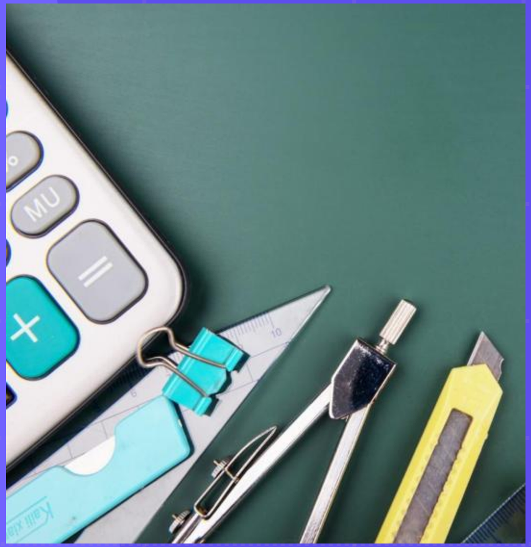
The 100 m sprint is a race run on a straight section of track. During a race the velocity, $v$, measured in metres per second, of an athlete is given by

$$v(t) = -10e^{-0.8t} - 0.05e^{0.2t} + 10.05$$

where $t$ is the time, in seconds, measured from the moment the athlete starts to move from the start line.

(a) Determine the acceleration of the athlete three seconds after moving from the start line. (2 marks)

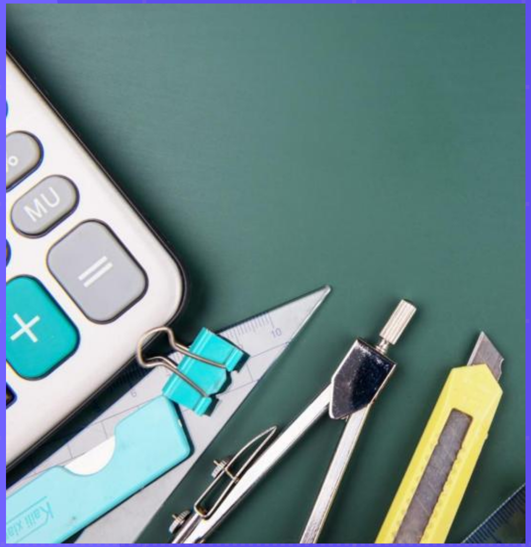
(b) Using calculus, determine the maximum velocity of the athlete during the race, and the time, $t$, at which it is achieved. (4 marks)



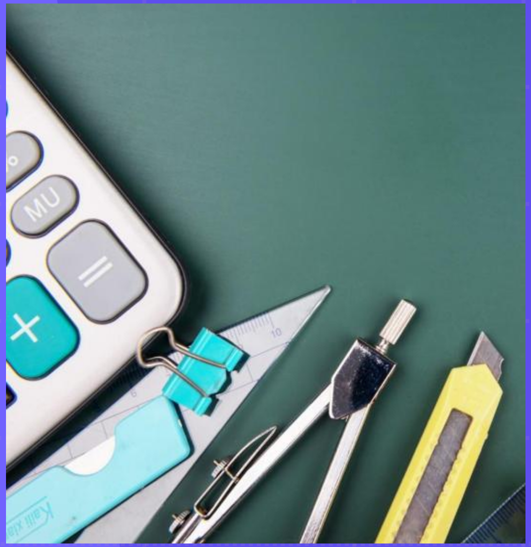
# WACE 2022 Question 11 (U3T2)

(c) The displacement, $x$, of the athlete is 0 m at the start of the race. Determine an expression for the displacement of the athlete during the race. (3 marks)

(d) Determine the time, $t$, at which the athlete finishes the 100 m race. (2 marks)



# WACE 2022 Question 3 (U3T3)



# WACE 2022 Question 3 (U3T3)

**Question 3** **(6 marks)**

The tables below display the partially completed probability distribution and cumulative distribution for a discrete random variable $X$.

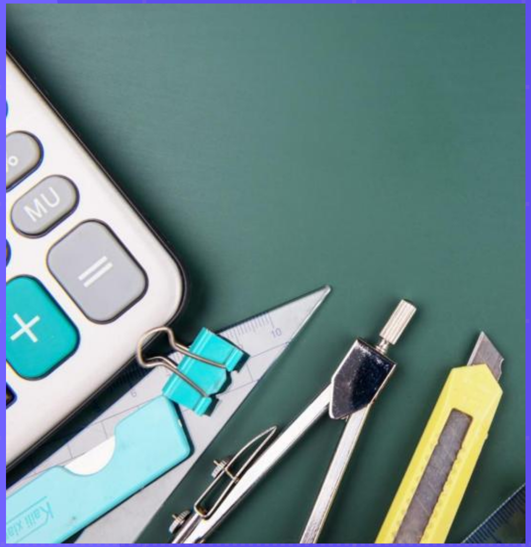
| $x$ | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| $\mathrm{P}(X = x)$ | 0.2 | 0.15 | | | 0.05 |

| $x$ | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| $\mathrm{P}(X \le x)$ | 0.2 | | 0.6 | 0.95 | |

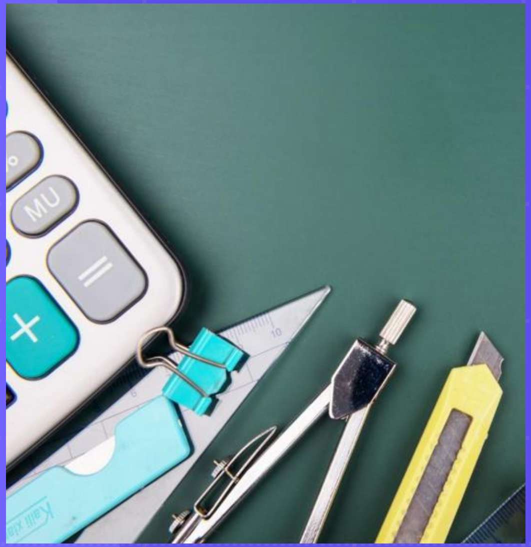
(a) Complete the missing probability entries in each of the tables above. (2 marks)

(b) Evaluate $\mathrm{P}(2 \le X \le 4)$. (2 marks)

(c) Evaluate $\mathrm{P}(X = 1 \mid X \le 3)$. (2 marks)



# WACE 2022 Question 3 (U3T3)



# WACE 2023 Question 1 (U4T1)

**Question 2** **(14 marks)**

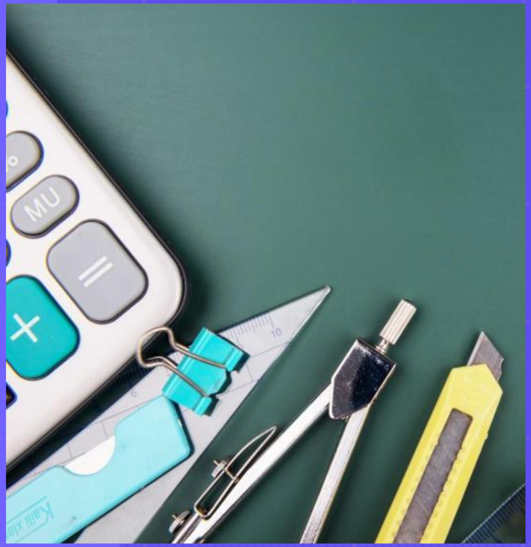
Let $p = \ln(2)$, $q = \ln(3)$ and $r = \ln(5)$.

(a) Express each of the following in terms of $p$, $q$ and/or $r$.

(i) $\ln(6)$ (2 marks)

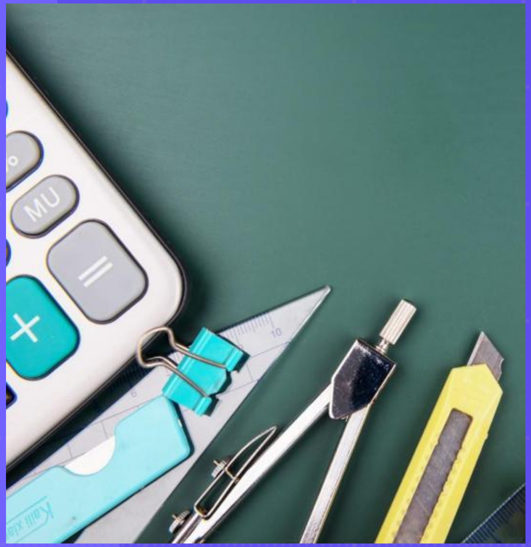
(ii) $\ln(6.25)$ (3 marks)

(iii) $\int_2^3 \frac{d}{dx} \ln(x)\, dx$ (2 marks)



# WACE 2023 Question 1 (U4T1)

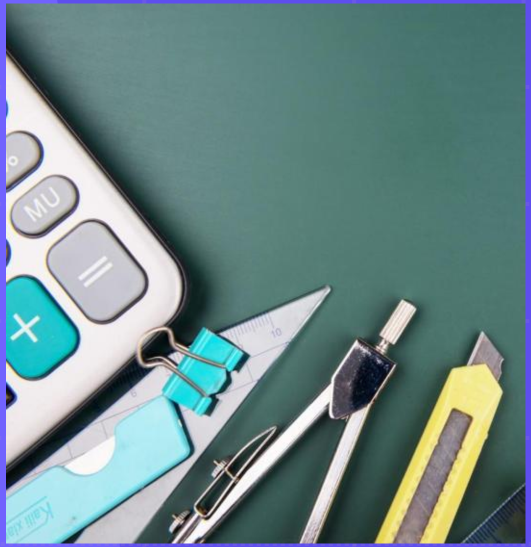
(b) Evaluate $e^{p+q}$. (2 marks)



# WACE 2023 Question 1 (U4T1)

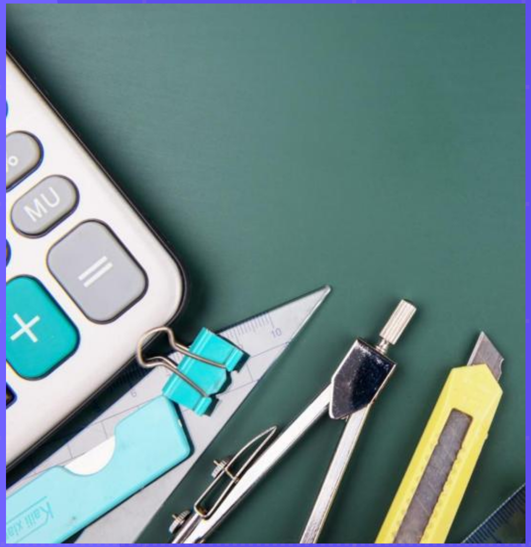
(c) (i) Determine $\frac{d}{dx}(x\ln(x))$. (1 mark)

(ii) Hence show that $\int \ln(x)\,dx = x\ln(x) - x + c$ where $c$ is a constant. (2 marks)



# WACE 2023 Question 1 (U4T1)

(iii) Evaluate $\int_1^3 \ln(x)\,dx$ in terms of $p, q$ and/or $r$. (2 marks)

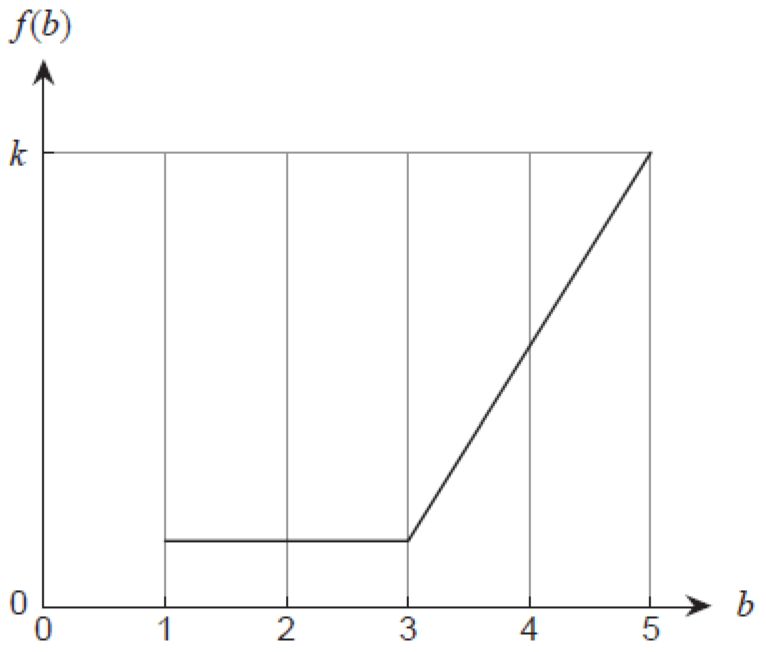


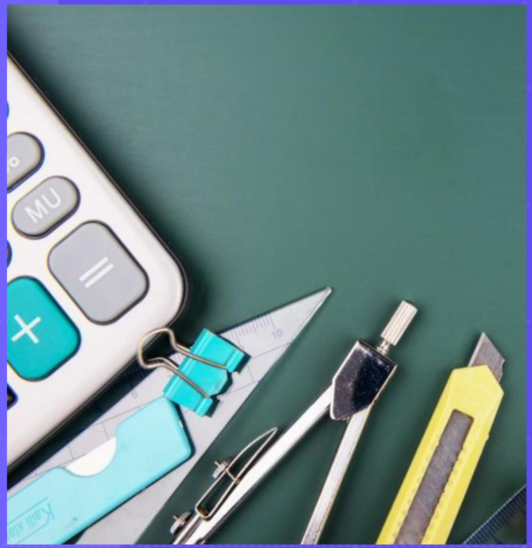
# WACE 2023 Question 11 (U4T2)

**Question 11** **(13 marks)**

Mrs Euler is having her car serviced at BIMDAS Mechanics. She drops her vehicle off at 8 am and is told that her car will be ready for collection at some time between 1 pm and 5 pm that day.

Let the random variable $B$ denote the time after noon (12 pm) at which a vehicle is ready for collection at BIMDAS Mechanics. The probability density function for $B$ is shown in the graph below.



# WACE 2023 Question 11 (U4T2)

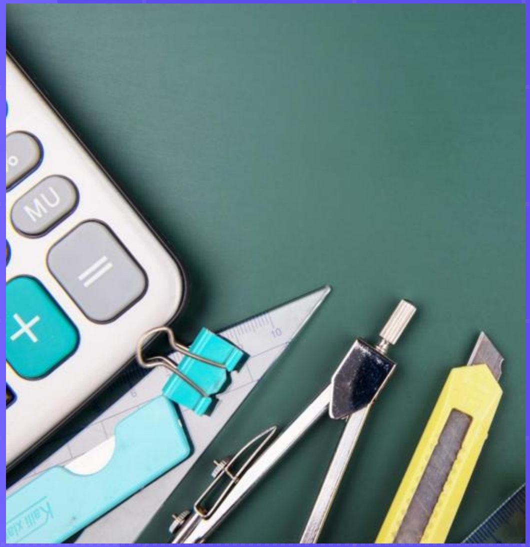
The probability of a vehicle being ready for collection between 2 pm and 3 pm is 0.1.

(a) Determine the value of $k$. (2 marks)

(b) An incomplete expression for the probability density function of $B$ is given below. Fill in the boxes to complete the missing parts of the expression. (2 marks)

$$f(b) = \begin{cases} 0.1, & \boxed{\qquad\qquad} \\ \boxed{\qquad\qquad}, & 3 \le b \le 5 \\ 0, & \text{otherwise} \end{cases}$$

(c) Determine the expected time that Mrs Euler's vehicle will be ready for collection at BIMDAS Mechanics. (3 marks)

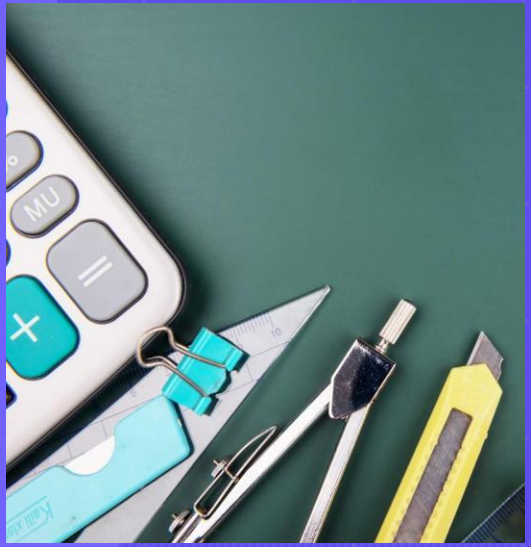


# WACE 2023 Question 11 (U4T2)

Mr Euler is also having his car serviced, but by Addition Autos. He drops his vehicle off at 8 am and is told that his car will be ready for collection at some time between 1 pm and 5 pm that day.

Let the random variable $A$ denote the time after noon (12 pm) that a vehicle is ready for collection at Addition Autos. The cumulative distribution function for $A$ is given by

$$P(A \leq a) = \begin{cases} 0, & a < 1 \\ \dfrac{10a - a^2 - 9}{16}, & 1 \leq a \leq 5 \\ 1, & a > 5 \end{cases}$$



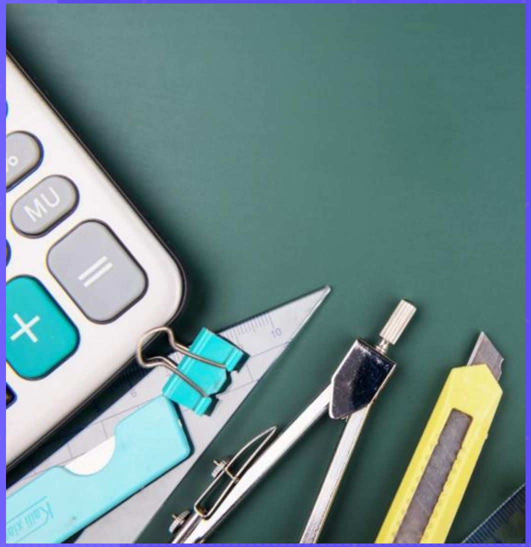
# WACE 2023 Question 11 (U4T2)

(d) Determine the probability that Mr Euler's vehicle will be ready to collect

(i) by 3 pm. (1 mark)

(ii) between 3 pm and 4 pm. (2 marks)

(e) Determine the expected time at which Mr Euler's vehicle will be ready for collection at Addition Autos. (3 marks)



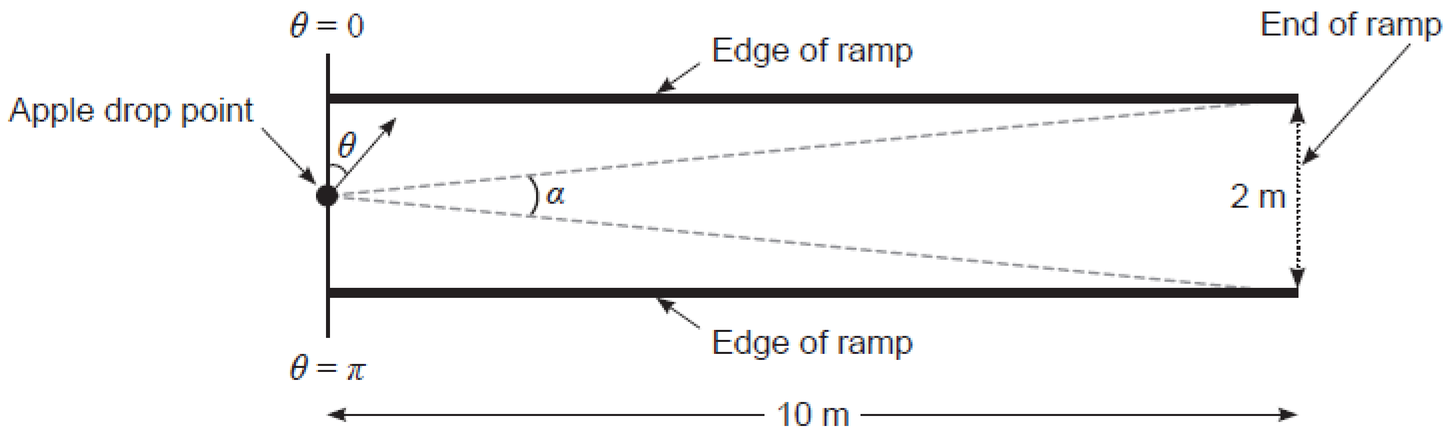
# WACE 2023 Question 13 (U4T3)

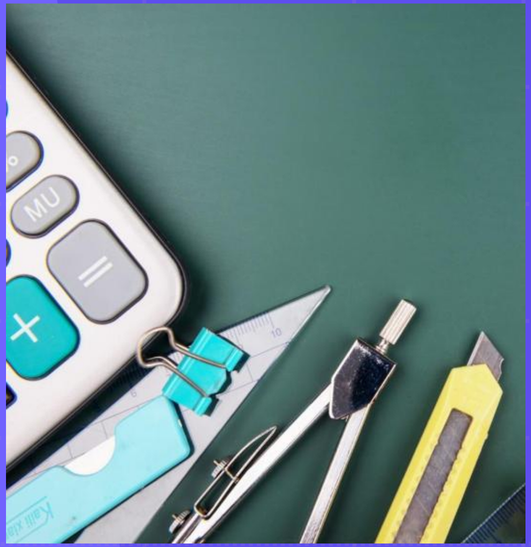
**Question 13** **(9 marks)**

While leaving a shopping centre a mathematician accidentally drops a bag of apples at the top of a ramp of length 10 m and width 2 m. The diagram below shows the top view of the ramp. Four of the apples roll safely to the end of the ramp, while six roll off an edge and splatter on the ground below.

The mathematician decides to create a simple model by assuming that the:

- apples roll independently of one another along straight lines from the apple drop point
- direction each apple rolls, $\theta$, is an angle measured about the apple drop point and is uniformly distributed over $0 \leq \theta \leq \pi$.



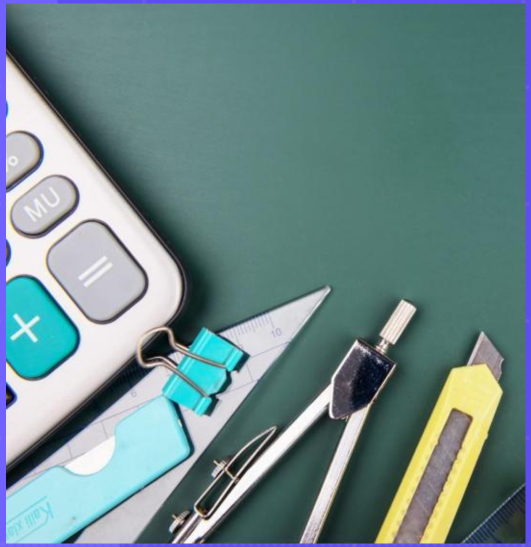
# WACE 2023 Question 13 (U4T3)

Apples that roll along a line within the sector marked by $\alpha$ will arrive safely at the end of the ramp, while others will roll off the edge.

(a) (i) Determine the value of $\alpha$. (2 marks)

(ii) Hence show that the probability, $p$, of an apple rolling safely to the end of the ramp is $p = 0.063$ (rounded to three decimal places). (1 mark)

(b) Determine the probability that, of the 10 apples, four or more make it safely to the end of the ramp. (2 marks)

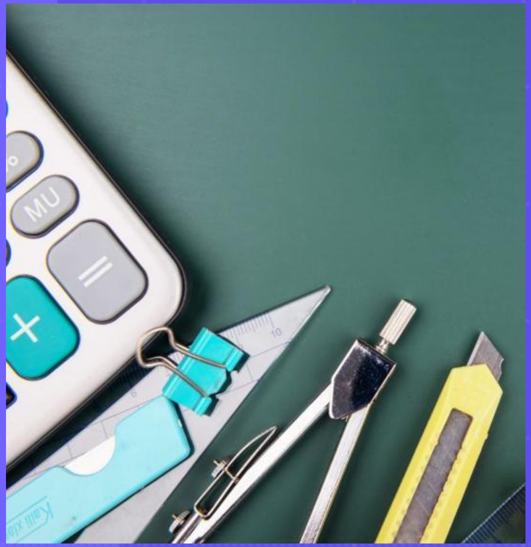


# WACE 2023 Question 13 (U4T3)

The mathematician decides to purchase another 20 bags of apples, i.e. 200 apples, return to the top of the ramp, and break each bag open one at a time. After the experiment a total of 63 apples have rolled safely to the end of the ramp.

(c) Using the sample of 200 apples, calculate a 99% confidence interval for the population proportion of apples that will roll safely to the end of the ramp. (2 marks)

(d) What does the confidence interval from part (c) suggest about the validity of the model assumptions used to calculate the probability in part (a)(ii)? (2 marks)



# What’s next?

Well… you’ll all have to go home… I guess? You can’t stay here!

Recognise where you are on the learning journey and be realistic about what you can achieve in the next four weeks.

# What’s next?

Revise the things you know are going to feature in the exam, ignore the dot points about ‘develop’ or ‘investigate’

Past exam questions are one of the best vehicles to become more familiar with what is required and what is likely to reappear this year!

# Finally.....

Thank you

For coming along
For being present here today
For furthering your mathematics
For challenging yourself to choose the harder maths

# The way to get started is to quit talking and begin doing.

Walt Disney

# Don’t aim for perfection, aim for progress.

Unknown

Go forth and be wonderful people, the world needs you now more than ever before!

Chris Simpson '26